SNS COLLEGE OF TECHNOLOGY

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DEPARTMENT OF AEROSPACE ENGINEERING

TORSION (GATE Question)

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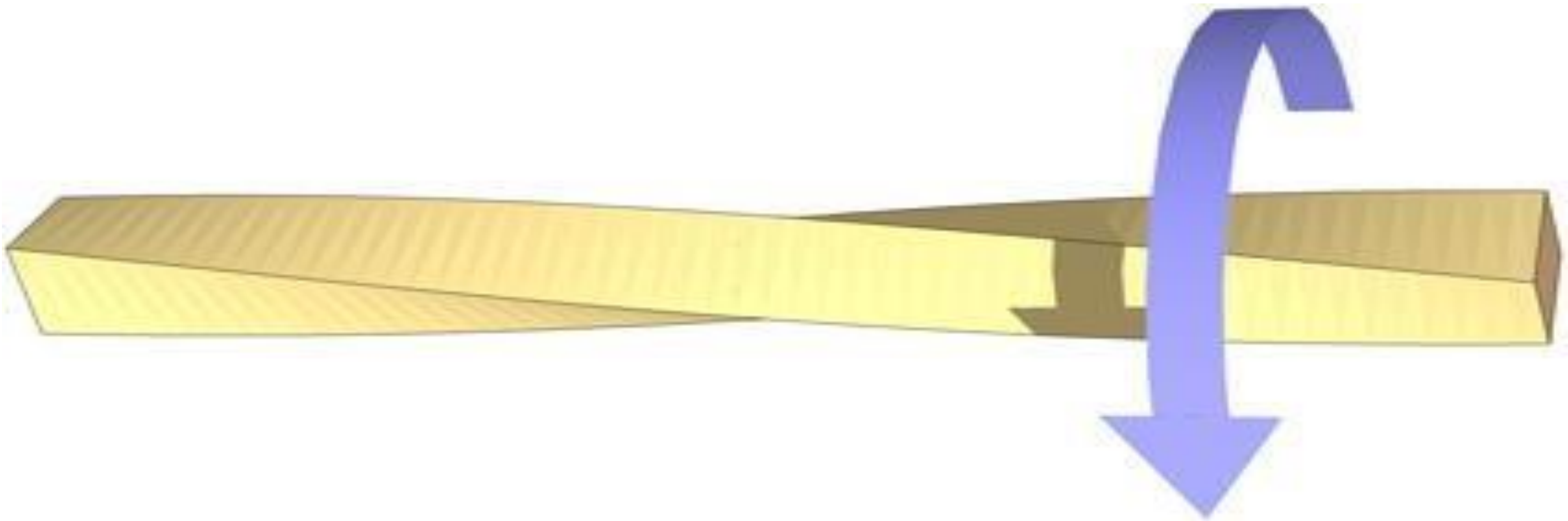
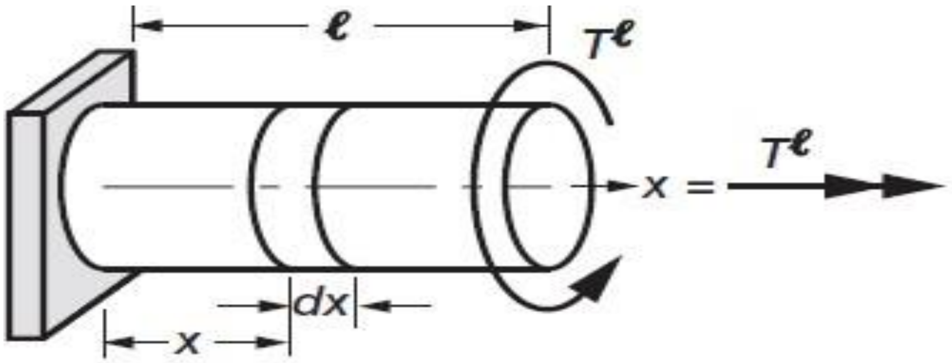
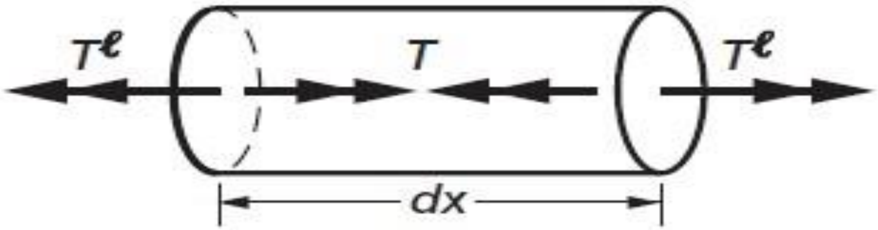
- Torsion is the moment applied in a plane containing the longitudinal axis of the beam or shaft.

e.g. Shaft transmitting torque or power, I beams, Portico beams, curved beams, closed coil springs.

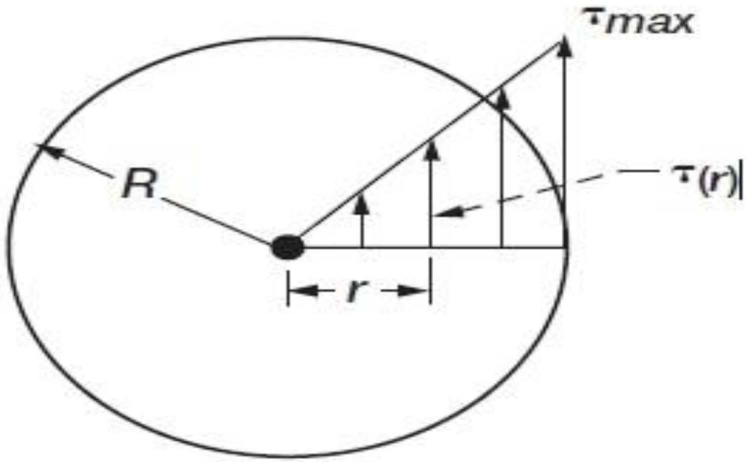




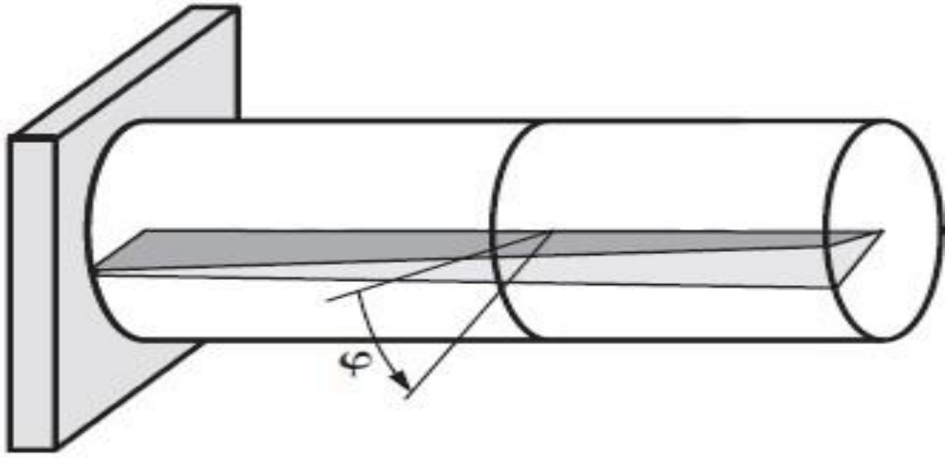
(a) Shaft under torque.



(b) Internal torque T.



(c) Variation of shear stress in cross-section.



(d) Deformation, or angle of twist, φ.

Assumptions

- The material of the shaft is uniform throughout
- Circular sections remain circular even after twisting
- Plane sections remain plane and do not twist or warp
- Stresses do not exceed the proportionality limit
- Shaft is loaded by twisting couples in planes that are perpendicular to the axis of the shaft
- Twist along the shaft is uniform
- The distance between any two normal cross sections remains the same

Torsion Equation: $$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{R}$$

$$T = \tau.\frac{J}{R} \Rightarrow T = \tau.Z_p$$

T : Torque or twisting moment

D : Diameter of the shaft

J : Polar moment of inertia

τ : Shear Stress

G : Modulus of Rigidity

θ : Angle of the twist

l : Length of the shaft

R : Radius of the shaft

For the shaft of given material, the maximum permissible shear stress (τ) is constant.

$$T \propto Z_p$$

The torsional resistance of the shaft is proportional to the polar modulus of the shaft.

Polar modulus of the section is a measure of the strength of shaft in torsion.

Polar moment of inertia for solid circular shaft, $J = \frac{\pi D^4}{32}$

Polar moment of inertia for hollow circular shaft, $J = \frac{\pi(D^4 - d^4)}{32}$

For a given shaft, J and R are constants.

$$\text{Polar modulus, } Z_p = \frac{J}{R}$$

$$\text{For solid circular section, } Z_p = \frac{\pi D^3}{16}$$

$$\text{For hollow circular section, } Z_p = \frac{\pi (D^4 - d^4)}{16D}$$

$$\frac{T}{J}=\frac{G\theta}{l} \qquad \theta=\frac{Tl}{GJ}$$

For a given shaft G, l and J are constants.

$$\theta \propto T$$

The angle of twist θ is directly proportional to the twisting moment T.

