Transformer and motional EMF

The negative sign in equation was introduced by Lenz in order to comply with the polarity of the induced emf. The negative sign implies that the induced emf will cause a current flow in the closed loop in such a direction so as to oppose the change in the linking magnetic flux which produces it. (It may be noted that as far as the induced emf is concerned, the closed path forming a loop does not necessarily have to be conductive).

If the closed path is in the form of N tightly wound turns of a coil, the change in the magnetic flux linking the coil induces an emf in each turn of the coil and total emf is the sum of the induced emfs of the individual turns, i.e.,

For a circuit with a single turn (N = 1)

$$V_{\text{emf}} = -\frac{d\Psi}{dt}$$

In terms of E and B this can be written as

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

where ψ has been replaced by

$$\int_S \mathbf{B} \cdot d\mathbf{S}$$

and S is the surface area of the circuit bounded by a closed path L.. The equation says that in time-varying situation, both electric and magnetic fields are present and are interrelated.

The three ways of induced EMF are,

1. By having a stationary loop in a time-varying B field.

2. By having a time-varying loop area in a static B field.

3. By having a time-varying loop area in a time-varying B field.

Stationary loop in a time-varying B field (Transformer emf)

Consider a stationary conducting loop in a time-varying magnetic B field. The equation (i) becomes

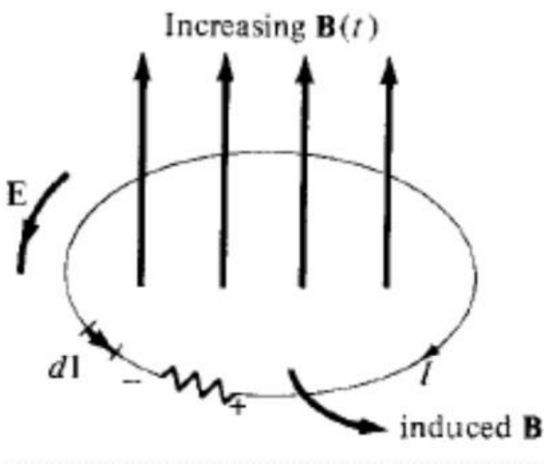

Fig 4.1 Stationary loop in a time-varying B field

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

This emf induced by the time-varying current in a stationary loop is often referred to as transformer emf in power analysis since it is due to the transformer action. By applying Stokes's theorem to the middle term, we get

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

This is one of the Maxwell's equations for time-varying fields. It shows that the time-varying field is not conservative.

Moving loop in static B field (Motional emf)

When a conducting loop is moving in a static B field, an emf is introduced in the loop. The force on a charge moving with uniform velocity u in a magnetic field B is

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$$

The motional electric field Em is defined as

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{Q} = \mathbf{u} \times \mathbf{B}$$

Consider a conducting loop moving with uniform velocity u, the emf induced in the loop is

$$V_{\text{emf}} = \oint_L \mathbf{E}_m \cdot d\mathbf{l} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

This kind of emf is called the motional emf or flux-cutting emf. Because it is due to the motional action. eg,. Motors, generators Consider a rod moving between a pair of rails Here B and perpendicular u are so the force can be given by

$$\mathbf{F}_m = I\ell \times \mathbf{B}$$

$$F_m = I\ell B$$

The equation (i) becomes

$$V_{emf} = uB\ell$$

By applying Stokes's theorem to equation (i), we get

$$\int_S (\nabla \times \mathbf{E}_m) \cdot d\mathbf{S} = \int_S \nabla \times (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{E}_m = \nabla \times (\mathbf{u} \times \mathbf{B})$$

Moving loop in time-varying field

Consider a moving conducting loop in a time-varying magnetic field Then both transformer emf and motional emf are present. Thus the total emf will be the sum of transformer emf and motional emf

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{u} \times \mathbf{B})$$