THIN CYLINDRICAL SHELLS

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Definition of cylindrical shell, Definition of longitudinal and hoop stress, Derive the expression for longitudinal, hoop and shear stress for seamless and seam shells. Longitudinal, hoop and volumetric strain and change in dimensions of a seamless shell subjected to internal fluid pressure, Design of thin cylindrical shells.

INTRODUCTION

This unit presents the analysis of thin and thick cylindrical shells subjected to fluid pressure. Steam boilers, reservoirs, reactors, nuclear containers tanks, working chambers of engines, etc. are the common examples. In this unit, stresses and strains induced in the walls of the cylinder will be found out based on the geometry of the shell and equilibrium of the forces involved. We shall begin by defining a thin cylinder identifying the assumptions made in the analysis. After finding the stresses in the material of the cylinder, strains will be calculated. Stresses in wire wound pipes will be considered.

We shall then see the limitations for treating a shell to be thin and look for the differences in the behaviour of a thick shell as against a thin shell. The stresses in a thick cylinder will be obtained based on a standard method involving certain assumptions.

Objectives

After studying this unit, you should be able to

- define a cylindrical shell and distinguish between thin and thick cylinders,

- identify the assumptions involved in the analysis of a thin cylinder,

- determine the stresses in a thin cylinder,

- find the strains and deformation in thin cylinder,

- find stresses in a wire wound pipe,

- make out the assumptions for analyzing a thick cylinder,

- derive the standard expressions for stresses in thick shell, and

- find the stress distribution across a compound cylinder.

THIN CYLINDERS

In this section, we shall derive expressions for the stresses and strains in thin cylinders and use them for working out related problems.

Stresses in Shafts and Shells

Assumptions

The following assumptions are made in order to derive the expressions for the stresses and strains in thin cylinders :

(a) The diameter of the cylinder is more than 20 times the thickness of the shell.

(b) The stresses are uniformly distributed through the thickness of the wall.

(c) The ends of the cylindrical shall are not supported from sides.

(d) The weight of the cylinder and that of the fluid contained inside are not taken into account.

(e) The atmosphere pressure is taken as the reference pressure.

Stresses

Consider a thin seamless cylindrical shell of nominal diameter d, and shell thickness which is containing some fluid at an internal pressure of p. The two ends of the cylinder are closed with walls perpendicular to the shell (Figure 3.1)
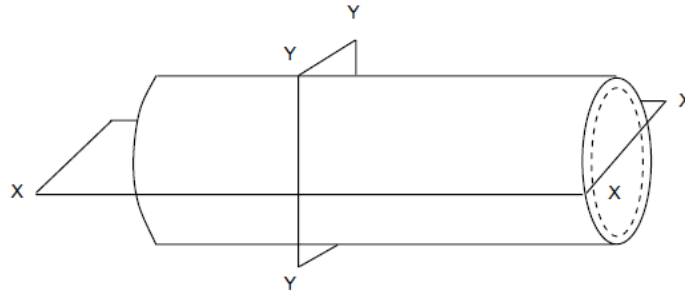

Figure 3.1

We shall consider a vertical plane YY which cuts the cylinder anywhere along the length. We shall consider the left portion of the cylinder and see the nature and magnitude of the internal stresses acting on the section. The stresses will be as shown in Figure 3.2.
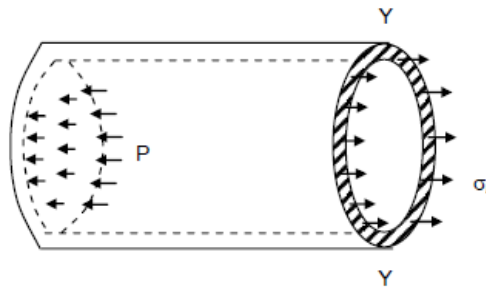

Figure 3.2

It can be seen that the internal stresses acts over the shaded annular portion of the cylinder, which is the wall area exposed due to the cutting the by plane Y-Y. The direction of these internal stresses will be clearly longitudinal as the exposed areas is in the vertical plane. In addition it can also be seen that these stresses develop owing to the unbalanced horizontal force acting on the left vertical wall of the cylinder, since the pressure acting on the curved walls balance each other. Thus the stresses will be tensile in nature so as to maintain equilibrium. The unbalanced force acting on the left wall is called the bursting force and the force due to internal stresses acting on the wall thickness of the cylinder is called the resisting force.