GJ : Torsional rigidity

$$\frac{GJ}{l}=\frac{T}{\theta}$$

If l and θ be unity, then $GJ=T$

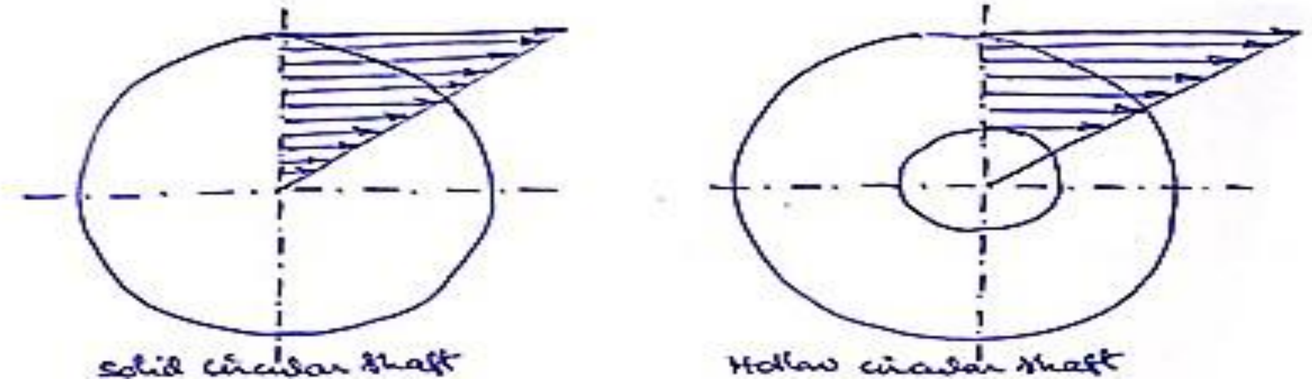
Torsional rigidity (GJ) is the torque that produces a twist of one radian in a shaft of unit length.

$$\tau=\frac{T}{J}.R$$

The maximum shear stress occurs on the cross section of the shaft at its outermost surface.

$$\tau = \frac{G\theta}{l}.r$$

Shearing stress varies directly as the distance r from the axis of the shaft. The shear stress distribution diagram in the plane of cross section is shown in fig.



Power transmitted by the shaft

Workdone by the torque = $T.2\pi N$ N-m/min

Power transmitted by the shaft = P kW

$= P\times10^3$ N-m/sec (1W = 1 J/sec = 1 Nm/sec)

$= 60\ P\times10^3$ N-m/min

Equating the workdone by the torque to Power transmitted by the shaft,

$$P = \frac{2\pi NT}{60000}\ \text{kW} \qquad P = \frac{2\pi NT}{45000}\ \text{HP}$$

P : Power transmitted by the shaft

N : Rotational speed of the shaft, rpm

T : Average or mean torque, Nm

Average or mean torque is considered for computing the power.

For determining the diameter of shaft, the maximum torque (and not average torque) is taken into account because the maximum shear stress developed is ensured to be within safe limits.

Stresses in shafts

- The maximum shear stress occurs at the outermost surface of the cross section.
- The maximum longitudinal shear stress occurs at the surface of the shaft on the longitudinal planes passing through the longitudinal axis of the shaft.
- The maximum tensile stress (major principal stress) occurs at planes 45° to the maximum shearing stress planes at the surface of the shaft.
- The maximum compressive stress (minor principal stress) occurs on the planes at 45° to the longitudinal and the cross sectional planes at the surface of the shaft. This stress is equal to the maximum shear stress on the cross section.
- Maximum shear stress on the cross section of the shaft is the significant stress for design.
- For most engineering materials, shear strength is small as compared to the tensile and compressive stresses.

MODULUS OF RUPTURE

Modulus of rupture is defined as the maximum fictitious shear stress required to rupture the shaft based on the ultimate torque.

$$\tau_r = \frac{T_u . R}{J}$$

τ_r : Modulus of rupture in torsion

T_u : Ultimate torque at failure

R : Outer radius of the shaft

J : Polar moment of inertia

The torsion formula is not applicable beyond the limit of proportionality.

The actual shear stress at the ultimate torque is uniformly distributed across the cross section.

Comparison of solid and Hollow shafts

a. Comparison by strength:

Both the solid and hollow shafts have same length, material, same weight and same maximum shear stress.

T_H : Torque transmitted by the hollow shaft

T_S : Torque transmitted by the solid shaft

$$\frac{T_H}{T_S} = \frac{n^2+1}{n\sqrt{n^2-1}}$$

D_H : External diameter of the hollow shaft

d_H : Internal diameter of the hollow shaft

$$n = \frac{D_H}{d_H}$$

Let $n = 2$

$$\frac{T_H}{T_S} = \frac{2^2+1}{2\sqrt{2^2-1}} = 1.44$$

- ✓ The torque transmitted by the hollow shaft is greater than the solid shaft of same weight.
- ✓ The hollow shaft is stronger than the solid shaft for a given weight (or cross sectional area).

b. Comparison by weight:

Both the solid and hollow shafts have the same length, material, same maximum shear stress and same torque.

W_H : Weight of hollow shaft

W_S : Weight of solid shaft

$$\frac{W_H}{W_S} = \frac{(n^2 - 1)\, n^{2/3}}{(n^4 - 1)^{2/3}}$$

Let $n = 2$

$$\frac{W_H}{W_S} = \frac{(2^2 - 1)(2)^{2/3}}{(2^4 - 1)^{2/3}} = 0.7829$$

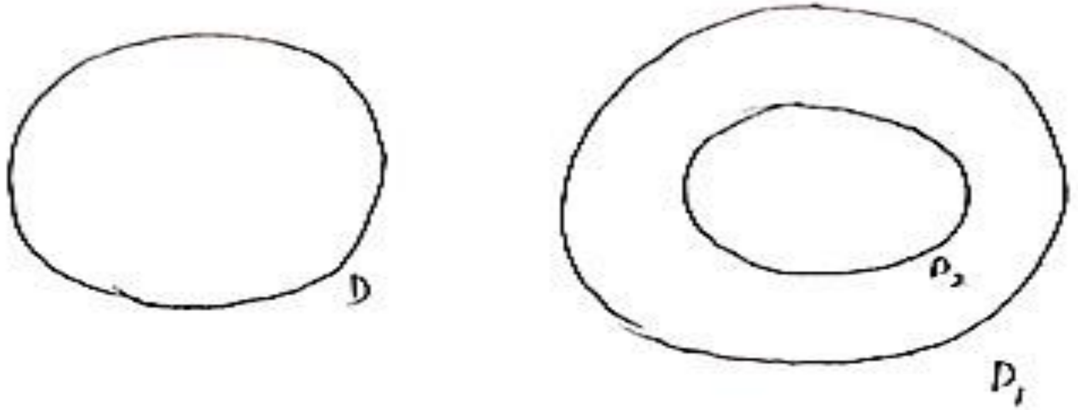
Weight of hollow shaft is less than that of solid shaft subjected to same torque.

Hollow shafts are economical compared to solid shafts in resisting torsion.

Comparision of Solid & Hollow Circular Shafts

Comparision of strength — a. Same material, same length, same weight :- Same Area

$\frac{T_H}{T_S}$



$n = \frac{D_1}{D_2}$

$D_1 = n D_2$

$W_S = W_H$

$A_S = A_H$

$\frac{\pi}{4} D^2 = \frac{\pi}{4}(D_1^2 - D_2^2)$

$D^2 = D_1^2 - D_2^2$

$D_1^2 = n^2 D_2^2 - D_2^2$

$D^2 = D_2^2(n^2 - 1)$

$D = D_2 \sqrt{n^2 - 1}$

$$T = T \cdot Z_P$$

$$\frac{T_H}{T_S} = \frac{\frac{\pi}{16 D_1}(D_1^4 - D_2^4)}{\frac{\pi}{16} D^3}$$

$$= \frac{D_1^4 - D_2^4}{D_1 \cdot D^3}$$

$$= \frac{D_2^4 (n^4 - 1)}{n D_2 \cdot D^3}$$

$$= \frac{(n^4 - 1)}{n} \cdot \frac{D_2^3}{D^3}$$

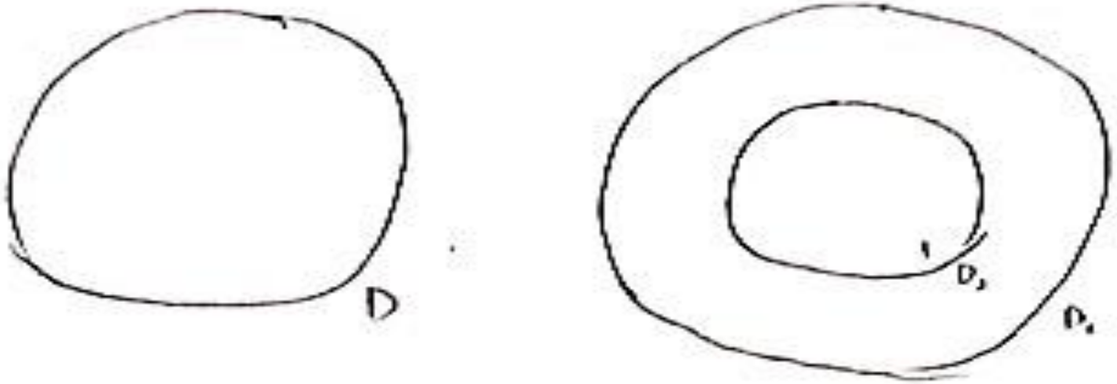
$$= \frac{n^4 - 1}{n} \cdot \frac{1}{(n^2 - 1)^{3/2}}$$

$$= \frac{(n^2 - 1)(n^2 + 1)}{n(n-1)\sqrt{n^2 - 1}}$$

$$\boxed{\frac{T_H}{T_S} = \frac{n^2 + 1}{n\sqrt{n^2 - 1}}}$$

6 Comparision of weight

same length, same material, same torque (l, τ, T are same)



