Determination of Centroid

$$\bar{y} = \frac{\int y\,dA}{A}$$

$$\bar{x} = \frac{\int x\,dA}{A}$$

Centroid of a Triangle

Consider the triangle ABC of base width b and height h as shown in Fig. Let us locate the distance of centroid from the base. Let b_1 be the width of elemental strip of thickness dy at a distance y from the base. Since ΔAEF and ΔABC are similar triangles, we can write:

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \left(\frac{h-y}{h}\right)b = \left(1-\frac{y}{h}\right)b$$

$\therefore$ Area of the element

$$= dA = b_1\,dy$$

$$= \left(1-\frac{y}{h}\right)b\,dy$$

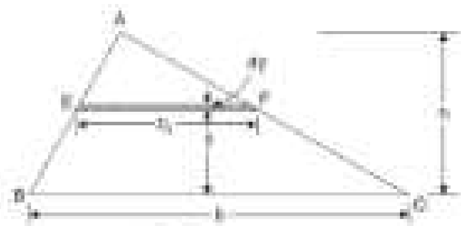

Fig.

Area of the triangle $\quad A = \frac{1}{2}bh$

$\therefore$ From eqn. 6.4

$$\bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{\int y\,dA}{A}$$

Now,

$$\int y\,dA = \int_0^h y\left(1-\frac{y}{h}\right)b\,dy$$

$$= \int_0^h \left(y - \frac{y^2}{h}\right)b\,dy$$

$$= b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h$$

$$= \frac{bh^2}{6}$$

$$\therefore \quad \bar{y} = \frac{\int y\,dA}{A} = \frac{bh^2}{6} \times \frac{1}{\frac{1}{2}bh}$$

$$\therefore \quad \bar{y} = \frac{h}{3}$$

Thus the centroid of a triangle is at a distance $\frac{h}{3}$ from the base (or $\frac{2h}{3}$ from the apex) of the triangle, where h is the height of the triangle.

Centroid of a Semicircle

Consider the semicircle of radius R as shown in Fig. Due to symmetry centroid must lie on y axis. Let its distance from diametral axis be $\bar{y}$. To find $\bar{y}$, consider an element at a distance r from the centre O of the semicircle, radial width being dr and bound by radii at θ and $\theta + d\theta$.

Area of element = $r\,d\theta\,dr$.

Its moment about diametral axis x is given by:

$$r d\theta \times dr \times r \sin\theta = r^2 \sin\theta\, dr\, d\theta$$

∴ Total moment of area about diametral axis,

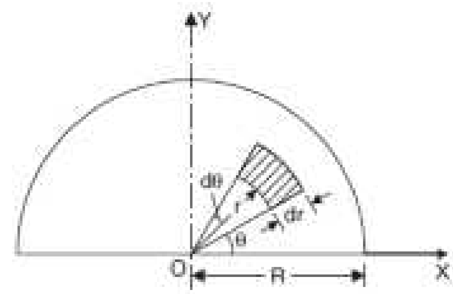

Fig. 4.26

$$\int_0^{\pi}\int_0^{R} r^2 \sin\theta\, dr\, d\theta = \int_0^{\pi}\left[\frac{r^3}{3}\right]_0^R \sin\theta\, d\theta$$

$$= \frac{R^3}{3}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{R^3}{3}[1+1] = \frac{2R^3}{3}$$

Area of semicircle $A = \frac{1}{2}\pi R^2$

$$\therefore \quad \bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{\frac{2R^3}{3}}{\frac{1}{2}\pi R^2}$$

$$= \frac{4R}{3\pi}$$

Thus, the centroid of the circle is at a distance $\frac{4R}{3\pi}$ from the diametral axis.