We can write the expressions for the bursting and resisting forces as below.

It can be clearly seen that the bursting force is caused due to the internal pressure acting on the vertical circular wall of nominal diameter d.

Hence, the bursting force $= p \times \dfrac{\pi d^2}{4}$.

The resisting force is generated by the longitudinal tensile stress σ_l acting on the vertical area exposed, of thickness t and diameter d.

Hence, the resisting force $= \sigma_l \times \pi\, dt$

For equilibrium, the resisting force should be equal to the bursting force.

Thus, we get $\qquad \sigma_l \times \pi\, dt = p \times \dfrac{\pi d^2}{4}$

or $\qquad\qquad\qquad \sigma_l = \dfrac{pd}{4t}$

This internal stress is called **longitudinal stress**, indicating the direction in which it is acting and its nature will be tensile.

We shall now consider a horizontal diametrical plane XX which cuts the cylindrical shell in two halves. The stresses have been shown in Figure 3.3.
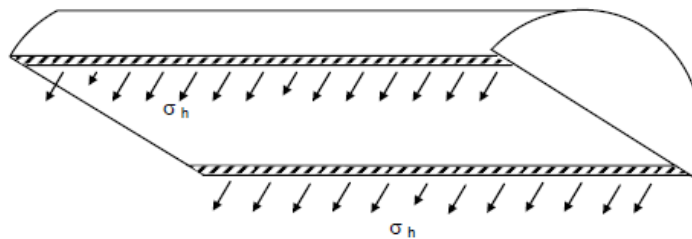

Figure 3.3

We shall consider the equilibrium of the top portion of the cylinder. The horizontal pressure acting on the two end walls will balance each other and hence, there will be no longitudinal stress in the wall of the cylinder. The pressure acting on the curved surface of the shell creates the bursting force for this free body diagram which should be balanced by the reacting force caused by the development of internal stresses along with wall thickness of the cylinder. Since the plane XX is horizontal, the cylinder's wall exposed by the cutting, will also be horizontal and it will be in the form of two rectangular strips, along with longitudinal direction of length l and thickness t. Thus, the stress acting on this strip will be in the vertical direction.

The pressure acting on the curved surface acts normal to the surface and hence, it will be acting in different direction at different points along the surface. The vertical component of the bursting force is obtained by considering an element at angle θ to the horizontal which is subtended by an angle $d\theta$ at the centre as shown in Figure 3.3. The length of this element may be the total length of the cylinder itself. The revolution of the elemental force in the vertical direction is shown in Figure 3.4.
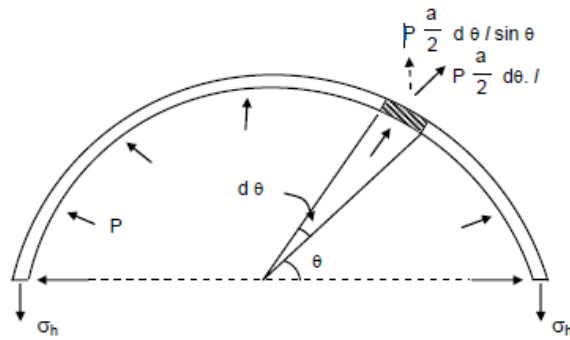
Figure 3.4

The radial force acting on the element,

$$p \times \frac{d}{2} \times d\theta \times l$$

The vertical component of the elemental force,

$$p \times \frac{d}{2} \times d\theta \times l \sin \theta$$

Hence, the total vertical component of the bursting force in the top portion of the cylindrical shell will be

$$P_v = \int_0^\pi p \times \frac{d}{2} \times l \sin \theta \, d\theta$$

$$= p \cdot d \cdot l$$

Since this resultant vertical component is acting upwards, the internal stress on the horizontal strip of dimensions l and t will be acting downwards indicating that the nature of this stress is tensile.