$\frac{D_1}{D_2} = n$

$D_1 = n D_2$

$T_S = T_H$

$\tau \cdot (Z_P)_S = \tau (Z_P)_H$

$$\frac{\pi D^3}{16} = \frac{\pi}{16 D_1}(D_1^4 - D_2^4)$$

$$D^3 = \frac{D_1^4 - D_2^4}{D_1}$$

$$D^3 = \frac{D_2^4 (n^4 - 1)}{n \cdot D_2}$$

$$D^3 = \frac{(n^4-1)}{n} D_2^3 \Rightarrow D = \frac{(n^4-1)^{1/3}}{n^{1/3}} D_2$$

$$\frac{W_S}{W_H} = \frac{A_S}{H} = \frac{\frac{\pi D^2}{4}}{\frac{\pi}{4}(D_1^2 - D_2^2)}$$

$$= \frac{D^2}{D_1^2 - D_2^2}$$

$$= \frac{D^2}{(n^2-1) D_2^2} = \frac{(n^4-1)^{2/3}}{n^{2/3}(n^2-1)}$$

$$\boxed{\frac{W_S}{W_H} = \frac{(n^4-1)^{2/3}}{n^{2/3}(n^2-1)}}$$

If $T_S = T_H \Rightarrow W_S > W_H$

If $W_S = W_H \Rightarrow T_H > T_S$

Hollow circular shafts are compared to solid section in resisting torsion.

- Strength comparison (same weight, material, length and τ_{max})

$$\frac{T_h}{T_s} = \frac{n^2+1}{n\sqrt{n^2-1}} \qquad \text{Where, } n = \frac{\text{External diameter of hollow shaft}}{\text{Internal diameter of hollow shaft}}$$

- Weight comparison (same Torque, material, length and τ_{max})

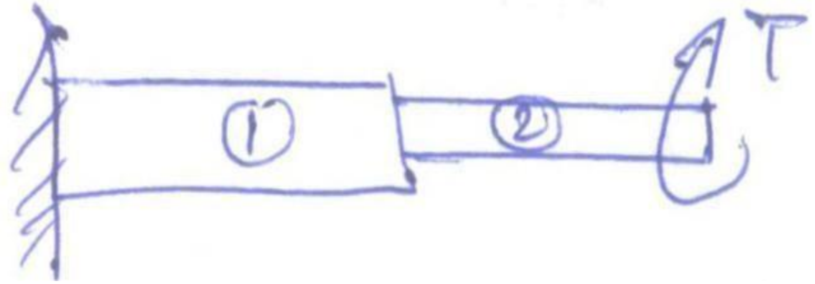
$$\frac{W_h}{W_s} = \frac{(n^2-1)n^{2/3}}{(n^4-1)^{2/3}} \qquad \text{Where, } n = \frac{\text{External diameter of hollow shaft}}{\text{Internal diameter of hollow shaft}}$$

- Strain energy comparison (same weight, material, length and τ_{max})

$$\frac{U_h}{U_s} = \frac{n^2+1}{n^2} = 1 + \frac{1}{n^2}$$

Composite shafts

a. Shafts in series



If the shafts are connected in series,

i. each shaft transmits the same torque, and

ii. the angle of twist is the sum of the angel of twist of each shaft

$$T_1 = T_2 = \ldots\ldots\ldots\ldots = T$$

$$\theta = \theta_1 + \theta_2 + \ldots\ldots\ldots$$

$$\theta = \frac{T\,l_1}{G_1 J_1} + \frac{T\,l_2}{G_2 J_2} + \ldots\ldots\ldots\ldots$$

$$= T\left(\frac{l_1}{G_1 J_1} + \frac{l_2}{G_2 J_2} + \ldots\ldots\right)$$

T : Torque transmitted by each shaft

$l_1, l_2, \ldots\ldots\ldots$: Respective lengths of shafts

$G_1, G_2, \ldots\ldots$: Respective modulii of rigidity

$J_1, J_2, \ldots\ldots$: Respective polar moment of inertia

If the shafts are made of same material,

$$G_1 = G_2 = \ldots\ldots = G$$

$$\theta = \frac{T}{G}\left(\frac{l_1}{J_1} + \frac{l_2}{J_2} + \ldots\ldots\ldots\right)$$

B. SHAFTS IN PARALLEL

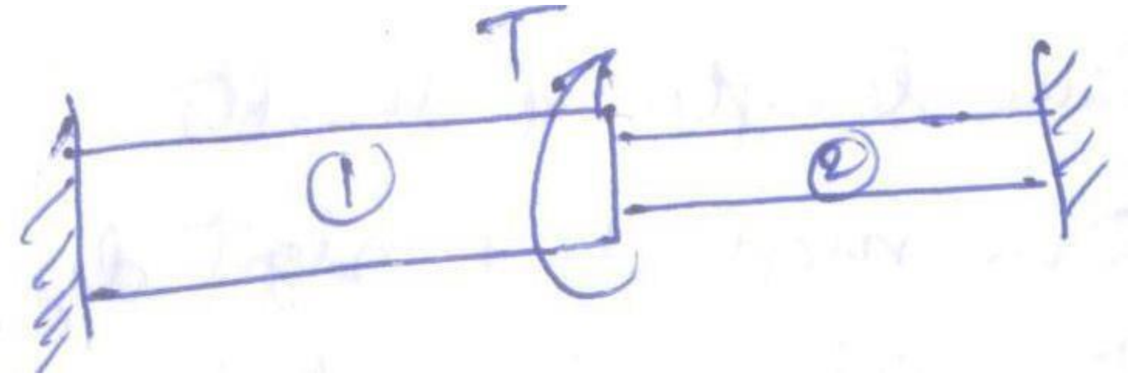
If the shafts are connected in parallel,

i. the angle of twist is same for each shaft

ii. the applied torque is divided between the shafts

$$\theta_1 = \theta_2 = \dots\dots\dots\dots = \theta$$

$$\frac{T_1 l_1}{G_1 J_1} = \frac{T_2 l_2}{G_2 J_2} = \dots\dots\dots\dots$$



$$T = T_1 + T_2 + \dots\dots\dots\dots$$

If the shafts are made of same material,

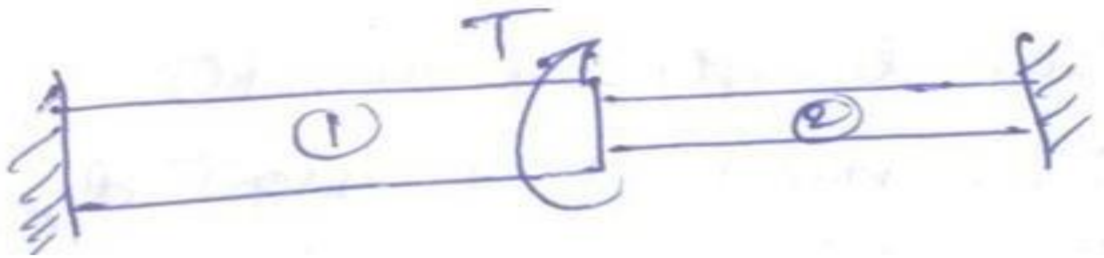
$$G_1 = G_2 = \dots\dots\dots\dots = G$$

$$\frac{T_1 l_1}{J_1} = \frac{T_2 l_2}{J_2} = \dots\dots\dots\dots \qquad \theta = \frac{TL}{G_1 J_1 + G_2 J_2}$$

