MODULE 6

DESIGN OF SHAFT

SHAFTS

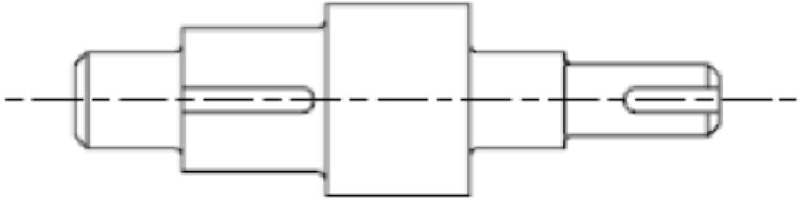
- Shaft is a rotating machine element which either receives power ,transmits power or both .
- Shafts are normally of circular cross section .
- Shaft may be solid or hollow depending upon the application.

Two types of shafts

1. Transmission shaft
2. Machine shaft :They are integral part of machine itself .

   Example – crankshaft in IC engines

Transmission shaft usually refers to a rotating machine element ,circular in cross section ,which supports transmission elements like gears, pulleys and sprockets and transmits power.



- Axle – non rotating member that carries no torque and is used to support rotating wheels, pulleys etc.
- Spindle –short rotating shaft used in machine tools.
- Countershaft –secondary shaft which is driven by main shaft supplies the power to machine component.

MATERIALS USED FOR SHAFTS

The material used for ordinary shafts is mild steel. When high strength is required, alloy steel such as nickel, nickel-chromium or chromium-vanadium steel is used.

Shafts are generally formed by hot rolling and finished to size by cold drawing or turning and grinding. Cold rolling produces a stronger shaft than hot rolling but has high residual stresses which cause distortion of the shaft when it is subjected to machining operations like slotting and milling.

Thus, most of the shafts after being hot rolled are turned and ground and are commonly referred to as T shafting and G shafting.

MAXIMUM PERMISSIBLE WORKING OR DESIGN STRESS

The following stresses are induced in the shafts:

- Shear stresses due to torque.
- Bending stresses (tensile or compressive) due to the elements mounted on the shaft (such as gears, pulleys, etc.)
- Stresses due to combined torsional and bending loads.

– *Working stresses for shaft* … **Table 3.5(b)/ Pg 57, DHB**

DESIGN OF SHAFT

1. Based on strength:
   a. Shaft subjected to torque (T) only
   b. Shaft subjected to bending moment (BM) only
   c. Shafts subjected to combined T and BM
   d. Shafts subjected to combined axial load (F), T and BM.
2. Based on rigidity:
   a. Torsional rigidity
   b. Lateral rigidity.

SHAFT DESIGN BASED ON STRENGTH

Shafts Subjected to Torque (T) Only

When the shaft is subjected to a twisting moment, then the diameter of the shaft may be obtained from the torsional equation and is given as follows:

a. **For hollow shaft:** Outside diameter of shaft is found as

$$d_o = \left[\frac{16T}{\pi\tau_d}\left(\frac{1}{1-K^4}\right)\right]^{1/3}$$ ... (Eq. 5.1) **3.4(a)/ Pg 50, DHB**

b. **For solid shaft:** Diameter of solid shaft is found by substituting $K = 0$ and $d_o = d$, in (Eq. 5.1)

i.e. $$d = \left[\frac{16T}{\pi\tau_d}\right]^{1/3}$$ ... (Eq. 5.2)

where d = diameter of solid shaft (mm)

d_o = outer diameter of hollow shaft (mm)

d_i = inner diameter of hollow shaft (mm)

a. **For hollow shaft:** Outside diameter of shaft is found as

$$d_o = \left[\frac{16T}{\pi\tau_d}\left(\frac{1}{1-K^4}\right)\right]^{1/3} \quad \text{... (Eq. 5.1) } \mathbf{3.4(a)/\ Pg\ 50,\ DHB}$$

b. **For solid shaft:** Diameter of solid shaft is found by substituting $K = 0$ and $d_o = d$, in (Eq. 5.1)

i.e.

$$d = \left[\frac{16T}{\pi\tau_d}\right]^{1/3} \quad \text{... (Eq. 5.2)}$$

$K = \dfrac{d_i}{d_o}$ = ratio of inner diameter to outer diameter of hollow shaft

T = torque transmitted (N·mm)