The resisting force $= \sigma_h \times l \times t \times 2$

Since the equilibrium is maintained by the action of bursting and resisting forces only, they must be equal.

$$\sigma_h \times l \times t \times 2 = P \, d \, l$$

$$\sigma_h = \frac{pd}{2t}$$

This stress is called the **hoop stress** acting in the circumferential direction and it will be tensile in nature.   .

It can be stated at this stage that the internal stresses in a thin cylindrical shell are acting in the longitudinal and circumferential directions and hence they are named as longitudinal stress and hoop stress and both are tensile in nature. Let us see some example for finding stresses in thin cylindrical shells.

Example 3.1

A cylindrical boiler is 2.5 m in diameter and 20 mm in thickness and it carries steam at a pressure of 1.0 N/mm². Find the stresses in the shell.

Solution

Solution

Diameter of the shell, $d = 2.5$ m $= 2500$ mm.

Thickness of the shell, $t = 20$ mm.

Internal pressure, $p = 1.0$ N/mm^2.

∴ Longitudinal stress $\dfrac{pd}{4t} = \dfrac{1.0 \times 2500}{4 \times 20} = 31.25$ N/mm^2 (tensile)

∴ Hoop stress $\dfrac{pd}{2t} = \dfrac{1.0 \times 2500}{2 \times 20} = 62.50$ N/mm^2 (tensile)

Example 3.2

A thin cylindrical vessel of 2 m diameter and 4 m length contains a particular gas at a pressure of 1.65 N/mm^2. If the permissible tensile stress of the material of the shell is 150 N/mm^2, find the maximum thickness required.

Solution

In a thin cylindrical shell, stress will be higher, since it is double that of longitudinal stress. Hence, maximum stress is reached in the circumferential direction.

Diameter of the shell, $d = 2$ m $= 2000$ mm.

Internal pressure, $p = 1.65$ N/mm^2.

Permissible tensile stress $= 150$ N/mm^2.

If thickness required is t, then

$$\frac{pd}{t} = 150$$

$$\frac{1.65 \times 2000}{2 \times t} = 150$$

$$t = 11 \text{ mm}$$

Thus, minimum thickness required is 11 mm.

Example 3.3

A cylindrical compressed air drum is 2 m in diameter with plates 12.5 mm thick. The efficiencies of the longitudinal (σ_l) and circumferential (σ_c) joints are 85% and 45% respectively. If the tensile stress in the plating is to be limited to 100 MN/m^2, find the maximum safe air pressure.

Solution

The efficiency of the joint influences the stresses induced. For a seamless shell (with no joints), efficiency is 100%. When the efficiency of joint is less than 100%, the stresses are increased accordingly.

Hence, if η is the efficiency of a joint in the longitudinal direction, influencing the hoop stress, then the stress will be given as,

$$\sigma_h = \frac{pd}{4t \times \eta_l}$$

Here, the diameter $d = 2$ m $= 2000$ mm.

Thickness, $t = 12.5$ mm.

Limiting tensile stress $= 100$ MN/m$^2 = 100$ N/mm^2.

Considering the circumferential joint which influences the longitudinal stress,

$$\frac{pd}{4t \times \eta_l} = 100$$

$$\frac{p \times 2000}{4 \times 12.5 \times 0.45} = 100$$

$$p = 1.125 \text{ N/mm}^2$$

Similarly, considering the longitudinal joint which influences the hoop stresses,

$$\frac{pd}{2t \times \eta_c} = 100$$

$$\frac{p \times 2000}{2 \times 12.5 \times 0.85} = 100$$

$$p = 1.063 \text{ N/mm}^2$$

Evidently, safe pressure is governed by hoop stress.

