Waves in free space, conductors, dielectrics

Waves in free space

Electromagnetic waves can transport energy from one point to another point. The electric and magnetic field intensities asscociated with a travelling electromagnetic wave can be related to the rate of such energy transfer.

Let us consider Maxwell's Curl Equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Using vector identity

$$\nabla . \left(\vec{E} \times \vec{H}\right) = \vec{H} . \nabla \times \vec{E} - \vec{E} . \nabla \times \vec{H}$$

the above curl equations we can write

$$\nabla . \left(\vec{E} \times \vec{H}\right) = -\vec{H} . \frac{\partial \vec{B}}{\partial t} - \vec{E} . \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$$or, \nabla . \left(\vec{E} \times \vec{H}\right) = -\vec{H} . \frac{\partial \vec{B}}{\partial t} - \vec{E} . \vec{J} - \vec{E} . \frac{\partial \vec{D}}{\partial t}$$ ....(6.35)

In simple medium where $\in, \mu$ and σ are constant, we can write

$$\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right)$$

$$\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mu E^2\right)$$ and $$\vec{E}\cdot\vec{J} = \sigma E^2$$

$$\therefore \nabla\cdot\left(\vec{E}\times\vec{H}\right) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\in E^2 + \frac{1}{2}\mu H^2\right) - \sigma E^2$$

Applying Divergence theorem we can write,

$$\oint_S \left(\vec{E}\times\vec{H}\right)\cdot d\vec{S} = -\frac{\partial}{\partial t}\int_V \left(\frac{1}{2}\in E^2 + \frac{1}{2}\mu H^2\right)dV - \int_V \sigma E^2 dV$$ .... and magnetic fields and the term $\int_V \sigma E^2 dV$ represents the power dissipation within the volume. Hence right hand side of the equation (6.36) represents thetotal decrease in power within the volume under consideration......(6.36)

The term $\frac{\partial}{\partial t}\int_V \left(\frac{1}{2}\in E^2 + \frac{1}{2}\mu H^2\right)dV$ represents the rate of change of energy stored in the electric

The left hand side of equation (6.36) can be written as $\oint_S \left(\vec{E}\times\vec{H}\right)\cdot d\vec{S} = \oint_S \vec{P}\cdot d\vec{S}$

where $\vec{P} = \vec{E}\times\vec{H}$ (W/mt^2) is called the Poynting vector and it represents the power density vector associated with the electromagnetic field. The integration of the Poynting vector over any closed surface gives the net power flowing out of the surface. Equation (6.36) is referred to as Poynting theorem and it states that the net power flowing out of a given volume is equal to the time rate of decrease in the energy stored within the volume minus theconduction losses.

Definition

For time harmonic case, the time variation is of the form $e^{j\omega t}$, and we have seen thatinstantaneous value of a quantity is the real part of the product

of a phasor quantity and $e^{j\omega t}$ when $\cos \omega t$ is used as reference. For example, if weconsider the phasor

$$\vec{E}(z) = \hat{a}_x E_x(z) = \hat{a}_x E_0 e^{-j\beta z}$$

then we can write the instanteneous field as

$$\vec{E}(z,t) = \mathrm{Re}\left[\vec{E}(z)e^{j\omega t}\right] = E_0 \cos(\omega t - \beta z)\hat{a}_x \text{ ....(6.37)}$$

when E_0 is real.

Let us consider two instanteneous quantities A and B such that

$$A = \mathrm{Re}\left(Ae^{j\omega t}\right) = |A|\cos(\omega t + \alpha)$$

$$B = \mathrm{Re}\left(Be^{j\omega t}\right) = |B|\cos(\omega t + \beta)$$

where A and B are the phasor quantities.i.e,

$A = |A|e^{j\alpha}$

Therefore, $B = |B|e^{j\beta}$

$$AB = |A|\cos(\omega t + \alpha)|B|\cos(\omega t + \beta)$$

$$= \frac{1}{2}|A||B|\left[\cos(\alpha - \beta) + \cos(2\omega t + \alpha + \beta)\right] \text{ ....(6.39)}$$

Since *A* and *B* are periodic with period $T = \frac{2\pi}{\omega}$, the time average value of the product form AB, denoted by $\overline{AB}$ can be written as

$$\overline{AB} = \frac{1}{T}\int_0^T AB\,dt$$

$$\overline{AB} = \frac{1}{2}|A||B|\cos(\alpha - \beta)$$ ....(6.40)

