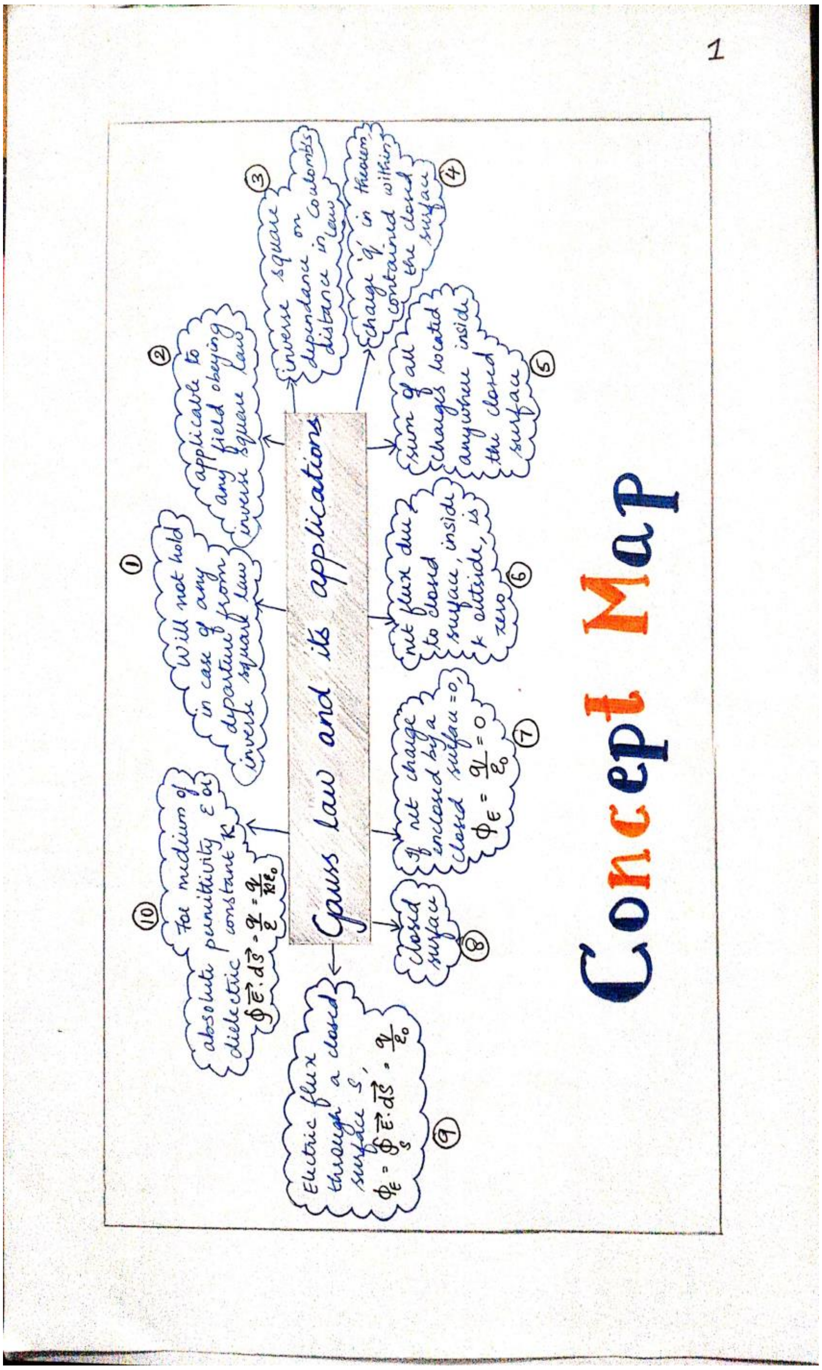
GAUSS LAW AND ITS APPLICATIONS



Terms, Definitions and Symbols:

ELECTRIC FLUX:

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. It is usually denoted by the Greek letter ϕ. and its unit is Nm^2C^{-1}. Electric flux is a scalar quantity and it can be positive or negative

GAUSS LAW:

Gauss law states that if a charge 'Q' is enclosed by an arbitary closed surface, then the total electric flux ϕ_E through the closed surface is $\phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

DATA TABLE, EQUATIONS AND FORMULAE:

ELECTRIC FLUX FOR UNIFORM ELECTRIC FIELD IN A REGION OF SPACE:

$$\phi_E = EA$$

$A \rightarrow$ Area

$E \rightarrow$ Electric field

AREA 'A' IS KEPT PARALLEL TO UNIFORM FIELD, THEN,

$$\phi_E = 0$$

IF AREA IS INCLINED AT AN ANGLE θ, THEN,

$$\phi_E = (E \cos\theta) A$$

FOR UNIFORM ELECTRIC FIELD, ELECTRIC FLUX,

$$\phi_E = \vec{E} \cdot \vec{A} = EA \cos\theta$$

ELECTRIC FLUX IN A NON UNIFORM ELECTRIC FIELD AND AN ARBITARY SHAPED AREA,

$$\phi_E = \sum_{i=1}^{n} \vec{E_i} \cdot \Delta\vec{A_i}$$

By taking limit $\Delta\vec{A_i} \to 0$, summation becomes integration. And so,

$$\phi_E = \int \vec{E} \cdot d\vec{A}$$

TOTAL ELECTRIC FLUX FOR CLOSED SURFACE,

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

TOTAL FLUX THROUGH THE CLOSED SURFACE OF THE SPHERE,

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E dA \cos\theta$$

ELECTRIC FIELD OF POINT CHARGE DIRECTED RADIALLY OUTWARD,

$$\phi_E = \oint E dA \quad (\text{since } \cos 0^\circ = 1)$$

E is uniform on the surface of the sphere,

$$\phi_E = E \oint dA.$$

GAUSS LAW:

$$\phi_E = \frac{Q}{\varepsilon_0}$$

$Q \to$ charge

$\varepsilon_0 \to$ permittivity of free space.

GAUSS LAW FOR CLOSED SURFACES:

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

TOTAL ELECTRIC FLUX IN CLOSED SURFACES (EXPANDED)

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_{\substack{\text{curved} \\ \text{surface}}} \vec{E} \cdot d\vec{A} + \int_{\substack{\text{Top} \\ \text{surface}}} \vec{E} \cdot d\vec{A} + \int_{\substack{\text{Bottom} \\ \text{surface}}} \vec{E} \cdot d\vec{A}$$

FOR CYLINDRICAL GAUSSIAN SURFACE,

$(\because)\ Q_{encl} = \lambda L$

$$E \int_{\substack{\text{curved} \\ \text{surface}}} dA = \frac{\lambda L}{\varepsilon_0}$$ $\lambda \rightarrow$ linear charge density

Here $\phi_E = \int_{\substack{\text{curved} \\ \text{surface}}} dA$ = total area of the curved surface $= 2\pi r L$.

Substituting,

$$E \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}$$

In vector form $\vec{E} = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r} \hat{r}$

ELECTRIC FIELD DUE TO CHARGED INFINITE PLANE SHEET:

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_{\substack{\text{curved} \\ \text{surface}}} \vec{E} \cdot d\vec{A} + \int_{P} \vec{E} \cdot d\vec{A} + \int_{P'} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

WHEN ELECTRIC FIELD IS PERPENDICULAR TO AREA AND PARALLEL TO SURFACE,

$$\phi_E = \int_P E\,dA + \int_{P'} E\,dA = \frac{Q_{encl}}{\varepsilon_0}$$

Since magnitude of electric field at both surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get

$$2E \int_P dA = \frac{\sigma A}{\varepsilon_0} \qquad \sigma \rightarrow \text{surface charge density}$$

The total area of surface either at P or P'

$$\int_P dA = A$$

Hence $2EA = \frac{\sigma A}{\varepsilon_0}$ (or) $E = \frac{\sigma}{2\varepsilon_0}$

In vector form, $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

ELECTRIC FIELD DUE TO TWO PARALLEL CHARGED INFINITE SHEETS:

$$E_{inside} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL:

Case (a):

AT A POINT OUTSIDE THE SHELL ($r > R$)

$$\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

Since $\vec{E}$ is same at all points,

$$E \oint_{\substack{\text{gaussian} \\ \text{surface}}} dA = \frac{Q}{\varepsilon_0}$$

But $\oint_{\substack{\text{Gaussian} \\ \text{surface}}} dA$ = total area of Gaussian surface

$= 4\pi r^2$. Substituting,

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \quad (\text{or}) \quad E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

In vector form, $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$

Case (b): AT A POINT ON THE SURFACE OF THE SPHERICAL SHELL ($r = R$)

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2} \hat{r}$$

Case (c): AT A POINT INSIDE THE SPHERICAL SHELL ($r < R$)

$$\oint_{\substack{\text{Gaussian} \\ \text{surface}}} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

Since gaussian surface encloses no charge, so $Q = 0$. Equation becomes,

$$E = 0 \quad (r < R)$$