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SAQ 1

(a) A closed cylindrical vessel with plane ends is made of steel plates 3 mm thick, the internal dimensions being 600 mm and 250 mm for length and diameter respectively. Determine the stresses in the material of the vessel, when the internal pressure is 2.0 N/mm^2.

(b) A copper tube of 50 mm internal diameter, 1.2 m long and 12.5 mm thickness has a closed ends and filled with a gas under pressure. If the safe tensile stress for copper is 60 N/mm^2, find the maximum pressure that can be applied.

(c) A boiler of 2 m diameter is to be made from mild steel plates. Taking the efficiencies of longitudinal and circumferential joints as 75% and 50% respectively, find the thickness of plate required if the internal pressure, that will develop in the boiler is 1.5 N/mm^2 and the permissible tensile stress for mild steel is 150 N/mm^2.

Strains

At any point in a thin cylindrical shell with an internal pressure p, we have obtained the expressions for stresses along longitudinal direction and circumferential direction. In order to obtain the strain along any direction, we have to see the state of stress at any point. In the three mutually perpendicular directions, the stresses are as follows :

Stress along the radial direction $= p$ (compressive)

Stress along the circumferential direction $= \sigma_h = \dfrac{pd}{2t}$ (tensile)

Stress along the longitudinal direction $= \sigma_l = \dfrac{pd}{4t}$ (tensile)

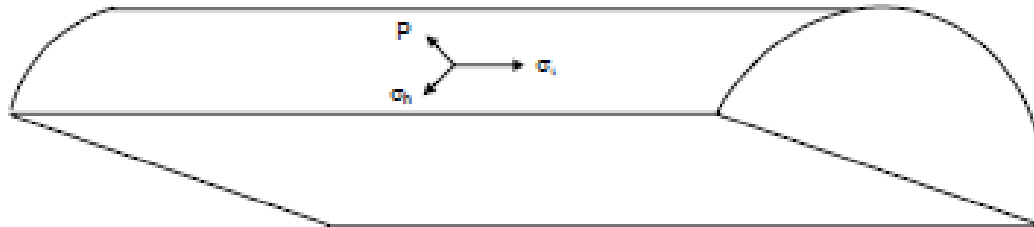The state of stress is shown in Figure 11.5.

Figure 11.5

These are the principal stresses acting at the point considered. However, when $\dfrac{d}{t}$ is very large making the shell thin, the radial pressure p will be very small compared to the longitudinal and hoop stresses. Hence, this compressive stress can be neglected at any point for the purpose of working out the strain, which is going to be still smaller. This assumption leaves only the two tensile stresses at any point, mutually perpendicular to each other. If E is the Young's modulus of the material of the shell and v, its Poisson's ratio, then the expression for the strains in the two direction are obtained as follows :

Longitudinal strain $\varepsilon_l = \dfrac{\sigma_l}{E} - v\,\dfrac{\sigma_h}{E}$

$$= \dfrac{pd}{4t\,E} - v\,\dfrac{pd}{2t\,E}$$

$$= \dfrac{pd}{4t\,E}\,(1 - 2v)$$

Hoop strain $\varepsilon_h = \dfrac{\sigma_h}{E} - v\,\dfrac{\sigma_l}{E}$

$$= \dfrac{pd}{2t\,E} - v\,\dfrac{pd}{4t\,E}$$

$$= \dfrac{pd}{4t\,E}\,(2 - v)$$

Using these expressions, we may proceed to obtain the changes in length and diameter of the cylinder.

Change in length = Longitudinal strain X Original length = $\varepsilon_l \times l$

However, it can be noted that since the circumference is a constant product of diameter, i.e. $C = \Pi\, d$, the diametrical strain will be the same as the circumferential strain. Thus, change in diameter = Hoop strain x Original diameter.

Change in Volume of Cylinder

To find the volumetric change (ratio of change in volume to be original volume) the expression for the volume of the cylinder will be considered.