τ_d = design shear stress (MPa) ... **Tb. 3.5(b)/ Pg 57, DHB**

$= \dfrac{\tau_e B}{R K_s}$

R = reliability factor ... **Tb. 2.7/ Pg 33, DHB**

B = size factor ... **2.9(b)/ Pg 22, DHB**

τ_e = shear stress at elastic limit (MPa)

K_s = theoretical stress concentration factor for shear stress

Shafts Subjected to Bending Moment (BM) Only

When the shaft is subjected to a pure bending moment, then the maximum stress (tensile or compressive) may be obtained from by the bending equation and is given below.

a. **For hollow shaft:** Outside diameter of shaft is found as

$$d_o = \left[\frac{32M}{\pi\sigma_d}\left(\frac{1}{1-K^4}\right)\right]^{1/3} \quad \ldots \quad \textbf{3.4(b)/ Pg 50, DHB}$$

b. **For solid shaft:** Diameter of solid shaft is found by substituting $K = 0$ and $d_o = d$, in (Eq. 5.3)

i.e. $$d = \left[\frac{32M}{\pi\sigma_d}\right]^{1/3}$$

where σ_d = design stress (MPa) ... **Tb. 3.5(b)/ Pg 57, DHB**

$$= \frac{\sigma_e B}{RK_t}$$

σ_e = stress at elastic limit (MPa)

K_t = theoretical stress concentration factor for normal stress (tensile or bending).

Shafts Subjected to Combined T and BM

When the shaft is subjected to combined twisting moment and bending moment, the design is based on either the maximum normal stress theory or the maximum shear stress theory.

a. **For hollow shaft:**

i. According to maximum normal stress theory, outside diameter of shaft is found as:

$$d_o = \left[\frac{16}{\pi\sigma_{max}}\left(M + \sqrt{M^2 + T^2}\right)\left(\frac{1}{1 - K^4}\right)\right]^{1/3}$$

... **3.5(a)/ Pg 50, DHB**

ii. According to maximum shear stress theory, outside diameter of shaft is found as:

$$d_o = \left[\frac{16}{\pi\tau_{max}}\left(\sqrt{M^2 + T^2}\right)\left(\frac{1}{1 - K^4}\right)\right]^{1/3}$$

... **3.5(b)/ Pg 50, DHB**

Shaft subjected to combined T and BM (continuation)

b. **For solid shaft:** Diameter of solid shaft is found by substituting $K = 0$ and $d_o = d$, in the above equation

i. According to maximum normal stress theory, diameter of shaft is found as:

$$d = \left[\frac{16}{\pi \sigma_{max}} \left(M + \sqrt{M^2 + T^2} \right) \right]^{1/3} \quad \text{... (Eq. 5.7)}$$

ii. According to maximum shear stress theory, diameter of shaft is found as:

$$d = \left[\frac{16}{\pi \tau_{max}} \left(\sqrt{M^2 + T^2} \right) \right]^{1/3} \quad \text{... (Eq. 5.8)}$$

where σ_{max} = maximum normal stress (MPa)

τ_{max} = maximum shear stress (MPa)

Note:

- $$\sigma_{max} = \sigma_1 = \frac{\sigma_e \text{ or } \sigma_y}{n}$$

- $$\tau_{max} = \frac{\tau_e \text{ or } \tau_y}{n} = \frac{\sigma_e \text{ or } \sigma_y}{2n}$$

where σ_e or σ_y = yield strength of the material

n = factor of safety

$\tau_e = \frac{\sigma_e}{2}$, since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension

- The term $\left(M + \sqrt{M^2 + T^2}\right)$ is called the equivalent bending moment.
- The term $\left(\sqrt{M^2 + T^2}\right)$ is called the equivalent twisting moment.

SHAFT DESIGN BASED ON RIGIDITY

Here the shaft acts like a beam which deflects transversely and at the same time is a torsion bar that deflects torsionally.

Torsional Rigidity (Shafts as Torsion Bars)

A shaft is said to be torsionally rigid, if it does not twist too much under the action of external torque.

Examples: Cam shaft, machine tool spindles, etc.