Further, considering the phasor quantities *A* and *B*, we find that

$$AB^* = |A|e^{j\alpha}|B|e^{-j\beta} = |A||B|e^{j(\alpha-\beta)}$$

and $\mathrm{Re}(AB^*) = |A||B|\cos(\alpha - \beta)$, where * denotes complex conjugate.

$$\therefore \overline{AB} = \frac{1}{2}\mathrm{Re}(AB^*)$$ ....(6.41)

The poynting vector $\vec{P} = \vec{E}\times\vec{H}$ can be expressed as

$$\vec{P} = \hat{a}_x(E_yH_z - E_zH_y) + \hat{a}_y(E_zH_x - E_xH_z) + \hat{a}_z(E_xH_y - E_yH_x)$$

...................................(6.42)

If we consider a plane electromagnetic wave propagating in +z direction and has only E_x component, from (6.42) we can write:

$$\vec{P}_z = E_x(z,t)H_y(z,t)\hat{a}_z$$

Using (6.41)

$$\vec{P}_{zav} = \frac{1}{2}\text{Re}\left(E_x(z)H_y^*(z)\hat{a}_z\right)$$

$$\vec{P}_{zav} = \frac{1}{2}\text{Re}\left(E_x(z)\times H_y(z)\right)$$ ....(6.43)

where $\vec{E}(z) = E_x(z)\hat{a}_x$ and $\vec{H}(z) = H_y(z)\hat{a}_y$, for the plane wave under consideration.

For a general case, we can write

$$\vec{P}_{av} = \frac{1}{2}\text{Re}\left(\vec{E}\times\vec{H}^*\right)$$ ....(6.44)

We can define a complex Poynting vector

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*$$

and time average of the instantaneous Poynting vector is given by $\vec{P}_{av} = \text{Re}\left(\vec{S}\right)$.

5.1.1 Waves in loss dielectrics

The polarisation of a plane wave can be defined as the orientation of the electric field vector as a function of time at a fixed point in space. For an electromagnetic wave, the specification of the orientation of the electric field is sufficent as the magnetic field components are related to electric field vector by the Maxwell's equations.

Let us consider a plane wave travelling in the +z direction. The wave has both E_x and E_y components.

$$\vec{E} = \left(\hat{a}_x E_{ox} + \hat{a}_y E_{oy}\right)e^{-j\beta z}$$

....(6.45)

The corresponding magnetic fields are given by,

$$\vec{H} = \frac{1}{\eta}\hat{a}_z \times \vec{E}$$

$$= \frac{1}{\eta}\hat{a}_z \times \left(\hat{a}_x E_{ox} + \hat{a}_y E_{oy}\right) e^{-j\beta z}$$

$$= \frac{1}{\eta}\left(-E_{oy}\hat{a}_x + E_{ox}\hat{a}_x\right) e^{-j\beta z}$$

Depending upon the values of E_{ox} and E_{oy} we can have several possibilities:

1. If $E_{oy} = 0$, then the wave is linearly polarised in the *x*-direction.

2. If $E_{oy} = 0$, then the wave is linearly polarised in the *y*-direction.

3. If E_{ox} and E_{oy} are both real (or complex with equal phase), once again we get a linearly polarised wave with the axis of polarisation inclined at an angle $\tan^{-1}\frac{E_{oy}}{E_{ox}}$, with respect to the x-axis. This is shown in fig 6.4.

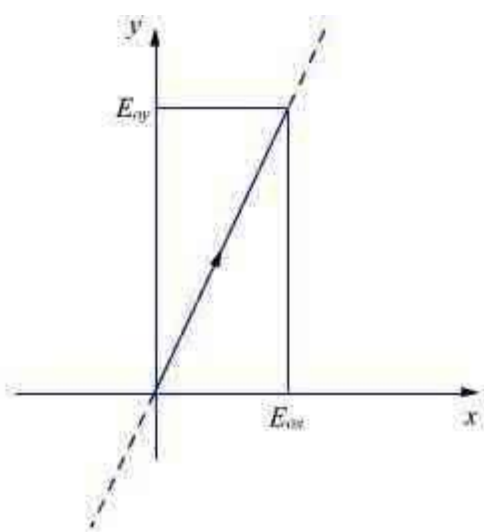


Fig 6.4 : Linear Polarisation

4. If *Eox* and *Eoy* are complex with different phase angles, $\vec{E}$ will not point to a single spatial direction. This is explained as follows:

Let $E_{ox} = |E_{ox}| e^{ja}$

$E_{oy} = |E_{oy}| e^{jb}$

Then,

$$E_x(z,t) = \text{Re}\left[|E_{ox}| e^{ja} e^{-j\beta z} e^{j\omega t}\right] = |E_{ox}| \cos(\omega t - \beta z + a)$$

and

$$E_y(z,t) = \text{Re}\left[|E_{oy}| e^{jb} e^{-j\beta z} e^{j\omega t}\right] = |E_{oy}| \cos(\omega t - \beta z + b)$$

....................................(6.46)

To keep the things simple, let us consider a =0 and $b = \frac{\pi}{2}$. Further, let us study the nature of the electric field on the z =0 plain.

From equation (6.46) we find that,

$$E_x(o,t) = |E_{ox}| \cos \omega t$$

$$E_y(o,t) = |E_{oy}| \cos\left(\omega t + \frac{\pi}{2}\right) = |E_{oy}|(-\sin \omega t)$$

$$\therefore \left(\frac{E_x(o,t)}{|E_{ox}|}\right)^2 + \left(\frac{E_y(o,t)}{|E_{oy}|}\right)^2 = \cos^2 \omega t + \sin^2 \omega t = 1$$ ....(6.47)

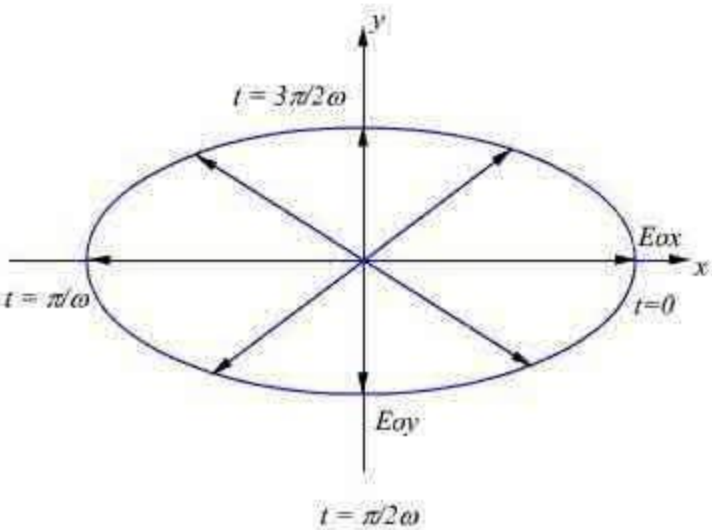
and the electric field vector at z = 0 can be written as

$$\vec{E}(o,t) = |E_{ox}| \cos(\omega t)\hat{a}_x - |E_{oy}| \sin(\omega t)\hat{a}_y$$ ....(6.48)

Assuming $|E_{ox}| > |E_{oy}|$, the plot of $\vec{E}(o,t)$ for various values of t is hown in

figure 6.5.

Figure 6.5 : Plot of *E*(*o*,*t*)



From equation (6.47) and figure (6.5) we observe that the tip of the arrow representing electric field vector traces qn ellipse and the field is said to be elliptically polarised.

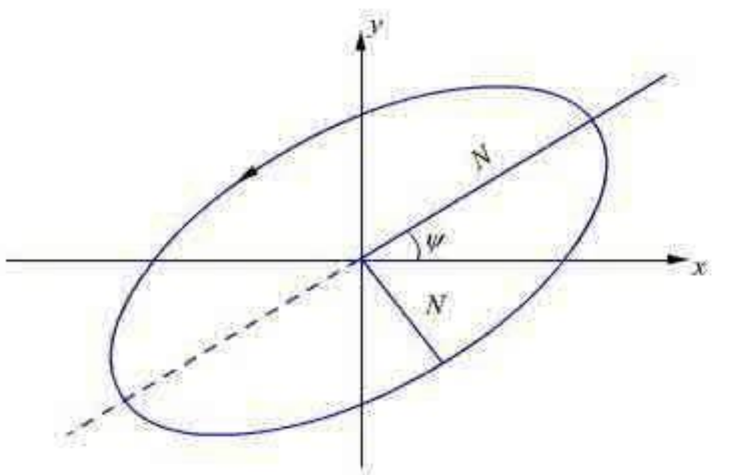


Figure 6.6: Polarisation ellipse

The polarisation ellipse shown in figure 6.6 is defined by its axial ratio(M/N, the ratio of semimajor to semiminor axis), tilt angle ψ (orientation with respect to xaxis) and sense of rotation(i.e., CW or CCW).

