Parallel Axis Theorem

Moment of inertia about any axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between the two parallel axis. Referring to Fig. the above theorem means:

$$I_{AB} = I_{GG} + A\,y_c^2$$

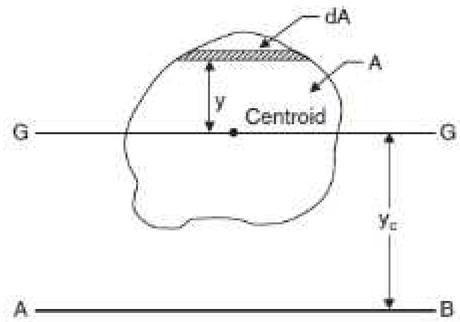

Fig.

where

I_{AB} = moment of inertia about axis AB

I_{GG} = moment of inertia about centroidal axis GG parallel to AB.

A = the area of the plane figure given and

y_c = the distance between the axis AB and the parallel centroidal axis GG.

Proof: Consider an elemental parallel strip dA at a distance y from the centroidal axis (Fig.).

Then,
$$I_{AB} = \Sigma(y + y_c)^2 dA$$
$$= \Sigma(y^2 + 2y\,y_c + y_c^2)dA$$
$$= \Sigma y^2 dA + \Sigma 2y\,y_c\,dA + \Sigma\,y_c^2 dA$$

Now,
$$\Sigma y^2 dA = \text{Moment of inertia about the axis } GG$$
$$= I_{GG}$$

$$\Sigma 2yy_c\,dA = 2y_c\,\Sigma y\,dA$$
$$= 2y_c A\,\frac{\Sigma ydA}{A}$$

In the above term $2y_c\,A$ is constant and $\frac{\Sigma ydA}{A}$ is the distance of centroid from the reference axis GG. Since GG is passing through the centroid itself $\frac{ydA}{A}$ is zero and hence the term $\Sigma 2yy_c dA$ is zero.

Now, the third term,
$$\Sigma y_c^2 dA = y_c^2 \Sigma dA$$
$$= Ay_c^2$$
$$\therefore \quad I_{AB} = I_{GG} + Ay_c^2$$

Note: The above equation cannot be applied to any two parallel axis. One of the axis (GG) must be centroidal axis only.