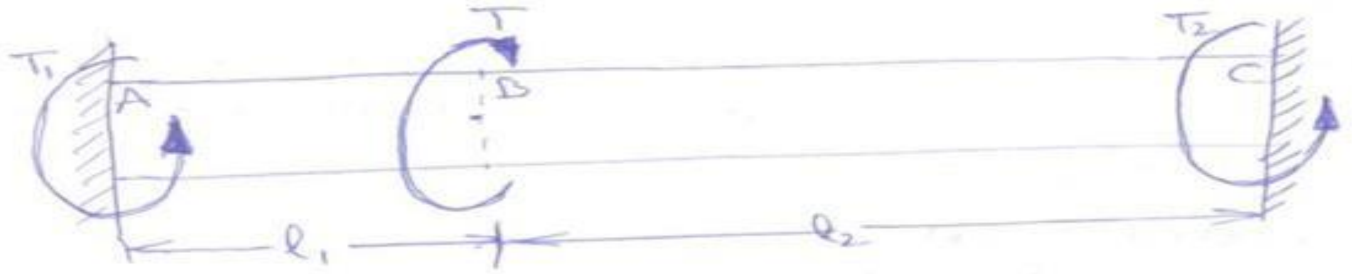
If the torque is shared equally by the two shafts,

$$T_1 = T_2 = T$$

$$\frac{l_1}{J_1} = \frac{l_2}{J_2} \Rightarrow l_1 J_2 = l_2 J_1$$



SHAFT FIXED AT ITS BOTH ENDS:



The angle of twist is same at B for the members BA and BC.

$$\theta_1 = \theta_2$$

$$\frac{T_1 l_1}{G_1 J_1} = \frac{T_2 l_2}{G_2 J_2}$$

If G and J are same for portions AB and BC,

$$T_1 l_1 = T_2 l_2 \quad \ldots\ldots\ldots\ldots(i)$$

Torque is distributed inversely proportional to their lengths. Torsion T is shared by the two members.

$$T = T_1 + T_2 \quad \ldots\ldots\ldots\ldots(ii)$$

From (i) and (ii)

$$T_1 = T\left(\frac{l_2}{l_1 + l_2}\right), \qquad T_2 = T\left(\frac{l_1}{l_1 + l_2}\right)$$

$T_1 > T_2$, torque required in a short portion of a shaft is more for the same angle of twist.

$\tau_1 > \tau_2$, the maximum shear stress developed in short portion of a shaft is more than that of a long portion of the shaft.

TORSIONAL RESILIENCE:

Torsional energy = Work done by the torque

= Average torque × Angle of twist

$$U = \frac{1}{2} T.\theta$$

For solid shaft, $U = \frac{\tau^2}{4G} \times \text{Volume of the shaft}$

Torsional resilience per unit volume $= \frac{\tau^2}{4G}$

For hollow shaft, $U = \frac{\tau^2}{4G} \cdot \frac{D^2 + d^2}{D^2} \times \text{Volume of the shaft}$

Torsional resilience per unit volume $= \frac{\tau^2}{4G} \cdot \frac{D^2 + d^2}{D^2}$

D : External diameter of hollow shaft

d : Internal diameter of hollow shaft

For a very thin hollow shaft D is nearly equal to d ,

$$U = \frac{\tau^2}{2G} \times \text{Volume of the shaft}$$

For the same volume, $\frac{\text{Strain energy stored in a solid shaft}}{\text{Strain energy stored in a hollow shaft}} = \frac{D^2}{D^2 + d^2}$

COMBINED BENDING AND TORSION:

When the shaft is subjected to combined bending and torsion, both the shear stress due to torsion and bending stress due to bending are induced in the shaft.

The principal stress are $\sigma_{max,min} = \frac{\sigma}{2} \pm \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

Bending stress, $\sigma = \frac{32M}{\pi D^3}$

Shear stress $\tau = \frac{16T}{\pi D^3}$

$$\sigma_{max,min} = \frac{16T}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

Equivalent bending moment, $M_e = \frac{1}{2}(M + \sqrt{M^2 + T^2})$

Equivalent torque, $T_e = (\sqrt{M^2 + T^2})$

Maximum shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$$\tau_{max} = \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$$

T_e is also used to evaluate the magnitude of the maximum shear stress.

$$\tan 2\theta = \frac{2\tau}{\sigma} = \frac{T}{M}$$

GATE CE

01. A circular solid shaft of span L= 5 m is fixed at one end and free at the other end. A twisting moment T= 100 kN-m is applied at the free end. The torsional rigidity *GJ* is 50000 KN-m^2/rad. Following statements are made for this shaft.

I. The maximum rotation is 0.01 rad

II. The torsional strain energy is 1 kN-m CE 2004

With reference to the above statements, which of the following applies?

a. Both statements are true

b. Statement I is true but II is false

c. Statement II is true but I is false

d. Both the statements are false

01. B

Length of shaft, L= 5 m

Twisting moment, T= 100 kNm

Torsion rigidity, GJ= 50000 kN-m^2/rad

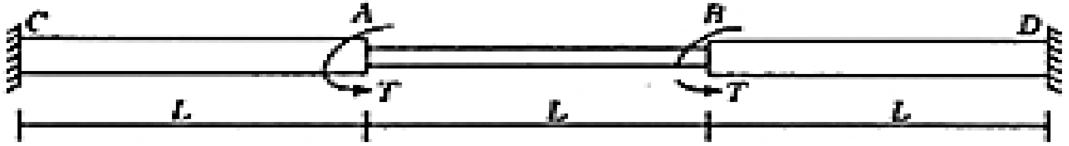
Torsion formula, $\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{R}$

Maximum rotation, $\theta = \frac{Tl}{GJ} = \frac{100 \times 5}{50000}$ =0.01 rad

Torsional strain energy, $U = \frac{1}{2} T.\theta = \frac{1}{2} \times 100 \times 0.01 = 0.5$ kNm

Hence, statement I is true but II is false.

02. A circular shaft shown in the figure is subjected to torsion T at two points A and B. The torsional rigidity of portions CA and BD is GJ_1 and that of portion AB is GJ_2. The rotations of shaft at point A and B are θ_1 and θ_2. The rotation θ_1 is



CE 2005

a. $\dfrac{TL}{GJ_1 + GJ_2}$ b. $\dfrac{TL}{GJ_1}$ c. $\dfrac{TL}{GJ_2}$ d. $\dfrac{TL}{GJ_1 - GJ_2}$

02. B

By symmetry, the shaft shows that there is no torsion on the portion AB. Hence Torque T is acting on each of the end portions AC and BD.

$$\frac{T}{J} = \frac{G\theta}{l} \quad \Rightarrow \theta = \frac{TL}{GJ}$$

For portion AC or BD, $\theta_1 = \dfrac{TL}{GJ_1}$

03. A long shaft of diameter d is subjected to twisting moment T at its ends. The maximum normal stress acting at its cross-section is equal to CE 2006

a. zero **b.** $\frac{16T}{\pi d^3}$ c. $\frac{32T}{\pi d^3}$ d. $\frac{64T}{\pi d^3}$

03. B

Let d : Diameter of the shaft

T : Twisting moment

σ : Maximum normal stress

τ : Maximum shear stress

Torsion formula is $\frac{T}{J} = \frac{G.\theta}{L} = \frac{\tau}{R}$

$$\tau_{max} = \frac{T}{J}.R = \frac{T}{\frac{\pi d^4}{32}}.\frac{d}{2} = \frac{16T}{\pi d^3}$$

Maximum normal stress is zero.

Due to twisting, the shaft is subjected to only shear stress.

04. The maximum and minimum shear stresses in a hollow circular shaft of outer diameter 20 mm and thickness 2 mm, subjected to a torque of 92.7 N-m will be CE 2007

a. 59 MPa and 47.2 MPa

b. 100 MPa and 80 MPa

c. 118 MPa and 160 MPa

d. 200 MPa and 160 MPa

04. B

Outer diameter of shaft, $D = 20$ mm

Thickness of the shaft, $t = 2$ mm

Inner diameter of shaft, $d = 20 - 2 \times 2 = 16$ mm

Torque, $T = 92.7$ Nm

Torsion equation is $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$

Polar moment of inertia, $J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(20^4 - 16^4) = 9274\ \text{mm}^4$

Maximum shear stress occurs at the outer surface and minimum shear stress occurs at inner surface of the shaft.

$\tau_{max} = \frac{92.7 \times 10^3}{9274} \times 10 = 100\text{N/mm}^2$ <> $\tau_{min} = \frac{92.7 \times 10^3}{9274} \times 8 = 80\text{N/mm}^2$

05. The maximum shear stress in a solid shaft of circular cross section having diameter d subjected to a torque T is τ. If the torque is increased by four times and the diameter of the shaft is increased by two times, the maximum shear stress in the shaft will be CE 2008

a. 2τ b. τ c. $\tau/2$ d. $\tau/4$

05. C

T : Torque

τ : Maximum shear stress

d : Diameter of the solid circular shaft

$T_1 = 4T$; $d_1 = 2d$; $R_1 = 2R$; τ_1 : ?

