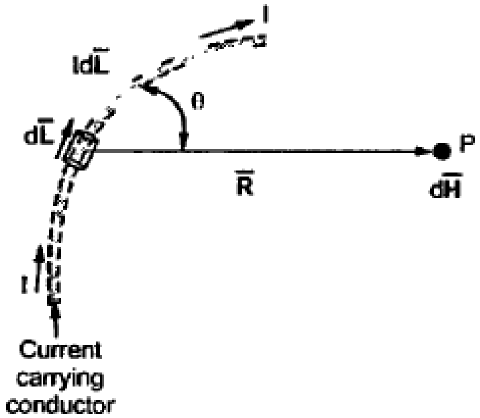
BIOT SAVART'S LAW

Biot-Savart Law

Consider a conductor carrying a direct current I and a steady magnetic field produced around it. The Biot-Savart law allows us to obtain the **differential magnetic field intensity** $d\overline{H}$, produced at a point P, due to a differential current element IdL.



Consider a differential length dL hence the differential current element is IdL. This is very small part of the current carrying conductor. The point P is at a distance R from the differential current element. The θ is the angle between the differential current element and the line joining point P to the differential current element.

The **Biot-Savart law** states that,

The magnetic field intensity $d\overline{H}$ produced at a point P due to a differential current element IdL is,

1. Proportional to the product of the current I and differential length dL.
2. The sine of the angle between the element and the line joining point P to the element.
3. And inversely proportional to the square of the distance R between point P and the element.

Mathematically, the Biot-Savart law can be stated as,

$$d\bar{H} \propto \frac{I\, dL \sin\theta}{R^2} \quad \text{... (1)}$$

$$\therefore \quad \boxed{d\bar{H} = \frac{k\, I\, dL \sin\theta}{R^2}} \quad \text{... (2)}$$

where $\quad k = $ Constant of proportionality

In SI units, $\quad k = \frac{1}{4\pi}$

$$\therefore \quad d\bar{H} = \frac{I\, dL \sin\theta}{4\pi R^2} \quad \text{... (3)}$$

Let us express this equation in vector form.

Let $\quad dL = $ Magnitude of vector length $d\bar{L}$ and

$\bar{a}_R = $ Unit vector in the direction from differential current element to point P

Then from rule of cross product,

$$d\bar{L}\times\bar{a}_R = dL\,|\bar{a}_R|\sin\theta = dL\sin\theta \quad \text{...}\ |\bar{a}_R| = 1$$

Replacing in equation (3),

$$d\bar{H} = \frac{I\, d\bar{L}\times\bar{a}_R}{4\pi R^2}\ \text{A/m} \quad \text{... (4)}$$

But $\quad \bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{\bar{R}}{R}$

Hence, $\quad \boxed{d\bar{H} = \frac{I\, d\bar{L}\times\bar{R}}{4\pi R^3}\ \text{A/m}} \quad \text{... (5)}$

The equations (4) and (5) is the mathematical form of Biot-Savart law.

According to the direction of cross product, the direction of $d\bar{H}$ is normal to the plane containing two vectors and in that normal direction which is along the progress of right handed screw, turned from $d\bar{L}$ through the smaller angle θ towards the line joining element to the point P. Thus the direction of $d\bar{H}$ is normal to the plane of paper. For the case considered, according to right handed screw rule, the direction of $d\bar{H}$ is going into the plane of the paper.

The entire conductor is made up of all such differential elements. Hence to obtain total magnetic field intensity $\bar{H}$, the above equation (4) takes the integral form as,

$$\boxed{\bar{H} = \oint \frac{I\, d\bar{L}\times\bar{a}_R}{4\pi R^2}} \quad \text{... (6)}$$

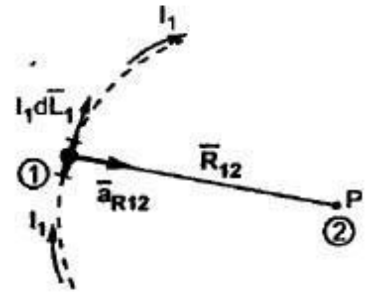


Fig. 7.7

The closed line integral is required to ensure that all the current elements are considered. This is because current can flow only in the closed path, provided by the closed circuit. If the current element is considered at point 1 and point P at point 2, as shown in the Fig. 7.7 then,

$$d\bar{H}_2 = \frac{I_1\, d\bar{L}_1 \times \bar{a}_{R12}}{4\pi R_{12}^2} \text{ A/m} \quad \text{... (7)}$$

where I_1 = Current flowing through dL_1 at point 1

$d\bar{L}_1$ = Differential vector length at point 1

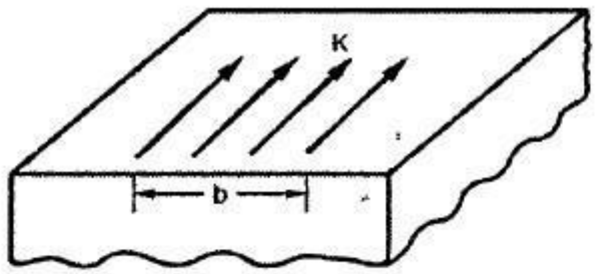
$\bar{a}_{R12}$ = Unit vector in the direction from element at point 1 to the point P at point 2

$$\bar{a}_{R12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{\bar{R}_{12}}{R_{12}}$$

$$\therefore \quad \boxed{\bar{H}_2 = \oint \frac{I_1\, d\bar{L}_1 \times \bar{a}_{R12}}{4\pi R_{12}^2} \text{ A/m}} \quad \text{... (8)}$$

This is called **integral form of Biot-Savart law.**

Biot-Savart Law Interms of Distributed Sources



Surface current density

Consider a surface carrying a uniform current over its surface as shown . Then the **surface current density** is denoted as $\overline{K}$ and measured in amperes per metre (A/m). Thus for uniform current density, the current I in any width b is given by I = Kb, where width b is perpendicular to the direction of current flow.

Thus if dS is the differential surface area considered of a surface having current density $\overline{K}$ then,

$$I\, d\overline{L} = \overline{K}\, dS \qquad \text{... (9)}$$

If the current density in a volume of a given conductor is $\overline{J}$ measured in A/m^2 then for a differential volume dv we can write,

$$I\, d\overline{L} = \overline{J}\, dv \qquad \text{... (10)}$$

Hence the Biot-Savart law can be expressed for surface current considering $\overline{K}$ dS while for volume current considering $\overline{J}$ dv.

$\therefore$

$$\overline{H} = \int_S \frac{\overline{K} \times \overline{a}_R\, dS}{4\pi R^2} \ \text{A/m} \qquad \text{... (11)}$$

and

$$\overline{H} = \int_{vol} \frac{\overline{J} \times \mathbf{a}_R\, dv}{4\pi R^2} \ \text{A/m} \qquad \text{... (12)}$$