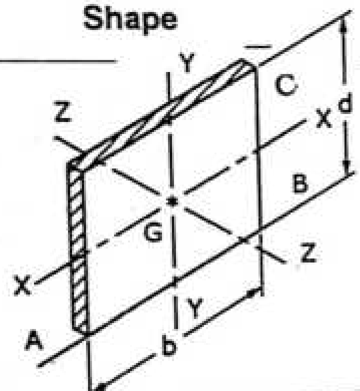
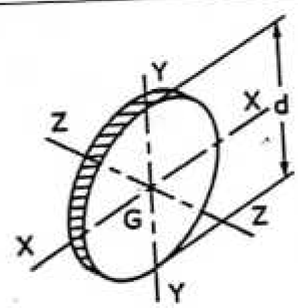
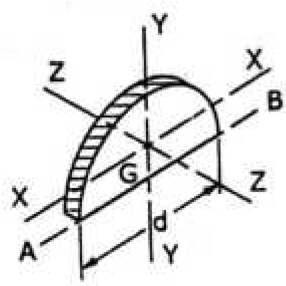
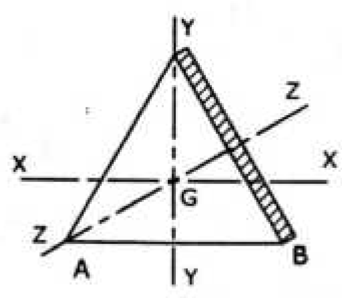
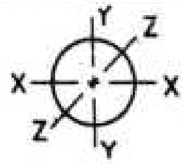
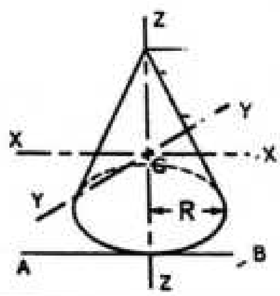
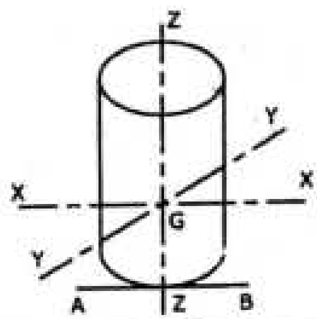
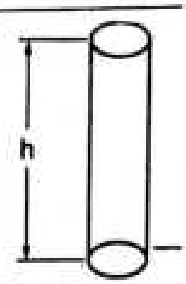
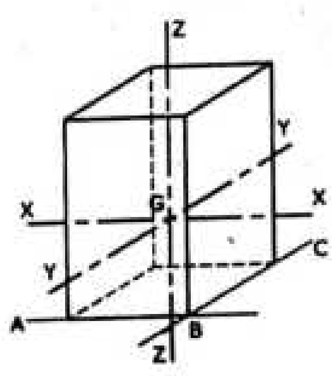
Table . Mass Moment of Inertia of Thin plates

Sl. No.	Name	Shape	Mass M.I
1	Rectangular Plate $M = \rho\, b\, d\, t$		$I_{xx} = Md^2/12$ $I_{yy} = Mb^2/12$ $I_{zz} = \frac{M}{12}(b^2 + d^2)$ $I_{AB} = Md^2/3$ $I_{BC} = Mb^2/3$
2	Circular disc $M = \rho\, t \frac{\pi d^2}{4}$ (or) $\rho\, t\, \pi r^2$		$I_{xx} = I_{yy}$ $= \frac{Md^2}{16} \left(\text{or } \frac{Mr^2}{4}\right)$ $I_{zz} = \frac{Md^2}{8} \left(\text{or } \frac{Mr^2}{2}\right)$
3.	Semi-circular disc $M = \rho t \pi \frac{d^2}{8}$ (or) $\rho\, t \frac{\pi r^2}{2}$		$I_{xx} = 0.0544\, Md^2$ (or) $0.2176\, Mr^2$ $I_{yy} = \frac{Md^2}{16} \left(\text{or } \frac{Mr^2}{4}\right)$ $I_{AB} = \frac{Md^2}{16} \left(\text{or } \frac{Mr^2}{4}\right)$
4.	Triangular plate $M = \frac{\rho\, t\, bh}{2}$		$I_{xx} = \frac{Mh^2}{18}$ $I_{yy} = \frac{Mb^2}{24}$ $I_{zz} = \frac{M}{18}\left(\frac{3b^2}{4} + h^2\right)$ $I_{AB} = \frac{Mh^2}{6}$