Torsion formula is, $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$; $\tau = \frac{T}{J}.R$

Polar moment of inertia, $J = \frac{\pi d^4}{32} = \frac{\pi R^4}{2}$

$$\frac{\tau_1}{\tau} = \left(\frac{T_1}{T}\right)\left(\frac{J}{J_1}\right)\left(\frac{R_1}{R}\right) = \frac{T_1}{T}.\left(\frac{D}{D_1}\right)^4.\frac{R_1}{R} = 4 \times \frac{1}{(2)^4} \times 2 = \frac{1}{2}$$

$$\tau_1 = \frac{\tau}{2}$$

06. A hollow circular shaft has an outer diameter of 100 mm and a wall thickness of 25 mm. The allowable shear stress in the shaft is 125 MPa. The maximum torque the shaft can transmit is CE 2009

a. 46 kNm b. 24.5 kNm c. 23 kNm d. 11.5 kNm

06. c

Outer diameter of the shaft, D = 100 mm

Thickness of the shaft, t = 25 mm

Internal diameter of the shaft, d = 50 mm

Allowable shear stress in the shaft, $\tau = 125$ MPa

Torque transmitted by the shaft, $T = ?$

Torsion formula, $\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}$

Outer radius of the shaft, R = 50 mm

Polar moment of Inertia, $J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(100^4 - 50^4) = 920.3 \times 10^4 \text{ mm}^4$

$T = \frac{\tau}{R}.J = \frac{125}{50} \times 920.3 \times 10^4 = 23.0 \times 10^6 \text{ Nmm} = 23.0 \text{ kNm}$

07. A sold circular shaft of diameter d and length L is fixed at one end and free at the other end. A torque T is applied at the free end. The shear modulus of the material is G. The angle of twist at the free ends is CE 2010

a. $\frac{16TL}{\pi d^4 G}$ **b.** $\frac{32TL}{\pi d^4 G}$ c. $\frac{64TL}{\pi d^4 G}$ d. $\frac{128TL}{\pi d^4 G}$

07. B

Diameter of solid circular shaft: d

Length of shaft:L

Applied torque:T

Shear modulus:G

Polar moment of inertia, $J = \frac{\pi d^4}{32}$

Torsion formula, $\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{R}$

Angle of twist, $\theta = \frac{Tl}{GJ} \Rightarrow \theta = \frac{TL}{G.\frac{\pi d^4}{32}} = \frac{32TL}{\pi d^4 G}$

08. A solid circular beam with radius of 0.25 m and length of 2 m is subjected to a twisting moment of 20 kNm about the z-axis at the free end, which is the only load acting as shown in the figure. The shear stress component T_{xy} at point 'M' in the cross-section of the beam at a distance of 1 m from the fixed end is

CE1 2018

a. 0.0 MPa b. 0.51 MPa c. 0.815 MPa d. 2.0 MPa

08. A

When the solid circular beam subjected to twisting moment about z axis, then the shaft is induced to shear stresses in zy and xz planes only. The shear stress in the xy plane is equal to zero.

39. A hollow circular shaft has an outer diameter of 100 mm and inner diameter of 50 mm. If the allowable shear stress is 125 MPa, the maximum torque (in kN-m) that the shaft can resist is CE2 2017

39. 23

Outer diameter of hollow shaft, $D_1 = 100$ mm

Inner diameter of hollow shaft, $D_2 = 50$ mm

Allowable shear stress, $\tau = 125$ MPa

Maximum torque in the shaft, T = ?

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow T = \frac{\tau}{R} J$$

$$T = \frac{\tau}{\frac{D_1}{2}} \cdot \frac{\pi}{32}\left[D_1^4 - D_2^4\right] = \frac{125 \times 2}{100} \cdot \left[\frac{\pi}{32}(100^4 - 50^4)\right] = 23.0 \times 10^6 \text{ Nmm} = 23.0 \text{ kNm}$$

GATE ME

01. The outside diameter of a hollow shaft is twice its inside diameter. The ratio of its torque carrying capacity to that of a solid shaft of the same material and the same outside diameter is : GATE ME 1993

a. $\frac{15}{16}$ b. $\frac{3}{4}$ c. $\frac{1}{2}$ d. $\frac{1}{16}$

01. A

D: Outside diameter of hollow shaft

d: inside diameter of hollow shaft

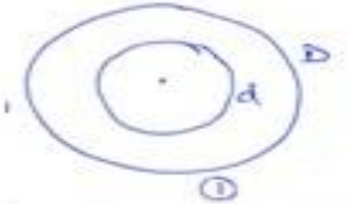
$B = 2d$

T_1 : Torque carrying capacity of a hollow shaft.

T_2 : Torque carrying capacity o f a solid shaft.

Torque, $T = \tau . Z$

$$\frac{T_1}{T_2} = \frac{Z_1}{Z_2} = \frac{\frac{\pi\left(D^4 - d^4\right)}{16D}}{\frac{\pi D^4}{16D}} = \frac{D^4 - d^4}{D^4} = 1 - \left(\frac{1}{2}\right)^4 = \frac{16-1}{16} = \frac{15}{16}$$



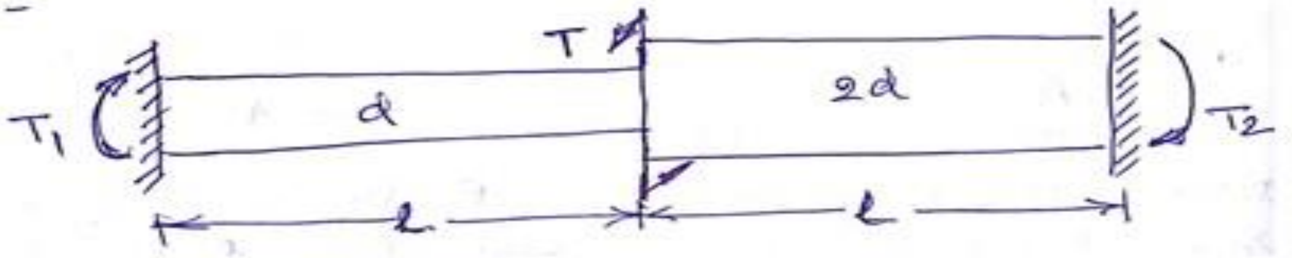
hollow shaft

Solid shaft

02. The compound shaft shown is built-in at the two ends. It is subjected to a twisting moment T at the middle. What is the ratio of the reaction torque T_1 and T_2 at the ends? GATE ME 1993

a. $\frac{1}{16}$ b. $\frac{1}{8}$ c. $\frac{1}{4}$ d. $\frac{1}{2}$

02. a



At the joint of the shaft, $\theta_1 = \theta_2$

$$\left(\frac{TL}{GJ}\right)_1 = \left(\frac{TL}{GJ}\right)_2$$

J: Polar moment of Inertia $= \frac{\pi d^4}{32}$

$$\frac{T.L}{G.\frac{\pi d^4}{32}} = \frac{T_2 L}{G.\frac{\pi (2d)^4}{32}}; \qquad T_1 = \frac{T_2}{16} \qquad \frac{T_1}{T_2} = \frac{1}{16}$$

03. Two shafts A and B are made of the same material .The diameter of shaft B is twice that of shaft A. The ratio of power which can be transmitted by shaft A to that of shaft B is: GATE ME 1994

a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{1}{8}$ d. $\frac{1}{16}$

03. C

d_A : diameter of shaft A

d_B : diameter transmitted by shaft $d_B = 2d_A$

$\frac{P_A}{P_B} = ?$ $P = \frac{2\pi\ NT}{60,000}$ kW

N: speed, rpm | T: Torque

$\frac{P_A}{P_B} = \frac{T_A}{T_B}$ $P \alpha T$ Torque, $T = \tau . Z$

$\frac{P_A}{P_B} = \frac{d_A^3}{d_B^3}$ $= \tau . \frac{\pi\ d^3}{16}$ For a given material τ is constant. $T\ \alpha\ d^3$