Volume is given by,

$$V = \frac{\pi d^2}{4} \times l$$

On differentiating, we get change in volume,

$$\delta V = \frac{\pi d^2}{4} \delta l + \frac{\pi}{4} \times l \times 2d \times \delta d$$

Hence, $\dfrac{\text{Ratio change in volume}}{\text{Original volume}}$

$$= \frac{\delta V}{V}$$

(We do not to define this ratio as volumetric strain, because the volume of the material that makes cylinder is not the internal volume of the cylinder. This volume is the volume of any substance contained in the space of the cylinder, and $\dfrac{\delta V}{V}$ with mean the volumetric strain of that substance. This strain then will be related to pressure and bulk modulus of the substance.)

$$= \frac{\dfrac{\pi d^2}{4} . \delta l + \dfrac{\pi}{4} . l . 2d . \delta d}{\dfrac{\pi d^2}{4} . l}$$

$$= \frac{\delta l}{l} + 2\frac{\delta d}{d} = \varepsilon_l + 2\varepsilon_h$$

Thus, $\qquad\qquad \varepsilon_v = \varepsilon_l + 2\varepsilon_h$

$$= \frac{pd}{4tE}(1-2v) + 2\frac{pd}{4tE}(2-v)$$

$$= \frac{pd}{4tE}(1-2v+4-2v)$$

$$= \frac{pd}{4tE}(5-4v)$$

Thus, the volumetric strain is obtained as the sum of longitudinal strain and twice the hoop strains. In terms of the pressure, diameter and thickness volumetric strain can be expressed as,

$$\varepsilon_v = \frac{pd}{4tE}(5-4v)$$

We shall now see some examples for finding out the strains and deformations in thin cylindrical shells.

Example 3.4

A cylindrical shell, 0.8 m in a diameter and 3 m long is having 10 mm wall thickness. If the shell is subjected to an internal pressure of 2.5 N/mm², determine change in diameter, change in length, and change in volume. Take E = 200 GPa and Poisson's ratio = 0.25.

Solution

Diameter of the shell, d = 0.8 m = 800 mm.

Thickness of the shell, t = 10 mm.

Internal pressure, p = 2.5 N/mm².

Hoop stress, $\qquad \sigma_h = \dfrac{pd}{2t}$

$$= \frac{2.5 \times 800}{2 \times 10} = 100 \text{ N/mm}^2$$

Longitudinal stress, $\sigma_l = \dfrac{pd}{4t}$

$$= \frac{2.5 \times 800}{4 \times 10} = 50 \text{ N/mm}^2$$

Hoop strain, $\varepsilon_n = \dfrac{1}{E}(\sigma_h - \varepsilon\sigma_l) = \dfrac{1}{2 \times 10^5}(100 - 0.25 \times 50)$

$$= 4.375 \times 10^{-4}$$

Longitudinal strain, $\varepsilon_l = \dfrac{1}{E}(\sigma_l - v\sigma_h) = \dfrac{1}{2 \times 10^5}(50 - 0.25 \times 100)$

$$= 1.25 \times 10^{-4}$$

$$\frac{\text{Change in volume}}{\text{Original volume}} = 2\varepsilon_h + \varepsilon_l = 2 \times 4.375 \times 10^{-4} + 1.25 \times 10^{-4}$$

$$= 10 \times 10^{-4} = 10^{-3}$$

Increase in diameter = Hoop strain × Original diameter

$$4.375 \times 10^{-4} \times 800 = 0.35 \text{ mm}$$

Increase in length = Longitudinal strain × Original length

$$1.25 \times 10^{-4} \times 3000 = 0.375 \text{ mm}$$

Increase in internal volume = $= \dfrac{\delta v}{v} \times$ Original length

Original volume $= \dfrac{\pi d^2}{4} \times l = \dfrac{\pi}{4} \times 800^2 \times 3000 = 1507 \times 10^6 \text{ mm}^3$

Increase in volume $= 10^{-3} \times 1507 \times 10^6 = 1507 \times 10^3 \text{ mm}^3$

Example 3.5

A copper tube of 50 mm diameter and 1200 mm length has a thickness of 1.2 mm with closed ends. It is filled with water at atmospheric pressure. Find the increase in pressure when an additional volume of 32 cc of water is pumped into the tube. Take E for copper = 100 GPa, Poisson's ratio = 0.3 and K for water= 2000 N/mm^2.

