POYNTING THEOREM &

POYNTING VECTOR

Pointing vector

If the medium 2 is not a perfect conductor (i.e. $\sigma_2 \neq \infty$) partial reflection will result. There will be a reflected wave in the medium 1 and a transmitted wave in the medium 2.Because of the reflected wave, standing wave is formed in medium 1.

From equation (6.49(a)) and equation (6.53) we can write

..................(6.59)

Let us consider $\vec{E}_1 = E_{io}\left(e^{-\gamma_1 z} + \Gamma e^{\gamma_1 z}\right)\hat{a}_x$ the media are dissipation less i.e.perfect dielectrics ($\sigma_1 = 0, \sigma_2 = 0$)

$$\gamma_1 = j\omega\sqrt{\mu_1\varepsilon_1} = j\beta_1 \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

$$\gamma_2 = j\omega\sqrt{\mu_2\varepsilon_2} = j\beta_2 \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \quad ....(6.60)$$

In this case both η_1 and η_2 become real numbers.

$$\begin{aligned}\vec{E}_1 &= \hat{a}_x E_{io}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)\\ &= \hat{a}_x E_{io}\left((1+T)e^{-j\beta_1 z} + \Gamma\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\right)\\ &= \hat{a}_x E_{io}\left(Te^{-j\beta_1 z} + \Gamma(2j\sin\beta_1 z)\right) \quad ....(6.61)\end{aligned}$$

From (6.61), we can see that, in medium 1 we have a traveling wave component withamplitude TE_{io} and a standing wave component with amplitude $2JE_{io}$.

The location of the maximum and the minimum of the electric and magnetic fieldcomponents in the medium 1from the interface can be found as follows.

The electric field in medium 1 can be written as

$$\vec{E}_1 = \hat{a}_x E_{io} e^{-j\beta_1 z}\left(1 + \Gamma e^{j2\beta_1 z}\right) ....(6.62)$$

If $\eta_2 > \eta_1$ i.e. $\Gamma > 0$

The maximum value of the electric field is

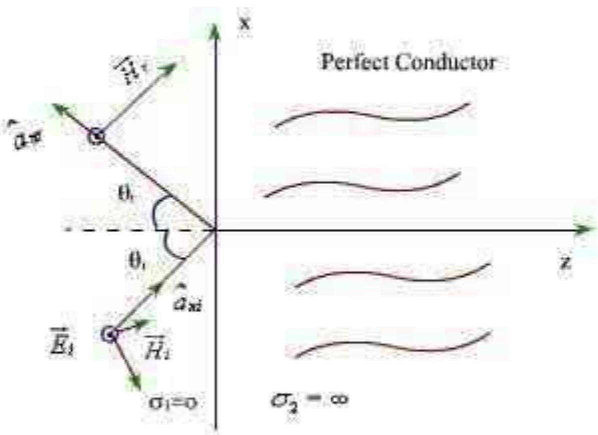
$\left|\vec{E}_1\right|_{max} = E_{io}\left(1+T\right)$ ....(6.63)

$2\beta_1 z_{max} = -2n\pi$

or $z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\pi}{2\pi/\lambda_1} = -\frac{n}{2}\lambda_1$, n = 0, 1, 2, 3........................(6.64)

The minimum value of $\left|\vec{E}_1\right|$ is

$\left|\vec{E}_1\right|_{min} = E_{io}\left(1-\Gamma\right)$ ....(6.65)



And this occurs when

$2\beta_1 z_{min} = -\left(2n+1\right)\pi$

or $z_{min} = -\left(2n+1\right)\frac{\lambda_1}{4}$, n = 0, 1, 2, 3............................ (6.66)

For $\eta_2 < \eta_1$ i.e. $\Gamma < 0$

The maximum value of $\left|\vec{E}_1\right|$ is $E_{io}(1-\Gamma)$ which occurs at the z_{min} locations and

the minimum value of $\left|\vec{E}_1\right|$ is $E_{io}(1+\Gamma)$ which occurs at z_{max} locations as given by theequations (6.64) and (6.66).

From our discussions so far we observe that $\frac{|E|_{max}}{|E|_{min}}$ can be written as

$$S = \frac{|E|_{max}}{|E|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$
.................(6.67)

The quantity *S* is called as the standing wave ratio.As $0 \le |\Gamma| \le 1$ the range of *S* is given by $1 \le S \le \infty$

From (6.62), we can write the expression for the magnetic field in medium 1 as

$$\vec{H}_1 = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1 z}\left(1-\Gamma e^{j2\beta_1 z}\right)$$
.................(6.68)

From (6.68) we find that $\left|\vec{H}_1\right|$ will be maximum at locations where $\left|\vec{E}_1\right|$ is minimum and vice versa.In medium 2, the transmitted wave propagates in the + z direction.