Sphere $M=\frac{4}{3}\pi R^3\rho$		$I_{xx} = I_{yy} = I_{zz}$ $= \frac{2}{5}MR^2$
Cone $M=\frac{1}{3}\pi R^2 h\rho$		$I_{xx} = I_{yy}$ $= \frac{3M}{20}\left[\frac{h^2}{4}+R^2\right]$ $I_{zz} = \frac{3}{10}MR^2$ $I_{AB} = \frac{3M}{5}\left(h^2+\frac{R^2}{4}\right)$
Cylinder $M=\pi R^2 h\rho$		$I_{xx} = I_{yy}$ $= \frac{M}{12}(3R^2+h^2)$ $I_{zz} = MR^2/2$ $I_{AB} = \frac{M}{12}(3R^2+4h^2)$
Slender rod $M = \pi h\rho$		$I_{xx} = I_{yy} = \frac{Mh^2}{12}$ $I_{zz} = 0$ $I_{AB} = \frac{Mh^2}{3}$
Prism $M = bdh\rho$		$I_{xx} = \frac{M}{12}(d^2+h^2)$ $I_{yy} = \frac{M}{12}(b^2+h^2)$ $I_{zz} = \frac{M}{12}(b^2+d^2)$ $I_{AB} = \frac{M}{12}(d^2+4h^2)$ $I_{BC} = \frac{M}{12}(b^2+4h^2)$

The machine part in Fig. (a) consists of a steel hemisphere joined to an aluminium cylinder into which a hole has been drilled. Determine the location of the center of mass. The mass densities for aluminum and steel are 2700 kg/m3 and 7850 kg/m3, respectively.

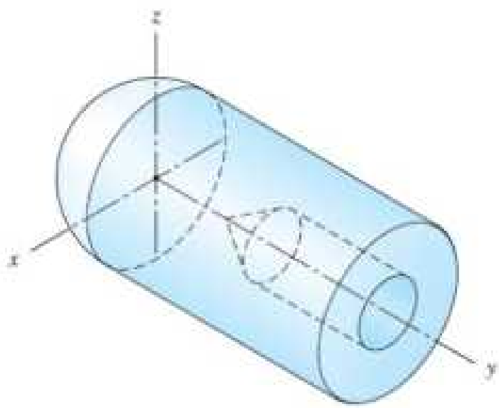


Fig. (a)

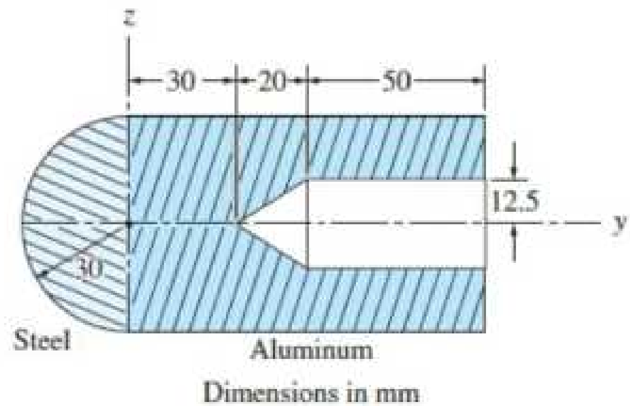
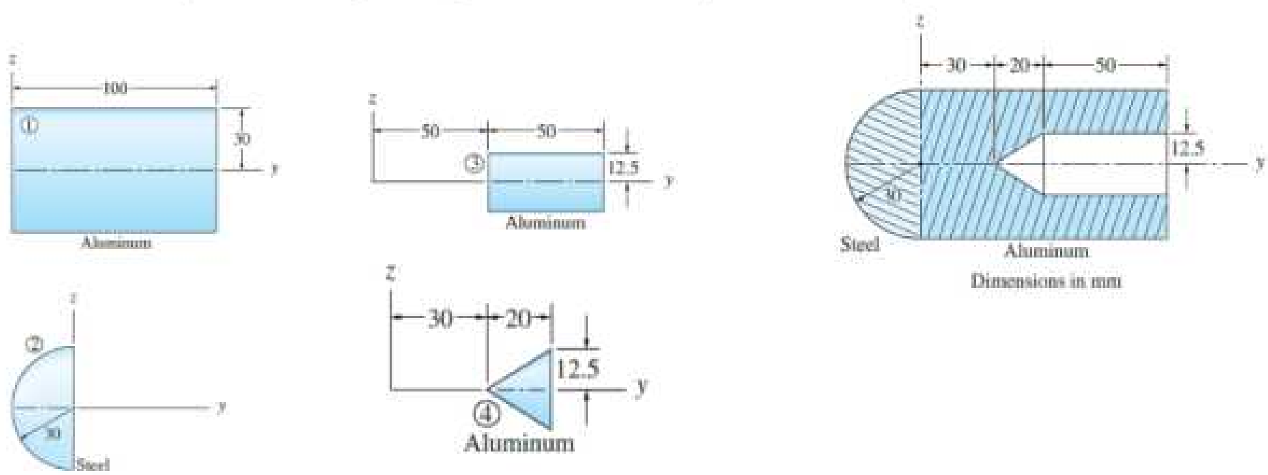