Solution

The additional quantity of water pumped in accounts for the change in volume of the shell as well as the compression of the water in it.

Hence, if p is the increase in pressure in water, then

,

$$\text{Hoop stress} = \frac{pd}{2t} = \frac{p \times 50}{2 \times 1.2} = 20.83\,p$$

$$\text{Longitudinal stress} = \frac{pd}{4t} = \frac{p \times 50}{4 \times 1.2} = 10.42\,p$$

$$\text{Hoop strain} = \frac{1}{E}[\sigma_h - v\sigma_l] = \frac{1}{E}[20.83\,p - 0.3 \times 10.42] = \frac{17.7\,p}{E}$$

$$\text{Longitudinal strain} = \frac{1}{E}[\sigma_l - v\sigma_h] = \frac{1}{E}[10.42\,p - 0.3 \times 20.83] = \frac{4.17\,p}{E}$$

$$\text{Volumetric strain of water} = 2\varepsilon_h + \varepsilon_l = \frac{2 \times 17.7\,p}{E} + \frac{4.17\,p}{E} = \frac{39.57\,p}{E} \text{ (increase)}$$

Due to compression in water, its volumetric strain $\dfrac{p}{K}$ (increase).

$\therefore$ Addition volume pumped

= Increase in volume of cylinder + Decrease in volume of water

i.e.
$$32 \times 10^3 = \frac{39.57\,p}{E} \times V + \frac{p}{K} V$$

$$= \left(\frac{39.57}{1 \times 10^5} + \frac{1}{2000} \right) \times p \times \frac{\pi d^2}{4} \times l$$

$$= 87.57 \times 10^{-5} \times p \times \frac{\pi}{4} \times 50^2 \times 1200$$

$$p = 15.51 \text{ N/mm}^2$$

SAQ 2

1. A cylindrical shell 3 m long and 1 m in diameter has 10 mm of metal thickness. Calculate the changes in dimensions and the volume of the cylinder when it is subjected to an internal pressure of 1.5 N/mm^2. Take E = 204 GPa and Poisson's ratio = 0.3.

2. A copper cylinder 900 mm lon g, 400 mm in diameter and 6 mm thick, with flat ends, is initially full of oil at atmospheric pressure. Calculate the volume of the oil which must be pumped into the cylinder in order to raise the pressure to 5 above atmospheric pressure. Take Young's modulus of copper= 1 x 10^5 N/mm^2, Poisson's ratio = 0.33 and Bulk modulus of oil = 2600 N/mm^2.

3. A thin cylindrical shell is subjected to internal fluid pressure, the ends being closed by two water tight pistons attached to a common piston rod and flanged ends. Find the increase in diameter in the two cases for the following data : Diameter = 200 mm; thickness = 5 mm; internal pressure = 3.5 N/mm^2; Young's Modulus = 210 GPa and Poisson's ratio = 0.3.

Wire Wound Pipes

In order to resist large internal pressures in a thin cylinder, it will be wound closely with a wire. These are called wire wound pipes. The tension in the wire, binding the cylinder will create initial compressive stresses in the pipe before applying the internal pressure. Thus, on application of the internal pressure, the cylinder and the wire jointly resisted the bursting force. The final stresses in the wire or the cylinder will be the sum of the initial stresses due to winding and the stresses induced due to the application of internal pressure. The relation between the stresses in the wire and cylinder are obtained by considering the strain at the common surface of the shell and the wire which should be the same for both. This is illustrated in the examples given below.

Example 3.6

A cast iron pipe of 200 mm internal diameter and 13 mm metal thickness is closely wound with a layer of 5 mm diameter steel wire under a tensile stress of 40 N/mm^2. Calculate the stresses, set up in the pipe and the wire, if water under a pressure of 2 N/mm^2 is admitted into the pipe. For cast iron, Young's Modulus is 101 GPa and Poisson's ratio is 0.3. For steel, Young's Modulus is 204 GPa.