Pointing Theorem

So far we have discuss the case of normal incidence where electromagnetic wave traveling in a lossless medium impinges normally at the interface of a second medium. In this section we shall consider the case of oblique incidence. As before, we consider two cases

i. When the second medium is a perfect conductor.
ii. When the second medium is a perfect dielectric.

A plane incidence is defined as the plane containing the vector indicating the direction of propagation of the incident wave and normal to the interface. We study two specific cases when the incident electric field $\vec{E}_i$ is perpendicular to the plane of incidence (perpendicular polarization) and $\vec{E}_i$ is parallel to the plane of incidence (parallel polarization). For a general case, the incident wave may have arbitrary polarization butthe same can be expressed as a linear combination of these two individual cases.

Plane wave reflection

i. Perpendicular Polarization

As the EM field inside the perfect conductor is zero, the interface reflects the incident plane wave. $\hat{a}_{ni}$ and $\hat{a}_{nr}$ respectively represent the unit vector in the direction of propagation of the incident and reflected waves, θ_i is the angle of incidence and θ_r is the angle of reflection.

We find that

$$\hat{a}_{ni} = \hat{a}_z \cos\theta_i + \hat{a}_x \sin\theta_i$$
$$\hat{a}_{nr} = -\hat{a}_z \cos\theta_r + \hat{a}_x \sin\theta_r \quad \text{............................(6.69)}$$

Since the incident wave is considered to be perpendicular to the plane of incidence, which for the present case happens to be xz plane, the electric field has only y- component.

Therefore,

$$\vec{E}_i(x,z) = \hat{a}_y E_{io} e^{-j\beta_1 \vec{a}_{ni}\cdot\vec{r}}$$
$$= \hat{a}_y E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

The corresponding magnetic field is given by

$$\vec{H}_i(x,z) = \frac{1}{\eta_1}\left[\hat{a}_n \times \vec{E}_i(x,z)\right]$$
$$= \frac{1}{\eta_1}\left[-\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z\right] E_{io} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

...........................(6.70)

Similarly, we can write the reflected waves as

$$\vec{E}_r(x,z) = \hat{a}_y E_{ro} e^{-j\beta_1 \vec{a}_{nr}\cdot\vec{r}}$$
$$= \hat{a}_y E_{ro} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

....

Since at the interface z=o, the tangential electric field is zero.

$$E_{io} e^{-j\beta_1 x\sin\theta_i} + E_{ro} e^{-j\beta_1 x\sin\theta_r} = 0 \quad \text{..(6.72)}$$

Consider in equation (6.72) is satisfied if we have

$$E_{ro} = -E_{io}$$
$$\text{and } \theta_i = \theta_r$$

.... 73)

The condition $\theta_i = \theta_r$ is Snell's law of reflection.

$$\therefore \vec{E}_r(x,z) = -\hat{a}_y E_{io} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$ ....(6.74)

$$\text{and } \vec{H}_r(x,z) = \frac{1}{n_1}\left[\hat{a}_{nr} \times \vec{E}_r(x,z)\right]$$
$$= \frac{E_{io}}{n_1}\left[-\hat{a}_x\cos\theta_i - \hat{a}_z\sin\theta_i\right] e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

..................................(6.75)

The total electric field is given by

$$\vec{E}_1(x,z) = \vec{E}_i(x,z) + \vec{E}_r(x,z)$$
$$= -\hat{a}_y 2jE_{io}\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i}$$ ....(6.76)

Similarly, total magnetic field is given by

$$\vec{H}_1(x,z) = -2\frac{E_{io}}{n_1}\left[\hat{a}_x\cos\theta_i\cos(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} + \hat{a}_z j\sin\theta_i\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i}\right]$$

.............................(6.77)

From eqns (6.76) and (6.77) we observe that

1. Along z direction i.e. normal to the boundary y component of $\vec{E}$ and x component of $\vec{H}$ maintain standing wave patterns according to $\sin\beta_{1z}z$ and $\cos\beta_{1z}z$ where $\beta_{1z} = \beta_1\cos\theta_i$. No average power propagates along z as y component of $\vec{E}$ and x component of $\vec{H}$ are out of phase.
2. Along x i.e. parallel to the interfacey component of $\vec{E}$ and z component of $\vec{H}$ are in phase (both time and space) andpropagate with phase velocity

$$v_{p1x} = \frac{\omega}{\beta_{1x}} = \frac{\omega}{\beta_1\sin\theta_i}$$
$$\text{and } \lambda_{1x} = \frac{2\pi}{\beta_{1x}} = \frac{\lambda_1}{\sin\theta_i}$$ ....(6.78)

The wave propagating along the x direction has its amplitude varying with z and hence constitutes a **non uniform** plane wave. Further, only electric

field $\hat{E}_1$ is perpendicular to the direction of propagation (i.e. x), the magnetic field has component along the direction of propagation. Such waves are called transverse electric or TE waves.