Perpendicular Axis Theorem

The moment of inertia of an area about an axis pependicular to its plane (polar moment of inertia) at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point O and lying in the plane of the area.

Referring to Fig. , if z-z is the axis normal to the plane of paper passing through point O, as per this theorem,

$$I_{zz} = I_{xx} + I_{yy}$$

The above theorem can be easily proved. Let us consider an elemental area dA at a distance r from O. Let the coordinates of dA be x and y. Then from definition:

$$I_{zz} = \Sigma r^2 dA$$
$$= \Sigma(x^2 + y^2)dA$$
$$= \Sigma x^2 dA + \Sigma y^2 dA$$
$$I_{zz} = I_{xx} + I_{yy}$$

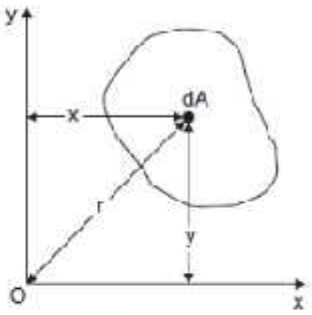

Fig.

Polar Moment of Inertia

Moment of inertia about an axis perpendicular to the plane of an area is known as *polar moment of inertia*. It may be denoted as J or I_{zz}. Thus, the moment of inertia about an axis perpendicular to the plane of the area at O in Fig. is called polar moment of inertia at point O, and is given by

$$I_{zz} = \Sigma r^2 dA$$

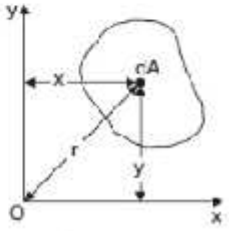

Fig. .