$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

04. A solid shaft can resist a bending moment of 3.0 kNm and a twisting moment of 4.0 kNm together , then the maximum torque that can be applied is

a. 7.0 kNm b. 3.5 kNm c. 4.5 kNm **d.** 5.0 kNm GATE ME 1996

04. D

Bending moment, twisting moment

M= 3 kNm T= 4 kNm

T_e : Equivalent torque

$$= \sqrt{M^2 + T^2} = \sqrt{3^2 + 4^2} = 5 \text{ kNm}$$

05. Maximum shear stress developed on the surface of a solid circular shaft under pure torsion is 240 MPa .If the shaft diameter is doubled then the maximum shear stress developed corresponding to the same torque will be GATE ME 2003

a. 120 MPa b. 60MPa c. 30 MPa d. 15 MPa

05. C

τ_{max} : Maximum shear stress= 240 MPa

Torsion formula is $\frac{T}{J} = \frac{\tau}{R}$

$$\tau = \frac{16T}{\pi\, d^3}$$

$$T = \tau . \frac{J}{R} = \tau . \frac{\pi\, d^3}{16}$$

$$\tau \;\alpha\; \frac{1}{d^3}$$

d_1 : diameter of shaft 1

d_2 : diameter of shaft 2

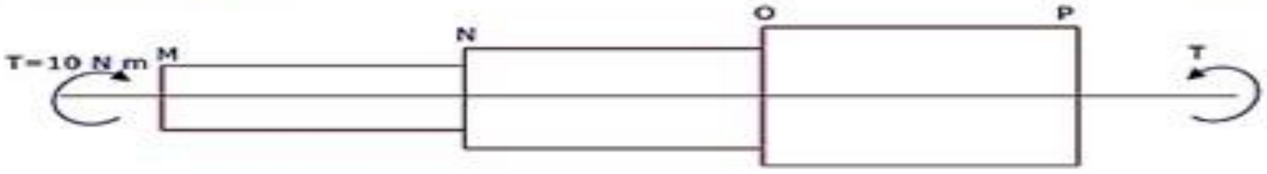
τ_1 : maximum shear stress in shaft 1

τ_2 : maximum shear stress in shaft 2

$$\frac{\tau_2}{\tau_1} = \left(\frac{d_1}{d_2}\right)^3 \qquad \frac{\tau_2}{\tau_1} = \left(\frac{d}{2d}\right)^3 = \frac{1}{8}$$

$$\tau_2 = \frac{\tau_1}{8} = \frac{240}{8} = 30\text{MPa}$$

06. A torque of 10 Nm is transmitted through a stepped shaft as shown in fig The torsional stiffnesses of individual sections of lengths MN, NO and OP are 20 Nm/rad, 30Nm/rad and 60Nm respectively. The angular deflection between the ends M and P of the shaft is GATE ME 2004



a. 0.5 rad   **b.** 1.0 rad   c. 5.0 rad   d. 10.0 rad

06. B.

T : Torque =10 Nm $\frac{T}{J} = \frac{G\theta}{l}$ $\theta = \frac{T}{K}$ $k = \frac{GJ}{l}$

$\theta_{MN} = \frac{10}{20} = \frac{1}{2}$ $\theta_{NO} = \frac{10}{30} = \frac{1}{3}$

θ : Angle of twist and k : Torsional stiffness

$\theta_{OP} = \frac{10}{60} = \frac{1}{6}$

θ : Angular deflection between the ends M and P

$$= \theta_{MN} + \theta_{NO} + \theta_{OP} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = \frac{6}{6} = 1 \text{ radian.}$$

07. A solid circular shaft of 60mm diameter transmits a torque of 1600NM. The value of maximum shear stress developed is GATE ME 2004

a. 37.72 MPa b. 47.72 MPa c. 57.72 MPa d. 67.72 MPa

07. a

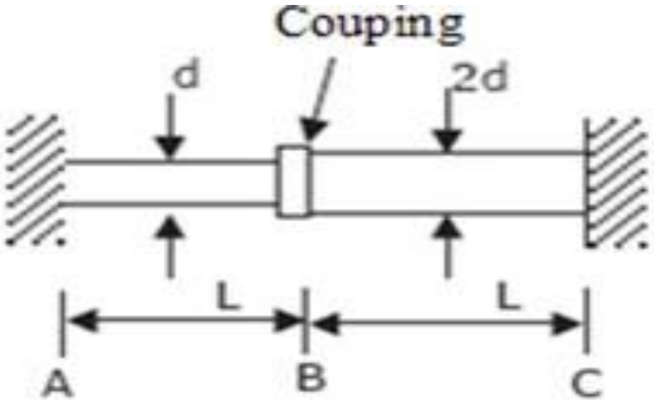
d: Diameter of the shaft = 60 mm

T: Torque = 1600 Nm

$$\tau_{max} : \text{ maximum shear stress} = \frac{16T}{\pi d^3} = \frac{16 \times 1600 \times 10^3}{\pi (60)^3}$$

$$\tau_{max} = 37.72 \text{ MPa}$$

08. The two shafts AB and BC, of equal length and diameters d and 2d, are made of the same material. They are joined at B through a shaft couplings, while the ends A and C are built –in (cantilevered). A twisting moment T is applied to the coupling. If T and T represents the twisting moments at the ends A and C, respectively, then GATE ME 2005



a. $T_C = T_A$ b. $T_C = 8T_A$ c. $T_C = 16T_A$ d. $T_A = 16T_C$

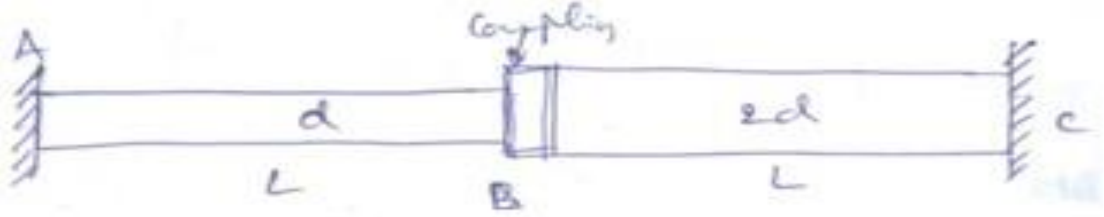
08. C

Angle of twist at the junction B is same in shaft AB and BC

$$\theta_{BA} = \theta_{BC}$$

$$\frac{T_A.L}{G.\frac{\pi d^4}{32}} = \frac{T_C.L}{G.\frac{\pi (2d)^4}{32}}$$

$$\frac{T_A}{d^4} = \frac{T_C}{16\, d^4} \Rightarrow T_c = 16T_A$$



09. For a circular shaft of diameter d subjected to torque T, the maximum value of the shear stress is : GATE ME 2006

a. $\frac{64T}{\pi d^3}$ b. $\frac{32T}{\pi d^3}$ c. $\frac{16T}{\pi d^3}$ d. $\frac{8T}{\pi d^3}$

09. c

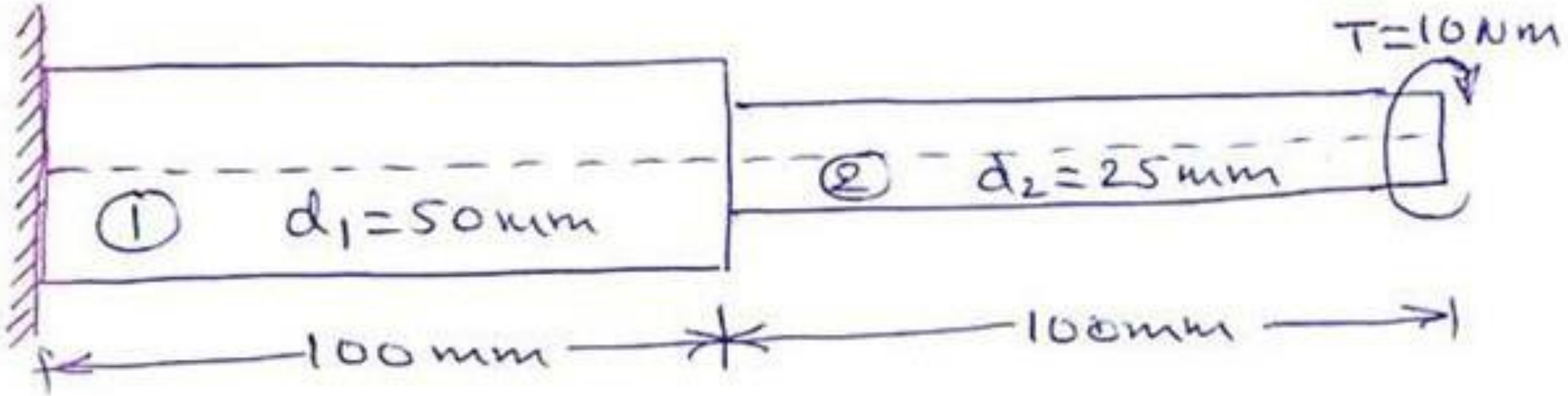
Torsion formula is $\frac{T}{J} = \frac{\tau}{R}$

τ_{max} : maximum shear stress $= \frac{T}{J}.R = \frac{T}{\frac{\pi d^4}{32}}.\frac{d}{2}$

$$\tau_{max} = \frac{16\,T}{\pi\, d^3}$$

End condition : both ends pinned

10. A stepped steel shaft shown below is subjected to 10 N-m torque. If the modulus of rigidity is 80 GPa, the strain energy in the shaft in N-mm is