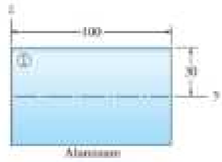
Fig. (b)

By symmetry, we note that $\bar{x} = \bar{z} = 0$. If the machine part were homogeneous, its center of mass would coincide with the centroid of the enclosing volume, and $\bar{y}$ could be determined using the method of composite volumes .
Because the machine part is not homogeneous, y must be determined by the method of composite bodies.



The part is composed of the four bodies shown in Fig. : the aluminum cylinder 1, plus the steel hemisphere 2, minus the aluminum cylinder 3, minus the aluminum cone 4. Because each of these bodies is homogeneous, each center of mass coincides with the centroid of the enclosing volume.

Aluminum Cylinder 1

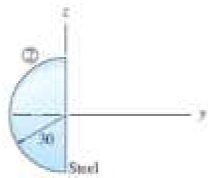


$$m = \rho V = \rho \pi R^2 h = 2700 \times \pi \times (0.030)^2 (0.100) = 0.7634\, kg$$

$$y_1 = \frac{100}{2} = 50\, mm\ [by\ symmetry]$$

$$my_1 = (0.7634)[50] = 38.17\, kg.mm$$

Steel Hemisphere 2

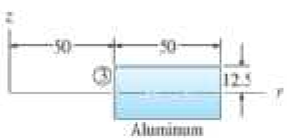


$$m = \rho V = \rho \frac{2\pi}{3} R^3 = 7850 \frac{2\pi}{3} (0.030)^3 = 0.4439\, kg$$

$$y_2 = -\frac{3}{8} R = -\frac{3}{8}(30) = -11.25\, mm$$

$$my_2 = (0.4439)(-11.25) = -4.994\, kg.mm$$

Aluminum Cylinder 3 (to be subtracted)

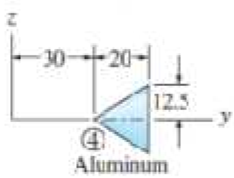


$$m = -\rho V = -\rho \pi R^2 h = -2700\pi (0.0125)^2 [0.050] = -0.06627\, kg$$

$$y_3 = 75\ mm\ \ [By\ symmetry]$$

$$my_3 = (-0.006627)(75) = -4.970\, kg.mm$$

Aluminum Cone 4 (to be subtracted)



$$m = -\rho V = -\rho \frac{\pi}{3} R^2 h = -2700 \frac{\pi}{3} [0.0125]^2 (0.020) = -0.008836\, kg$$

$$y_4 = 30 + \frac{3}{4}(20) = 45\, mm$$

$$my_4 = (-0.008836)(45) = -0.3976\, kg.\ mm$$

Totals

$$\sum m = [0.7634 + 0.4439 - 0.06627 - 0.008836] = 1.1322\, kg$$

$$\sum my_i = [38.17 - 4.994 - 4.970 - 0.3976] = 27.81\, kg.mm$$

Therefore, the coordinates of the mass center of the machine part are

$$\bar{x} = \bar{z} = 0;\ \bar{y} = \left[\frac{\sum m \bar{y}}{\sum m}\right] = \left[\frac{27.81}{1.1322}\right] = 24.6\, mm$$