Solution

Consider 10 mm length of the pipe. Let the initial stresses in the wire and the pipe be σ_w and σ_p, respectively, as shown in Figure 11.6.
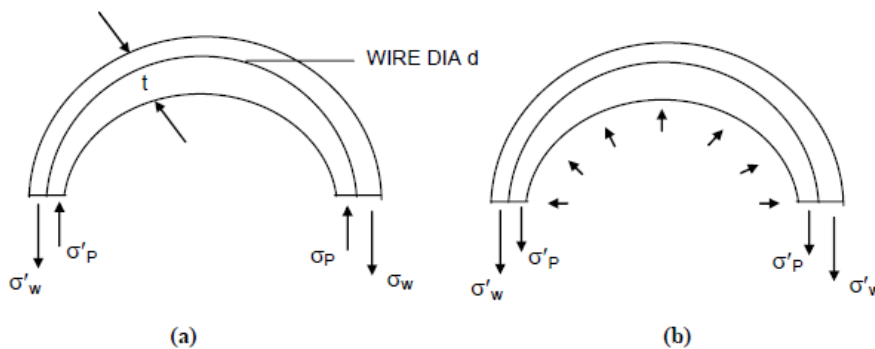

(a)   (b)

Figure 11.6

For Equilibrium

Tensile force in the wire = Compressive force in the pipe.

For considered length of 10 mm, two cross-sections of wire will be contributing the tensile forces since the diameter of the wire is 5 mm.

Here, $\sigma_w = 40$ N/mm^2.

∴ $2 \times \dfrac{10}{5} \times \dfrac{\pi}{4} \times 5^2 \times 40 = \sigma_p \times 2 \times 13 \times 10$

$\sigma_p = 12.8$ N/mm^2

After water is admitted,

let the stresses in wire and pipe be σ'_w and σ'_p, respectively.

Bursting force per 10 mm length $= p \cdot d \cdot l$

$$= 3 \times 200 \times 10 = 6000 \text{ N}$$

Resisting force due to pipe $= \sigma'_p \times 2 \times t \times l$

$$= \sigma'_p \times 2 \times 13 \times 10$$

$$= 260 \, \sigma'_p$$

Resisting force due to wire $= \sigma'_w \times 2 \times \dfrac{10}{5} \times \dfrac{\pi}{4} \times 5^2 = 78.54 \, \sigma'_w$

Since resisting force should be equal to bursting force, we get

$$78.54 \, \sigma'_w + 260 \, \sigma'_p = 6000 \qquad\qquad \ldots (1)$$

Longitudinal Stress $= \dfrac{pd}{4t}$

$$\sigma_{pl'} = \dfrac{3 \times 20000}{4 \times 13} = 11.54 \text{ N/mm}^2$$

Strain in the circumferential direction for the pipe and wire should be the same.

Strain in the wire

$$\dfrac{\sigma_w'}{E_w} = \dfrac{\sigma_w'}{101 \times 10^3}$$

Here, $E_w = 101 \times 10^3 \, \text{N/mm}^2$

Strain in the pipe $\dfrac{\sigma'_p}{E} - v\,\dfrac{\sigma_{pl'}}{E}$

$$\frac{\sigma_{p'}}{E} - v\,\frac{\sigma_{pl'}}{E} = \frac{\sigma'_p}{204 \times 10^3} - 0.3 \times \frac{11.54}{204 \times 10^3}$$

$$= \frac{1}{204 \times 10^3}\,(\sigma'_p - 3.462)$$

$$\frac{\sigma'_w}{101 \times 10^3} = \frac{1}{204 \times 10^3}\,(\sigma'_p - 3.462)$$

$$\sigma'_w = 0.495\,\sigma'_p - 1.714 \qquad\qquad \ldots (2)$$

Substituting Eq. (2) in Eq. (1), we get

$$\sigma'_p = 15.65 \text{ N/mm}^2 \qquad\qquad \text{(tensile)}$$

$$\sigma'_w = 24.60 \text{ N/mm}^2 \qquad\qquad \text{(tensile)}$$

Thus, Final stress in pipe $= -12.08 + 15.65 = 3.57$ N/mm^2 (tensile)

Final stress in wire $= 40 + 24.60 = 64.60$ N/mm^2 (tensile)

Strain in the circumferential direction for the pipe and wire should be the same.