Linear polarisation can be treated as a special case of elliptical polarisation, for which the axial ratio is infinite.

In our example, if $|E_{ox}| = |E_{oy}|$, from equation (6.47), the tip of the arrow representing electric field vector traces out a circle. Such a case is referred to as Circular Polarisation. For circular polarisation the axial ratio is unity.

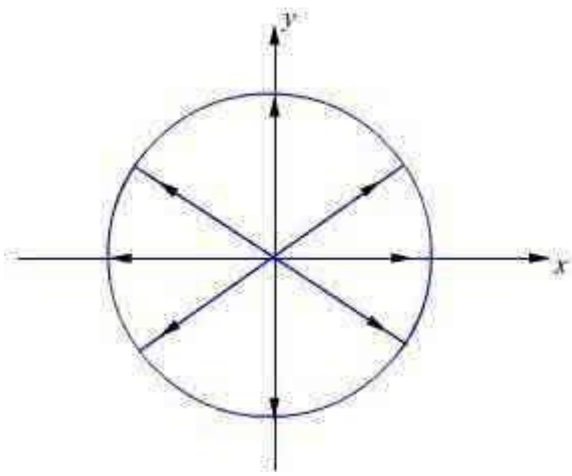


Figure 6.7: Circular Polarisation (RHCP)

Further, the circular polarisation is aside to be right handed circular polarisation (RHCP) if the electric field vector rotates in the direction of the fingers of the right hand when the thumb points in the direction of propagation-(same and CCW). If the electric field vector rotates in the opposite direction, the polarisation is asid to be left hand circular polarisation (LHCP) (same as CW).

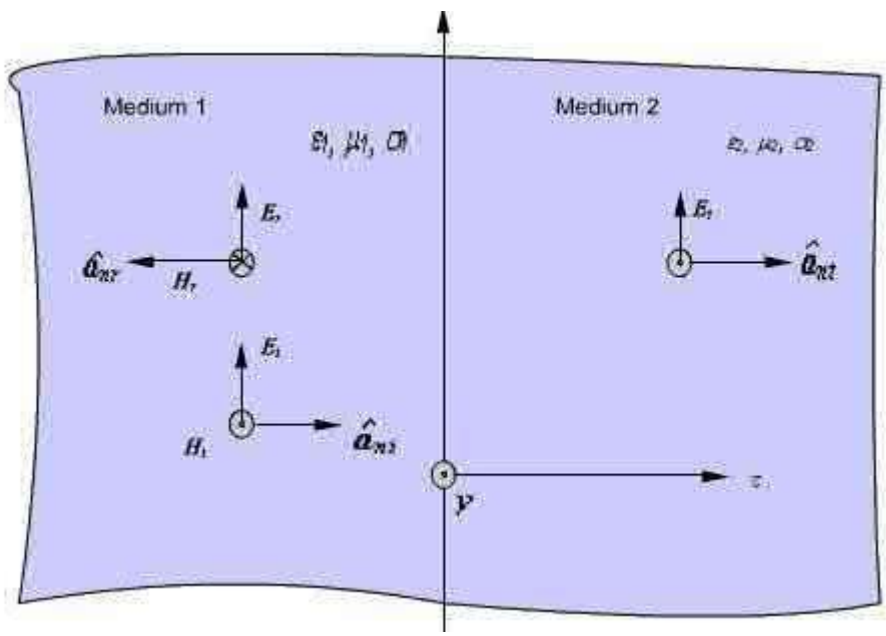
In AM radio broadcast, the radiated electromagnetic wave is linearly polarised with the $\vec{E}$ field vertical to the ground(vertical polarisation) where as TV signals are horizontally polarised waves. FM broadcast is usually carried out using circularly polarised waves.

In radio communication, different information signals can be transmitted at the same frequency at orthogonal polarisation (one signal as vertically polarised other horizontally polarised or one as RHCP while the other as LHCP) to increase capacity. Otherwise, same signal can be transmitted at orthogonal polarisation to obtain diversity gain to improve reliability of transmission.

