

SNS COLLEGE OF TECHNOLOGY



(An Autonomous Institution) Coimbatore-641035.

UNIT-I MULTIPLE INTEGRALS

Applications: Volume as triple integrals and solids of revolution

3] find the volume of the tetrahedrep bound
by the plane
$$\frac{x}{a} + \frac{y}{b} + \frac{x}{c} = 1$$

Soln.
Given $\frac{x}{a} + \frac{y}{b} + \frac{x}{c} = 1$
 $\frac{x}{a} = 1 - \frac{y}{b} - \frac{x}{c}$
CS Scanned with $x = a\left(1 - \frac{y}{b} - \frac{x}{c}\right)$



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UNIT-I MULTIPLE INTEGRALS Applications: Volume as triple integrals and solids of revolution Take x=0, $\frac{y}{b} + \frac{x}{c} = 1$ $y = 1 - \frac{\pi}{c}$ $y = b(1 - \frac{\pi}{c})$ Take ==08y=0 $\frac{x}{2} = 1$ Z = C $\begin{array}{cccc} x & \vdots & 0 & to & a \left(1 - \frac{y}{b} - \frac{x}{c}\right) \\ y & \vdots & 0 & to & b \left(1 - \frac{x}{c}\right) \\ z & \vdots & 0 & to & c \end{array}$ L9m9-18 : volume = SSS da dy da $= \int_{a}^{b} \int_{a} \left(1 - \frac{y}{b} - \frac{x}{c}\right) dx dy dx$ $\int \frac{1-\frac{x}{c}}{[x]} a\left(1-\frac{y}{b}-\frac{x}{c}\right) \\ \int \frac{1}{[x]} dy dx$ $b\left(1-\frac{x}{c}\right)$ $\left[\overline{a}\left(1-\frac{y}{b}-\frac{x}{c}\right)-o\right] dy dx$ $\int \left[\left(1 - \frac{x}{c}\right) - \frac{y}{b} \right] dy dz$ $\left[\left(1-\frac{x}{c}\right)y-\frac{y^{2}}{2b}\right]$



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$$= a \int_{0}^{C} \left[\left(1 - \frac{x}{c}\right) b \left(1 - \frac{x}{c}\right) - \frac{1}{2b} b^{2} \left(1 - \frac{x}{c}\right)^{2} \right] dx$$

$$= a \int_{0}^{C} \left[\left(1 - \frac{x}{c}\right)^{2} b - \frac{b}{2} \left(1 - \frac{x}{c}\right)^{2} \right] dz$$

$$= \frac{ab}{2} \int_{0}^{C} \left(1 - \frac{x}{c}\right)^{2} dz$$

$$= \frac{ab}{2} \left[\frac{\left(1 - \frac{x}{c}\right)^{2}}{\left(-\frac{1}{c}\right)^{3}} \right]_{0}^{C}$$

$$= -\frac{abc}{6} \left[0 - 1 \right]$$
Volume = $\frac{abc}{6}$
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