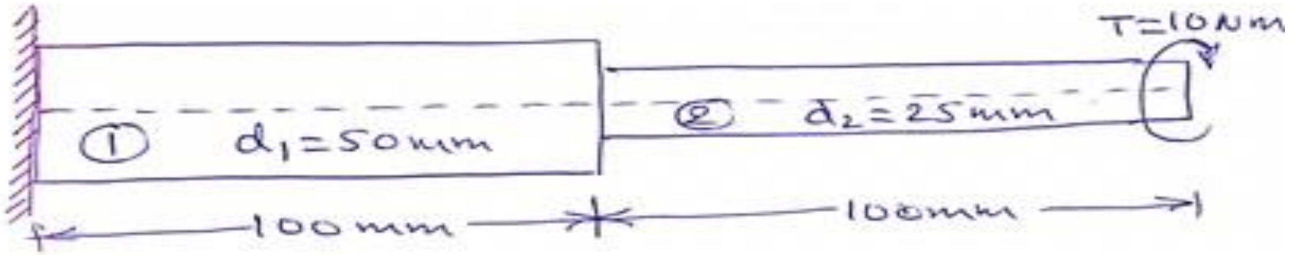
GATE ME 2007

a. 4.12 b. 3.46 c. 1.73 d. 0.86

10. C

Strain energy due to torsion,

$$U = \int_0^L \frac{T^2}{2GJ}.dx$$



$$U = \frac{T^2 L}{2GJ}$$

Torque, T=10 Nm

Modulus of rigidity, G=80 GPa

Polar moment of Inertia, $J = \frac{\pi d^4}{32}$

Strain energy in the shaft, $U = U_1 + U_2$

$$U = \frac{T^2}{2G}\left[\frac{L_1}{J_1} + \frac{L_2}{J_2}\right] = \frac{(10\times10^3)^2\times100}{2\times80\times10^3}\left[\frac{32}{\pi(50)^4} + \frac{32}{\pi(25)^4}\right]$$

U= 1.73 Nmm

11. A solid circluar shaft of diamter 100 mm is subjected to an axial stress of 50 MPa. It is furthur subjected to a torque of 10 kNm . The maximum principal stress experienced on the shaft is closest to GATE ME 2008

a. 41 MPa **b.** 82 MPa c. 164 MPa d. 204 MPa

11. B

Diameter of shaft, d = 100 mm

Axial stress, σ = 50 MPa

Torque, T =10 kNm

Shear stress due to torque, $\tau = \frac{T}{I_p}$

$$\tau = \frac{16T}{\pi d^3} = \frac{16\times10\times10^6}{\pi(100)^3} = 50.93 \text{ N/mm}^2$$

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{50+0}{2} + \frac{1}{2}\sqrt{(50-0)^2 + 4(50.93)^2} = 25+56.7; \quad \sigma_1 = 81.7 \text{ N/mm}^2$$

12. A solid circular shaft of diameter d is subjected to a combined bending moment M and torque, T. The material property to be used for designing the shaft using the relation $\frac{16}{\pi d^3}\sqrt{M^2+T^2}$ is

GATE ME 2009

a. ultimate tensile strength (S_u)

b. tensile yield strength (S_y)

c. torsional yield strength (S_{sy})

d. endurance strength (S_e)

12. C

Diameter of solid shaft = d   Bending moment = M   Torque = T

Bending stress, $\sigma = \frac{M}{Z} = \frac{32M}{\pi d^3}$

Shear stress, $\tau = \frac{T}{J}.R = \frac{16T}{\pi d^3}$

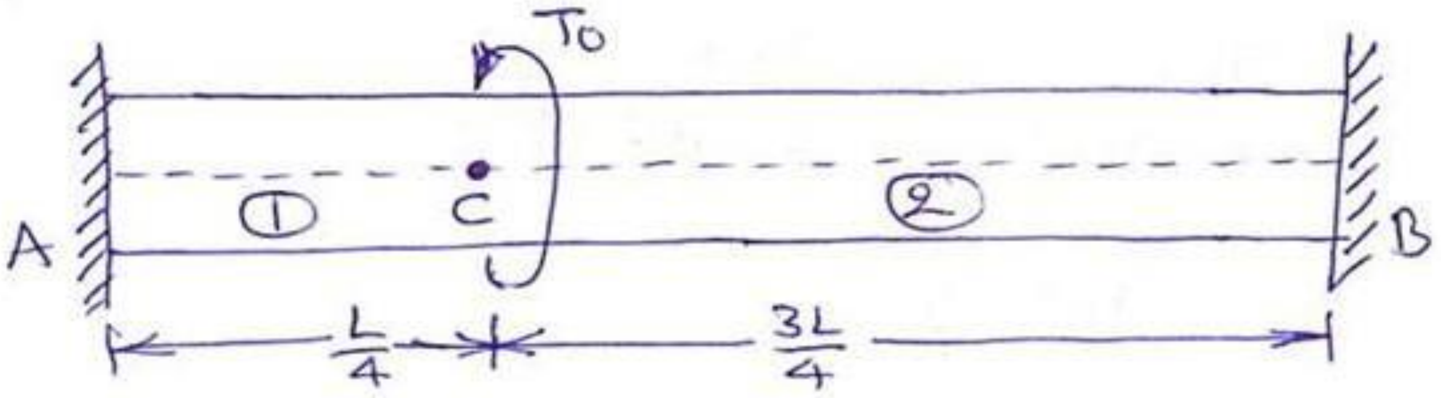
Principal stresses, $$\sigma_{1,2} = \frac{\frac{32M}{\pi d^3} \pm \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)}}{2}$$

$$\sigma_{1,3} = \frac{16}{\pi d^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

Maximum shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$

Therefore, Torsional yield strength, $S_{sy} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$

13. A solid shaft of diameter d and length L is fixed at both the ends. A Torque, T_0 is applied at a distance, $\frac{L}{4}$ from the left end as shown in the figure given below.



The maximum shear stress in the shaft is GATE ME 2009

a. $\frac{16T_0}{\pi d^3}$ **b.** $\frac{12T_0}{\pi d^3}$ c. $\frac{8T_0}{\pi d^3}$ d. $\frac{4T_0}{\pi d^3}$

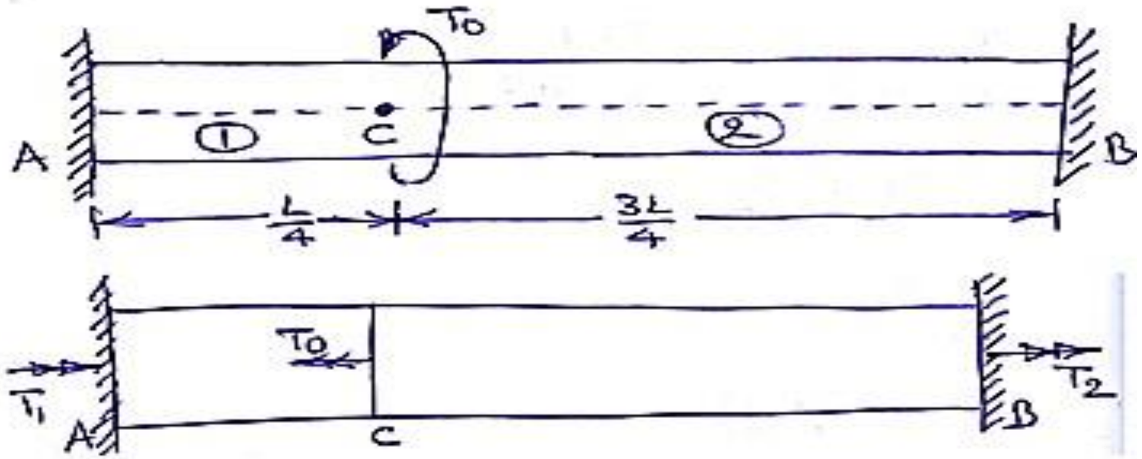
13. B

Equilibrium equation is $T_1 + T_2 = T_0$.......(1)

Torsion formula is $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$

At C, $\theta_1 = \theta_2$

For the some diameter and material of the shaft, Torque is inversely proportional to the length.



$T \propto \frac{1}{L}$

$T_1 L_1 = T_2 L_2$

$T_1 \frac{L}{4} = T_2 . \frac{3}{4} L$

$T_1 = 3T_2$......(2)

$3T_2 + T_2 = T_0 \Rightarrow T_2 = \frac{T_0}{4}, \; T_1 = \frac{3}{4} T_0$

$T_1 > T_2$

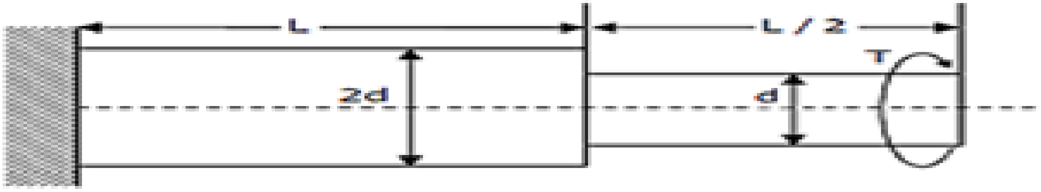
Also shear stress is proportional to the torque $\tau \propto T$

Therefore, the maximum shear stress induced is the bar due to torque T_1 .