5.1.2 in lossless dielectrics

We have considered the propagation of uniform plane waves in an unbounded homogeneous medium. In practice, the wave will propagate in

bounded regions where several values of ε, μ, σ will be present. When plane wave travelling in one medium meets a different medium, it is partly reflected and partly transmitted. In this section, we consider wave reflection and transmission at planar



boundary between two media.

Fig 6.8 : Normal Incidence at a plane boundary

Case1: Let z = 0 plane represent the interface between two media. Medium 1 is characterised by $(\varepsilon_1, \mu_1, \sigma_1)$ and medium 2 is characterized by $(\varepsilon_2, \mu_2, \sigma_2)$.

Let the subscripts '*i*' denotes incident, '*r*' denotes reflected and '*t*' denotes transmitted field components respectively.

The incident wave is assumed to be a plane wave polarized along *x* and travelling in medium 1 along $\hat{a}_z$ direction. From equation (6.24) we can write

$\vec{E}_i(z) = E_{io} e^{-\gamma_1 z} \hat{a}_x$ ....(6.49.a)

$\vec{H}_i(z) = \frac{1}{\eta_i} \hat{a}_z \times E_{io}(z) = \frac{E_o}{\eta_i} e^{-\gamma_1 z} \hat{a}_y$ ....(6.49.b)

where $\gamma_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\varepsilon_1)}$ and $\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_2}}$.

Because of the presence of the second medium at *z* =0, the incident wave will undergo partial reflection and partial transmission.

The reflected wave will travel along $\hat{a}_z$ in medium 1.
The reflected field components are:

$\vec{E}_r = E_{ro} e^{\gamma_1 z} \hat{a}_x$...(6.50a)

$\vec{H}_r = \frac{1}{\eta_1}\left(-\hat{a}_z\right) \times E_{ro} e^{\gamma_1 z} \hat{a}_x = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z} \hat{a}_y$ ....(6.50b)

The transmitted wave will travel in medium 2 along $\hat{a}_z$ for which the field componentsare

$$\vec{E}_t = E_{t0} e^{-\gamma_2 z} \hat{a}_x \quad \text{...(6.51a)}$$

$$\vec{H}_t = \frac{E_{t0}}{\eta_2} e^{-\gamma_2 z} \hat{a}_y \quad \text{...(6.51b)}$$

where $\gamma_2 = \sqrt{j\omega\mu_2(\sigma_2 + j\omega\varepsilon_2)}$ and $\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}}$

$\vec{E}_1 = \vec{E}_i + \vec{E}_r$ and $\vec{H}_1 = \vec{H}_i + \vec{H}_r$

and in medium 2,

$\vec{E}_2 = \vec{E}_t$ and $\vec{H}_2 = \vec{H}_t$

Waves in conductors

Applying boundary conditions at the interface z = 0, i.e., continuity of tangential field components and noting that incident, reflected and transmitted field components are tangential at the boundary, we can write

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_i(0)$$

& $$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0)$$

From equation 6.49 to 6.51 we get,

$E_{io} + E_{ro} = E_{to}$ ....(6.52a)

$\dfrac{E_{io}}{\eta_1} - \dfrac{E_{ro}}{\eta_1} = \dfrac{E_{to}}{\eta_2}$ ....(6.52b)

Eliminating E_{to} ,

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{1}{\eta_2}\left(E_{io} + E_{ro}\right)$$

or, $$E_{io}\left(\frac{1}{\eta_1} - \frac{1}{\eta_2}\right) = E_{ro}\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)$$

or, $E_{ro} = \tau E_{io}$

$\tau = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ ....(6.53)

is called the reflection coefficient. From equation (6.52), we can write

$$2E_{io} = E_{to}\left[1 + \frac{\eta_1}{\eta_2}\right]$$

or, $$E_{to} = \frac{2\eta_2}{\eta_1 + \eta_2} E_{io} = TE_{io}$$

$$T = \frac{2\eta_2}{\eta_1 + \eta_2} \qquad \text{...(6.54)}$$

is called the transmission coefficient.We observe that,

$$T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{\eta_2 - \eta_1 + \eta_1 + \eta_2}{\eta_1 + \eta_2} = 1 + \tau \qquad \text{...(6.55)}$$

The following may be noted

(i) both τ and T are dimensionless and may be complex

(ii) $0 \le |\tau| \le 1$