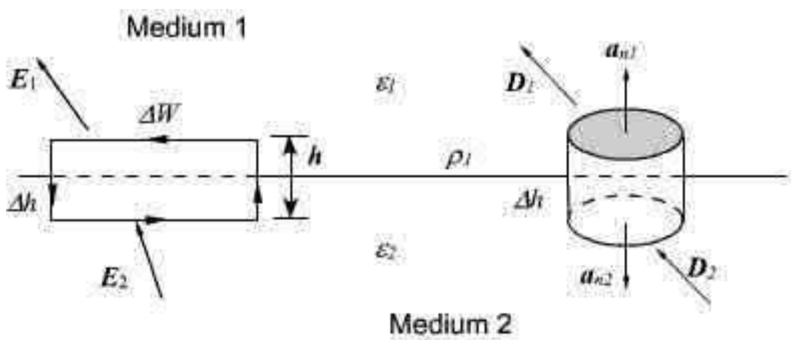
Boundary Conditions

Let us consider the relationship among the field components that exist at the interface betweentwo dielectrics as shown in the figure 2.17. The

permittivity of the medium 1 and medium 2 are ε_1 and ε_2 respectively and the interface mayalso have a net charge density ρ_s Coulomb/m.



Boundary Conditions at the interface between two dielectrics

We can express the electric field in terms of the tangential and normal

$$\vec{E_1} = \vec{E_{1t}} + \vec{E_{1n}}$$
$$\vec{E_2} = \vec{E_{2t}} + \vec{E_{2n}}$$

where E_t and E_n are the tangential and normal components of the electric field respectively.Let us assume that the closed path is very small so that over the elemental path length the variation of E can be neglected. Moreover very near to the

interface, $\Delta h \to 0$. Therefore

$$\oint \vec{E} d\vec{l} = E_{1t}\Delta w - E_{2t}\Delta w + \frac{h}{2}(E_{1n} + E_{2n}) - \frac{h}{2}(E_{1n} + E_{2n}) = 0$$

Thus, we have,

$E_{1t} = E_{2t}$ or $\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$ i.e. the **tangential component of an electric field is continuous acrossthe interface.**

For relating the flux density vectors on two sides of the interface we apply Gauss‟s law to asmall pillbox volume as shown in the figure. Once again as $\Delta h \rightarrow 0$, we can write

$$\oint \vec{D} \cdot d\vec{s} = (\vec{D_1} \cdot \hat{a}_{n2} + \vec{D_2} \cdot \hat{a}_{n1})\Delta s = \rho_s \Delta s$$

i.e., $D_{1n} - D_{2n} = \rho_s$

.e., $\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$

Thus we find that the **normal component of the flux density vector *D* is discontinuousacross an interface by an amount of discontinuity equal to the surface**
the interface.