A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.

Solution. Given: Pull = 180 N; Push = 220 N and angle at which force is inclined with horizontal plane $(\alpha) = 30°$

Let W = Weight of the body

R = Normal reaction, and

μ = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F_1) will act towards left as shown in Fig. 8.3. (*a*).

Resolving the forces horizontally,

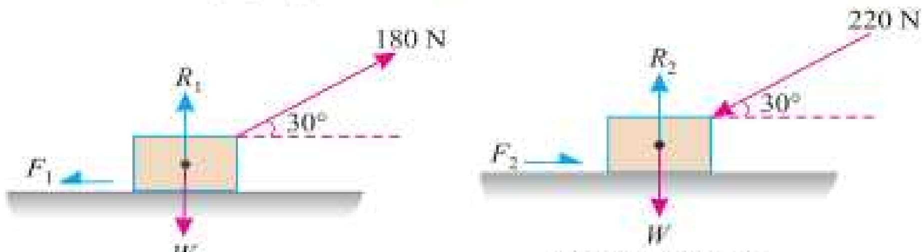
$$F_1 = 180 \cos 30° = 180 \times 0.866 = 155.9 \text{ N}$$

and now resolving the forces vertically,

$$R_1 = W - 180 \sin 30° = W - 180 \times 0.5 = W - 90 \text{ N}$$

We know that the force of friction (F_1),

$$155.9 = \mu R_1 = \mu (W - 90) \qquad ...(i)$$



(*a*) Pull of 180 N

(*b*) Pull of 220 N

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F_2) will act towards right as shown in Fig. 8.3 (*b*).

Resolving the forces horizontally,

$$F_2 = 220 \cos 30° = 220 \times 0.866 = 190.5 \text{ N}$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30° = W + 220 \times 0.5 = W + 110 \text{ N}$$

We know that the force of friction (F_2),

$$190.5 = \mu.R_2 = \mu (W + 110) \qquad ...(ii)$$

Dividing equation (*i*) by (*ii*)

$$\frac{155.9}{190.5} = \frac{\mu(W-90)}{\mu(W+110)} = \frac{W-90}{W+110}$$

$$W + 17\ 149 = 190.5\ W - 17\ 145$$

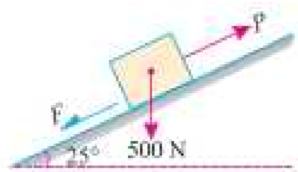
$$34.6\ W = 34\ 294$$

or $$W = \frac{34\ 294}{34.6} = 991.2 \text{ N} \quad \textbf{Ans.}$$

Now substituting the value of *W* in equation (*i*),

$$155.9 = \mu\ (991.2 - 90) = 901.2\ \mu$$

∴ $$\mu = \frac{155.9}{901.2} = 0.173 \quad \textbf{Ans.}$$



Determine the minimum and maximum values of P, for which the equilibrium can exist, if the angle of friction is 20°.

Solution. Given: Weight of the body (W) = 500 N ; Angle at which plane is inclined (α) = 25° and angle of friction (ϕ) = 20°.

Minimum value of P

We know that for the minimum value of *P*, the body is at the point of sliding downwards. We also know that when the body is at the point of sliding downwards, then the force

$$P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos\phi} = 500 \times \frac{\sin(25° - 20°)}{\cos 20°} \text{ N}$$

$$= 500 \times \frac{\sin 5°}{\cos 20°} = 500 \times \frac{0.0872}{0.9397} = 46.4 \text{ N} \quad \textbf{Ans.}$$

Maximum value of P

We know that for the maximum value of *P*, the body is at the point of sliding upwards. We also know that when the body is at the point of sliding upwards, then the force

$$P_2 = W \times \frac{\sin(\alpha + \phi)}{\cos\phi} = 500 \times \frac{\sin(25° + 20°)}{\cos 20°} \text{ N}$$

$$= 500 \times \frac{\sin 45°}{\cos 20°} = 500 \times \frac{0.7071}{0.9397} = 376.2 \text{ N} \quad \textbf{Ans.}$$