

#### SNS COLLEGE OF TECHNOLOGY



Coimbatore - 35

#### An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

#### DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

19ECT311 / Wireless Communication

III ECE/ VI SEMESTER

Unit II - MOBILE RADIO PROPAGATION

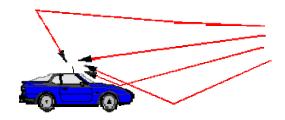
**Topic 9:** Rayleigh and Rician distribution



## Multipath fading



#### Multiple reflected waves arrive at the receiver



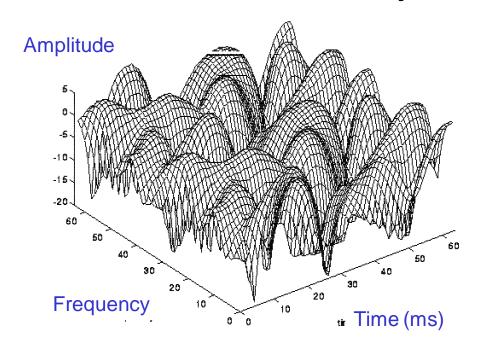
#### Narrowband model

- Different waves have different phases.
- These waves may cancel or amplify each other.
- •This results in a fluctuating ("fading") amplitude of the total received signal.



## Rayleigh Multipath Reception





The received signal amplitude depends on location and frequency

If the antenna is moving, the location *x* changes linearly

with time *t* (*x* = *v t*)

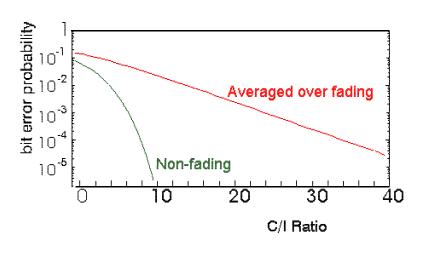
Parameters:

- probability of fades
- duration of fades
- bandwidth of fades



## Effect of Flat Fading







- In a fading channel, the BER only improves very slowly with increasing C/I
- Fading causes burst errors
- Average BER does not tell the full story
- Countermeasures to improve the slope of the curve

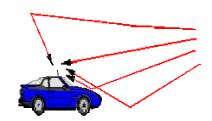


## Models for Multipath Fading

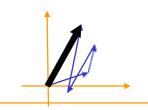


#### Rayleigh fading

- •(infinitely) large collection of reflected waves
- Appropriate for macrocells in urban environment
- •Simple model leads to powerful mathematical framework



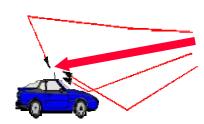
Transmit 
$$s(t) = \cos(\omega_c t)$$
,  
Receive  $v(t) = \sum_{n=1}^{N} \rho_n s(t-T_n)$ ,



N=4 waves with amplitude  $\rho_n$  and phase shift due to delay  $T_n$ 

#### Ricean fading

- •(infinitely) large collection of reflected waves plus line-of sight
- Appropriate for micro-cells
- Mathematically more complicated





## Rayleigh Model



**Use Central Limit Theorem** 

inphase  $s_i(t) = \zeta$  and

quadrature  $s_Q(t) = \xi$  components are zero- mean independently identically distributed (i.i.d.) jointly Gaussian random variables

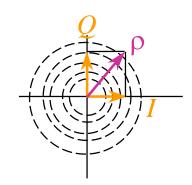
PDF:

$$f(\xi, \zeta) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{\xi^2 + \zeta^2}{2\sigma^2}\right\}$$

Conversion to polar co-ordinates:

Received amplitude  $\rho$ :  $\rho^2 = \zeta^2 + \xi^2$ .

$$\zeta = \rho \cos \phi;$$
  $\xi = \rho \sin \phi,$ 





## PDF of Rayleigh Amplitude



After conversion to polar co-ordinates:

 $f_{P,\Phi}(\rho,\phi) = \frac{\rho}{2\pi\sigma^2} \exp\left\{\frac{\rho^2}{2\sigma^2}\right\}$ 

Integrate this PDF over  $\phi$  from 0 to  $2\pi$ :

Rayleigh PDF of ρ

$$f_{\rho}(\rho) = \frac{\rho}{p} \exp\left(-\frac{\rho^2}{2\overline{p}}\right)$$

#### where

- p is the local mean power total scattered power ( $p = \sigma^2$ ).

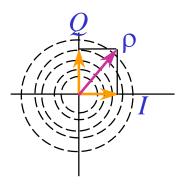


## Received Amplitudes









Probability
Density
Threshold

Amplitude

$$f_{P}(\rho) = \frac{\rho}{p} \exp\left\{-\frac{\rho^2}{2\overline{p}}\right\}.$$



### Received Power



Conversion from amplitude to power ( $p = \rho^2/2$ ) gives the exponential distribution:

$$f_p(p) = f_{\rho}(\rho) \left| \frac{d\rho}{d\overline{p}} \right| = \frac{1}{\overline{p}} \exp \left\{ -\frac{p}{\overline{p}} \right\}.$$

Exponential distributions are very convenient to handle mathematically.



## Who was Rayleigh?

- The basic model of <u>Rayleigh fading</u> assumes a received multipath signal to consist of a (theoretically infinitely) large number of reflected waves with independent and identically distributed inphase and quadrature amplitudes.
- This model has played a major role in our understanding of mobile propagation.
- The model was first proposed in a comment paper written by Lord Rayleigh in 1889, describing the resulting signal if many violinists in an orchestra play in unison, long before its application to mobile radio reception was recognized.