The angle of twist or angular deformation may be obtained from the torsional equation and is given as follows:

a. **For solid shaft:** $$\theta = \frac{584TL}{Gd^4} \text{ (deg)}$$ **3.2/ Pg 50, DHB**

b. **For hollow shaft:** $$\theta = \frac{584TL}{G(d_o^4 - d_i^4)} \text{ (deg)}$$ **3.2/ Pg 50, DHB**

where

θ = angle of twist (deg)

T = torsional moment (N·mm)

L = length of the shaft (mm)

- The standard and widely used relation is that the deflection should not exceed 1° in a length equal to twenty times the diameter of the shaft.
- The permissible amount of twist for machine tool spindles, e.g. cam shaft should not exceed 0.25° per meter length.
- For line shafts 2.5° to 3° per meter length may be used as the limiting value.

5.8.2 Lateral Rigidity (Shafts as Beams)

A shaft is said to be laterally rigid, if it does not deflect too much under the action of external forces and bending moment.

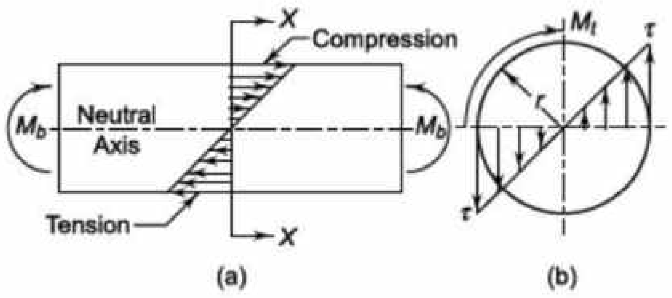
The lateral rigidity is important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in 'Strength of Materials'. But when the shaft is of variable cross-section, then the lateral deflection may be determined either by the graphical method or from the fundamental equation for the elastic curve of a beam as

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

COMPARISON BETWEEN SOLID SHAFT AND HOLLOW SHAFT

OBSERVATIONS

- Torsional shear stress and and bending stress are zero at shaft centre and negligibly small in the vicinity of shaft centre, where radius is small.
- As the radius increases, the resisting stresses due to external bending and torsional moments increase.
- Therefore outer fibres are more effective in resisting applied moments.
- In hollow shaft, the material at centre is removed and spread at large radius.
- Therefore hollow shafts are stronger than solid shafts having same weight.



(a) Distribution of Bending Stresses
(b) Distribution of Torsional Shear Stress

DISADVANTAGES OF HOLLOW SHAFT

1. Hollow shaft is costlier than solid shaft.
2. The diameter of hollow shaft is more than that of solid shaft and requires more space.

ADVANTAGES OF HOLLOW SHAFT OVER SOLID SHAFT

1. The strength of hollow shaft is more than that of solid shaft with same weight.
2. The stiffness of hollow shaft is more than that of solid shaft with same weight.
3. The natural frequency of hollow shaft is higher than that of solid shaft with same weight.

Compare the weight, strength and stiffness of a hollow shaft with that of a solid shaft having same material and length. OR

Prove that a hollow shaft is stronger and stiffer than a solid shaft of same length, weight and material.

VTU – June/ July – 13 Marks

Solution: $L = L_S = L_H$.

Let d = diameter of solid shaft

d_o = outer diameter of hollow shaft

d_i = inner diameter of hollow shaft

$K = \frac{d_i}{d_o}$ = ratio of inner diameter to outer diameter of hollow shaft

L = length of the shaft

τ_d = shear stress of shaft material

ρ = density of shaft material

W_H, W_S = weight of hollow shaft and solid shaft respectively

T_H, T_S = torque transmitted by hollow shaft and solid shaft respectively

S_H, S_S = stiffness of hollow shaft and solid shaft respectively.

a. **Comparison of weight:** We know that

$$\text{Weight} = \text{density} \times \text{volume} = \text{density} \times (\text{area} \times \text{length})$$

$$W = \rho AL$$

Thus for a solid shaft,

$$W_S = \rho A_s L = \rho L \frac{\pi d^2}{4} \quad \text{... Eq. (i)}$$

and for a hollow shaft,

$$W_H = \rho A_H L = \rho L \frac{\pi(d_o^2 - d_i^2)}{4} \quad \text{... Eq. (ii)}$$