Strain in the wire

$$\frac{\sigma_{w'}}{E_w} = \frac{\sigma_{w'}}{101 \times 10^3}$$

Here, $E_w = 101 \times 10^3$ N/mm^2

Example 3.7

A copper tube 38 mm external diameter and 35 mm internal diameter, is closely wound with a steel wire of 0.8 mm diameter. Estimate the tension at which the wire must have been wound if an internal pressure of 2 N/mm² produces a tensile circumferential stress of 7 N/mm² in the tube. Young's Modulus of steel is 1.6 times that of copper. Take Poisson's ratio of copper is 0.3.

Solution

Let σ_p and σ_w be the stresses in the pipe and the wire, respectively, before applying the internal pressure.

Considering for 1 mm length of pipe, we know,

Compressive force in pipe = Tensile force in wire.

$$\sigma_p \times 2 \times 1.5 = \sigma_w \times \frac{1}{0.8} \times 2 \times \frac{\pi}{4} \times 0.8^2$$

$$\sigma_p = 0.419 \, \sigma_w$$

Due to internal pressure alone, let the stresses be σ'_p and σ'_w. Then, we know,

Tensile force in pipe + Tensile force in wire = Bursting force

$$\sigma'_p \times 2 \times 1.5 + \sigma'_w \times \frac{1}{0.8} \times 2 \times \frac{\pi}{4} \times 0.8^2 = 2 \times 35$$

$$3\sigma'_p + 1.257 \, \sigma'_w = 70$$

Longitudinal stress $\dfrac{pd}{4t} = \dfrac{2 \times 35}{4 \times 1.5} = 11.67 \text{ N/mm}^2$

Hoop strain in the pipe = Strain in the wire at the junction

$$\frac{\sigma'_p}{E_c} - v\frac{\sigma_l}{E_c} = \frac{\sigma'_w}{E_s}$$

$$\frac{\sigma'_p}{E_c} - 0.3 \times \frac{11.67}{E_c} = \frac{\sigma'_w}{1.6 \, E_s}$$

$$\sigma'_p - 3.5 = 0.625 \, \sigma'_w$$

$$\sigma'_p - 0.625 \, \sigma'_w = 3.5$$

Solving for σ'_p and σ'_w,

$$\sigma'_p = 15.44 \text{ N/mm}^2$$

$$\sigma'_w = 19.11 \text{ N/mm}^2$$

∴   Final tensile stress in the pipe, $15.44 - \sigma_p = 7$.

Using the relation between initial stresses, $\sigma_p = 8.44 \text{ N/mm}^2$.

∴   Tension in the wire $= 20.15 \text{ N/mm}^2$.

SAQ 3

(a)   A cast iron pipe having an internal diameter of 300 mm has wall thickness of 6 mm and is closely wound with a single layer of steel wire of 3 mm diameter under a tensile stress of 8 N/mm². Calculate the stresses in the pipe and the wire, when the internal pressure inside the pipe is 1 N/mm². Take *E* for steel = 210 GPa, *E* for cast iron = 101 GPa and Poisson's ratio = 0.3.

(b)   A cylindrical thin walled vessel with closed ends made of an alloy has internal diameter of 200 mm and a wall thickness of 5 mm. The cylinder is strengthened by surrounding it with a single layer of steel wire of diameter 1.25 mm closely wound under tension. Determine the minimum tension under which the wire must be wound if the hoop tension in the cylinder is not to exceed 50 N/mm² when the vessel is subjected to an internal pressure of 4 N/mm². Take *E* for steel is 200 GPa   and   for   the   alloy   is   100   GPa   and   Poisson's   ratio   is   0.3