[1] Lord Rayleigh, "On the resultant of a large number of vibrations of the same pitch and of arbitrary phase", Phil. Mag., Vol. 10, August 1880, pp. 73-78 and Vol. 27, June 1889, pp. 460-469.

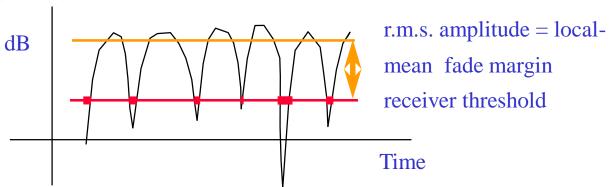
Lord Ravleigh (John William Strutt) was an English physicist (1877 - 1919) and a Nobel Laureate (1904) who made a number of contributions to wave physics of sound and optics.



## Fade Margin



Fade margin is the ratio of the average received power over some threshold power, needed for reliable communication.



PDF of signal amplitude

Fade margin

probability



## Average BER

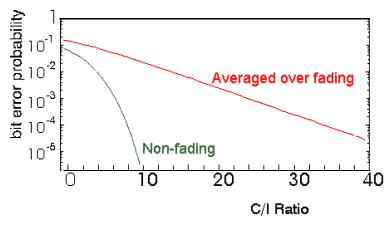


The BER for BPSK with known instantaneous power *p* 

$$P = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

The BER averaged over an exponential distribution

$$\overline{P} = \int_{0}^{\infty} \frac{1}{p} \exp\left\{-\frac{p}{p}\right\} \frac{1}{2} \operatorname{erfc} \sqrt{\frac{pT_b}{N_0}} dp = \frac{1}{2} - \frac{1}{2} \sqrt{\frac{\overline{p}T_b}{N_0 + \overline{p}T_b}}$$





## **ACTIVITY**





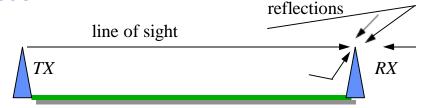
Activity: Draw a logo which may describe your character or things you like.



## Ricean Multipath Reception



Narrowband propagation model:



Transmitted carrier  $s(t) = \cos(\omega_t t)$ 

#### Received carrier

$$v(t) = C\cos\omega_{c}t + \sum_{n=1}^{N} \rho_{n}\cos(\omega_{c}t + \phi_{n}),$$

#### where

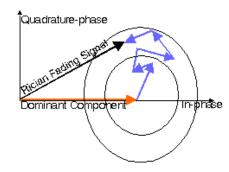
- C is the amplitude of the line-of-sight component
- $\rho_n$  is the amplitude of the *n*-th reflected wave
- $\phi_n$  is the phase of the *n*-th reflected wave



## Ricean Multipath Reception



#### Received carrier:



$$v(t) = C \cos \omega_c t + \sum_{n=1}^{N} \sum_{n=1}^{N} \rho_n \cos(\omega_c t + \phi_n),$$

#### where

 $\zeta$  is the in-phase component of the reflections

 $\xi$  is the quadrature component of the reflections.

I is the total in-phase component  $(I = C + \zeta)$ 

Q is the total quadrature component  $(I = C + \zeta)$ 



## Ricean Amplitude



#### After conversion to polar co-ordinates:

$$f_{P,\Phi}(\rho,\phi) = \frac{\rho}{2\pi\sigma^2} \exp\left\{-\frac{\rho^2 + C^2 - 2\rho C\cos\phi}{2\sigma^2}\right\}$$

Integrate this PDF over  $\phi$  from 0 to  $2\pi$ : Ricean PDF of  $\rho$ 

$$f_{\rho}(\rho) = \frac{\rho}{\overline{q}} \exp\left(-\frac{\rho^2 + C^2}{2\overline{q}}\right) I_0\left(\frac{\rho C}{\overline{q}}\right),$$

#### where

- $I_0(.)$  is the modified Bessel function of the first kind and zero order
- q is the total scattered power  $(q = \sigma^2)$ .



### Ricean Phase



#### After conversion to polar co-ordinates:

$$f_{P,\Phi}(\rho,\phi) = \frac{\rho}{2\pi\sigma^2} \exp\left\{-\frac{\rho^2 + C^2 - 2\rho C\cos\phi}{2\sigma^2}\right\}$$

#### Integrate this PDF over $\rho$



## Ricean *K*-factor



Definition:  $K = \text{direct power } C^2/2 \text{ over scattered power } q$ 

Measured values

 $K = 4 \dots 1000$  (6 to 30 dB) for micro-cellular systems

Light fading (*K*-> infinity)

- Very strong dominant component
- Ricean PDF approaches Gaussian PDF with small  $\sigma$

Severe Fading (K = 0):

Rayleigh Fading





# How do systems handle outages?

- Analog
  - Fast moving User experiences a click
  - Slow moving user experiences a burst of noise
- GSM
  - Speech extrapolation
- DECT
  - Handover to other base station if possible
  - Handover to different frequency
- WLAN / cellular CDMA
  - Large transmit bandwidth to prevent that the full signal vanishes in a fade



#### Assessment



- ➤ Link budget consists of calculation of
  - a) Useful signal power
  - b) Interfering noise power
  - c) Useful signal & Interfering noise power
  - d) Signal and Noise
- ➤ Link budget can help in predicting
  - a) Equipment weight and size
  - b) Technical risk
  - c) Prime power requirements
  - d) Equipment weight and size, Technical risk and Prime power requirements.
- > Space loss occurs due to decrease in
  - a) Electric field strength
  - b) Efficiency
  - c) Phase
  - d) Signal power







## Thank you