UNIT 5 – Multiple Integrals

Change of order of integration.

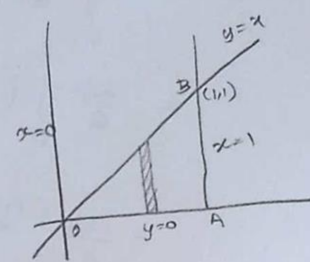
① Change the order of integration for $\int_0^1\int_0^x f(x,y)\,dx\,dy$

Solution :-

Given integral is not in the correct order

let us rearrange it

$$I = \int_0^1\int_0^x f(x,y)\,dy\,dx.$$



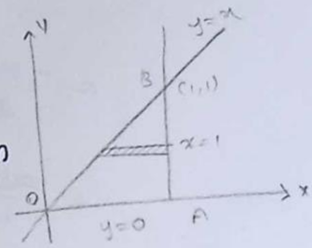
Given :- $y = 0$ to $y = x$

$x = 0$ to $x = 1$

Inner limit is wrt y

$\therefore$ It is a vertical strip

Now to change the order of integration we have to draw a horizontal strip



x limits : $x = y$ to $x = 1$

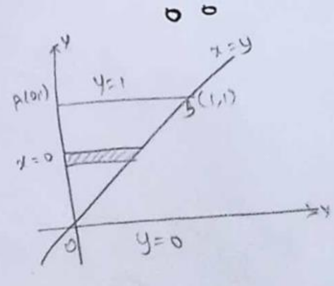
y limits : $y = 0$ to $y = 1$

$$\therefore I = \int_0^1\int_y^1 f(x,y)\,dx\,dy.$$

2. Change the order of integration in $\int_0^1\int_0^y f(x,y)\,dx\,dy$

$I = \int_0^1\int_0^y f(x,y)\,dx\,dy.$

Given limits:



x limit : $x = 0$ to $x = y$

y limit : $y = 0$ to $y = 1$

Inner limit is wrt 'x'

$\therefore$ Given limit is a horizontal strip

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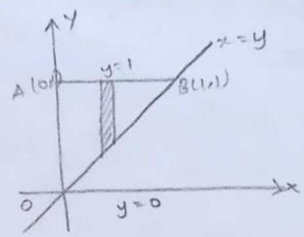
Now to change the order of integration we have to draw a vertical strip



x limit : $x=0$ to $x=1$

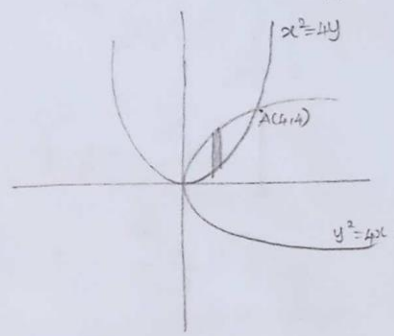
y limit : $y=x$ to $y=1$

$$I = \int_0^1 \int_x^1 f(x,y)\, dy\, dx.$$

3. Evaluate by changing the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\,dx$

Solution :-

$$I = \int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\,dx$$



Given limits :

x limits $x=0$ to $x=4$

y limits $y=\frac{x^2}{4}$ to $y=2\sqrt{x}$

i.e) $x^2=4y$ to $y^2=4x$

Inner limit is wrt 'y'

$\therefore$ It is a vertical strip

To find point A:

Solving $x^2=4y$ ⟶ ① and $y^2=4x$ ⟶ ②

$x^2=4y$

Squaring , $x^4=16y^2$

$x^4=16(4x)$ using ②

$x^4=64x$

$x^3=64$

$\boxed{x=4}$

A(4,4).

sub x in ②

$y^2=4x$

$y^2=4(4)$

$y^2=16$

$\boxed{y=4}$

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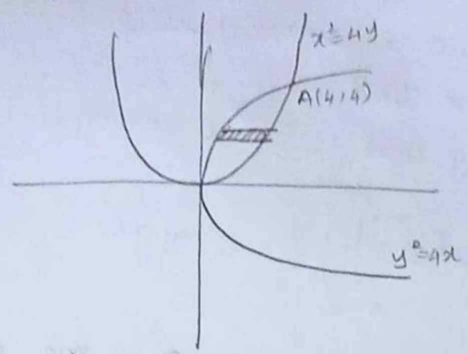
Now to change the order of integration draw a horizontal strip



x limits :

$x = \frac{y^2}{4}$ to $x = 2\sqrt{y}$

$y=0$ to $y=4$

$$\therefore I = \int_0^4 \int_{y^2/4}^{2\sqrt{y}} dx\, dy$$

$$= \int_0^4 \int_{y^2/4}^{2\sqrt{y}} [x]_{y^2/4}^{2\sqrt{y}} dy \quad = \int_0^4 [2\sqrt{y} - y^2/4]\, dy$$

$$= \left[\frac{2y^{3/2}}{3/2} - \frac{1}{4}\frac{y^3}{3} \right]_0^4$$

$$= \left[\frac{2(4)^{3/2}}{3/2} - \frac{1}{4}\frac{(4)^3}{3} \right]$$

$$= \left[\frac{4}{3}(8) - \frac{16}{3} \right] = \frac{32-16}{3}$$

$$I = \frac{16}{3}$$

4. Change the order of integration in $\int_0^a \int_{x^2/a}^{2a-x} xy\, dy\, dx$ and then evaluate. $\frac{3}{8}a^4$

The region of integration R is bounded by the curve $y = \frac{x^2}{a}$ i.e. the parabola $x^2=ay$, the line $y = 2a-x$, ie $x+y = 2a$ and the lines $x=0$ and $x=a$.