Dividing Eq. (ii) by Eq. (i) yields

$$\frac{W_H}{W_S} = \frac{\rho L[\pi(d_o^2 - d_i^2)/4]}{\rho L(\pi d^2/4)} = \frac{d_o^2 - d_i^2}{d^2}$$

$$\therefore \quad \frac{W_H}{W_S} = \frac{d_o^2(1 - K^2)}{d^2} \quad \text{... Eq. (iii)}$$

b. **Comparison of strength:** For a hollow shaft,

$$d_o = \left[\frac{16T_H}{\pi\tau_d}\left(\frac{1}{1-K^4}\right)\right]^{1/3} \qquad \text{... 3.4(a)/ Pg 50, DHB}$$

$$T_H = \left(\frac{\pi}{16}\right) d_o^3 \tau_d (1-K^4) \qquad \text{... Eq. (iv)}$$

For a solid shaft substituting $K = 0$ and $d_o = d$, in Eq. (iv), we have

$$T_s = \left(\frac{\pi}{16}\right) d^3 \tau_d \qquad \text{... Eq. (v)}$$

Dividing Eq. (iv) by Eq. (v) yields

$$\frac{T_H}{T_s} = \frac{(\pi/16) d_o^3 \tau_d (1-K^4)}{(\pi/16) d^3 \tau_d}$$

$$\therefore \quad \frac{T_H}{T_s} = \frac{d_o^3 (1-K^4)}{d^3} \qquad \text{... Eq. (vi)}$$

Since the material is same, equating Eqs. (i) and (ii), we have

$$\rho L[\pi(d_o^2 - d_i^2)/4] = \rho L(\pi d^2/4)$$

$$d_o^2(1 - K^2) = d^2$$

$$d_o\sqrt{(1 - K^2)} = d \qquad \text{... Eq. (iii-a)}$$

Substituting Eq. (iii-a) in Eq. (vi), we have

$$\frac{T_H}{T_S} = \frac{d_o^3(1 - K^4)}{\left[d_o\sqrt{(1 - K^2)}\right]^3} = \frac{(1 - K^4)}{\left[\sqrt{(1 - K^2)}\right]^3} = \frac{(1 + K^2)(1 - K^2)}{(1 - K^2)\sqrt{(1 - K^2)}}$$

$$\therefore \quad \frac{T_H}{T_S} = \frac{(1 + K^2)}{\sqrt{(1 - K^2)}} \qquad \text{... Eq. (vi-a)}$$

c. **Comparison of stiffness:** Stiffness is defined as

$$S = \frac{T}{\theta} \quad \text{... Eq. (vii)}$$

We know that $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$ **... 1.3(b)/ Pg 3, DHB**

$$S = \frac{T}{\theta} = \frac{GJ}{L}$$

For a solid shaft, $J = \left[\frac{\pi d^4}{32}\right]$ and for a hollow shaft $J = \left[\frac{\pi(d_o^4 - d_i^4)}{32}\right]$

... Tb. 1.3(b)/ Pg 14, DHB

For a hollow shaft,

$$S_H = \frac{G}{L}\left[\frac{\pi(d_o^4 - d_i^4)}{32}\right] = \frac{G}{L}\left[\frac{\pi d_o^4(1 - K^4)}{32}\right] \quad \text{... Eq. (viii)}$$

For a solid shaft substituting $K = 0$ and $d_o = d$ in Eq. (viii), we have

$$S_S = \frac{G}{L}\left[\frac{\pi d^4}{32}\right] \qquad \text{... Eq. (ix)}$$

Dividing Eq. (viii) by Eq. (ix) yields

$$\frac{S_H}{S_S} = \frac{(G/L) \times [\pi d_o^4 (1 - K^4)/32]}{(G/L) \times (\pi d^4/32)}$$

$$\therefore \quad \frac{S_H}{S_S} = \frac{d_o^4 (1 - K^4)}{d^4} \qquad \text{... Eq. (x)}$$

Substituting Eq. (iii-a) in Eq. (vi), we have

$$\frac{S_H}{S_S} = \frac{d_o^4 (1 - K^4)}{\left[d_o \sqrt{(1 - K^2)}\right]^4} = \frac{(1 - K^4)}{\left[\sqrt{(1 - K^2)}\right]^4} = \frac{(1 + K^2)(1 - K^2)}{(1 - K^2)^2}$$

$$\therefore \quad \frac{S_H}{S_S} = \frac{(1 + K^2)}{(1 - K^2)} \qquad \text{... Eq. (x-a)}$$