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16.\frac{3}{4}.T_0}{\pi d^3} = \frac{12T_0}{\pi d^3}$$

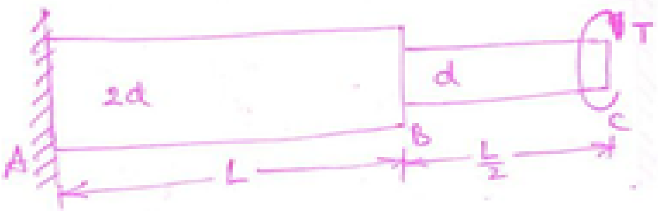
14. A torque T is applied at the free end of a stepped rod of circular cross-sections as shown in the figure. The shear modulus of the material of the rod is G. The expression for d to produce an angular twist θ at the free end is GATE ME 2011



a. $[32\,TL/\pi\theta G]^{1/4}$ b. $[18\,TL/\pi\theta G]^{1/4}$ c. $[16\,TL/\pi\theta G]^{1/4}$ d. $[2\,TL/\pi\theta G]^{1/4}$

14. B

θ : Angle of twist at the free end $= \theta_1 + \theta_2$



Torsion formula: $\dfrac{T}{J} = \dfrac{G\theta}{L} = \dfrac{\tau}{R}$

Angle of twist, $\theta = \dfrac{TL}{GJ}$ J : Polar modulus $= \dfrac{\pi d^4}{32}$

$$\theta = \left(\frac{TL}{GJ}\right)_{AB} + \left(\frac{TL}{GJ}\right)_{BC} = \frac{TL}{G.\frac{\pi}{32}(2d)^4} + \frac{T.L/2}{G.\frac{\pi d^4}{32}} = \frac{TL}{G.\pi d^4}\left[\frac{32}{16} + \frac{32}{2}\right]$$

$$\theta = \frac{18\,T.L}{G.\pi d^4}\,; \qquad d = \left(\frac{18\,TL}{\pi\theta G}\right)^{1/4}$$

15. A solid circular shaft needs to be designed to transmit a torque of 50Nm.If the allowable shear stress of the material is 140MPa,assuming a factor of safety of 2, the minimum allowable design diameter in mm is GATE ME 2012

a. 8 **b. 16** c. 24 d. 32

15. B

Torque, T = 50 N m

Allowable shear stress, $\tau_{max} = 140$ MPa

Factor of safety, F = 2

Diameter of the shaft, $d = ?$

τ_{max} : Maximum shear stress $= \dfrac{16T}{\pi d^3}$

$$\frac{140}{2} = \frac{16 \times 50 \times 10^3}{\pi d^3}$$

d = 15.4 mm

16. Two solid circular shafts of radii R_1 and R_2 are subjected to same torque. The maximum shear stresses developed in the two shafts are τ_1 and τ_2. If $R_1/R_2 = 2$, then τ_2/τ_1 is ……… GATE ME3 2014

Ans: 7.9 to 8.1

16. **8**

Radius of two solid circular shafts: R_1 and R_2 two shafts are subjected to same torque. $T_1 = T_2$

Maximum shear stress in two shafts: τ_1 and τ_2

$$\frac{R_1}{R_2} = 2, \frac{\tau_2}{\tau_1} = ?$$

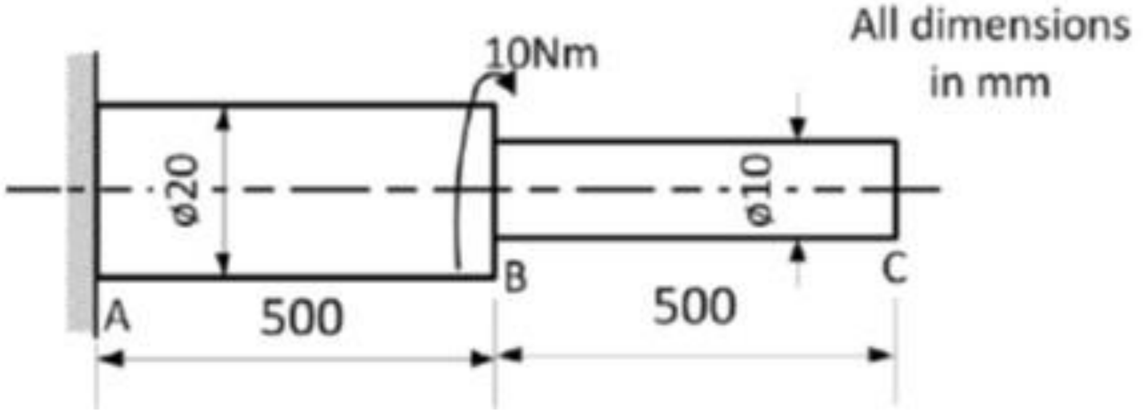
$$T = \tau.\frac{\pi D^3}{16} = \tau.\frac{\pi R^3}{2}$$

$$T_1 = T_2 \Rightarrow \tau_1.\frac{\pi R_1^3}{2} = \tau_2.\frac{\pi R_2^3}{2}$$

$$\frac{\tau_2}{\tau_1} = \frac{R_1^3}{R_2^3} \Rightarrow \frac{\tau_2}{\tau_1} = (2)^3 \Rightarrow \frac{\tau_2}{\tau_1} = 8$$

17. Consider a stepped shaft subjected to a twisting moment applied at B as shown in the figure. Assume shear modulus, G = 77 GPa. The angle of twist at C (in degrees) is….

GATE ME 2015



Ans: 0.22 to 0.25

17. 0.237

Modulus of rigidity, G = 77 GPa

Angle of twist is same between B and C since no torsion induced in BC

$$\Rightarrow \theta_R = \theta_B$$

Torque, T = 10 Nm

Diameter of shaft, D = 20 mm,   Length of shaft, L = 500 mm

Angle of wrist, $\theta_B = \dfrac{TL}{GJ}$

$$\theta_B = \frac{10\times10^3 \times 500}{77\times10^3 \times \frac{\pi(20)^4}{32}} \times \frac{180}{\pi} = 0.237^0$$

$$\theta_c = 0.237^0$$

18. A hollow shaft of 1 m length is designed to transmit a power of 30 kW at 700 rpm. The maximum permissible angle of twist in the shaft is 1°. The inner diameter of the shaft is 0.7 times the outer diameter. The modulus of rigidity is 80 GPa. The outside diameter (in mm) of the shaft is.... GATE ME 2015

Ans: 45 to 45

18. 44.52

Length of hollow shaft, L= 1m

Power transmitted, P = 30 kW

Angular speed, N = 700 rpm

The maximum permissible angle of twist, $\theta = 1^0$

Out diameter of shaft, $D_1 = D = ?$

Inner diameter of shaft, $D_1 = 0.7D$

Modulus of rigidity, G = 80 GPa

$$P = \frac{2\pi NT}{60000}$$

$$30 = \frac{2\pi \times 700 \times T}{60000} \Rightarrow \text{T} = 409.25\ \text{Nm}$$

$$\text{Polar moment of inertia, } J = \frac{\pi}{32}\left(D_1^4 - D_2^4\right)$$

$$= \frac{\pi}{32}\left[D^4 - (0.7D)^4\right] = 0.0746\,\text{D}^4$$

$$\theta = \frac{TL}{GJ} \Rightarrow 1 \times \frac{\pi}{180} = \frac{409.25 \times 10^3 \times 1 \times 10^3}{80 \times 10^3 \times 0.0746 D^4}$$

$$D^4 = 3929004.3 \Rightarrow \text{D} = 44.52\ \text{mm}$$

19. A hollow shaft ($d_0 = 2d_i$ where d_0 and d_i are the outer and inner diameters respectively) needs to transmit 20 kW power at 3000 RPM. If the maximum permissible shear stress is 30 MPa, d_0 is… GATE ME 2015

a. 11.29 mm **b.** 22.58 mm c. 33.87 mm d. 45.16 mm

19. B

Out diameter of shaft = d_0

Inner diameter of shaft = d_i

$d_0 = 2d_i$, $d_0 = ?$

Power transmitted, P = 20 kW

Angular speed, N = 3000 rpm

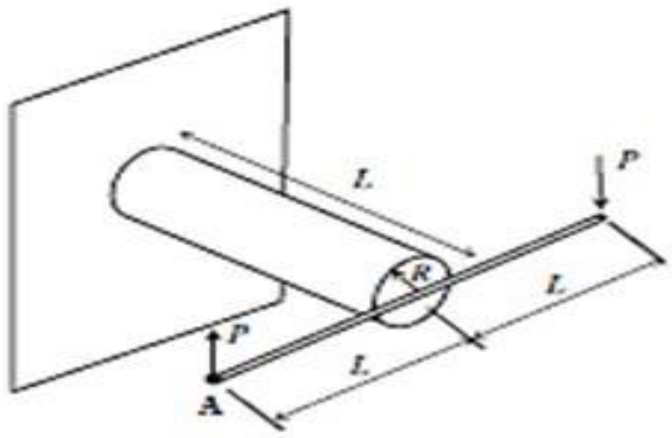
Maximum permissible shear stress, $\tau_{max} = 30\,\text{MPa}$

$$Z = \frac{\pi}{16d_0}\left(d_0^4 - d_i^4\right) = \frac{\pi}{16\,d_0}\left[d_0^4 - (0.5d_0)^4\right] = 0.1841d_0^3$$

$$p = \frac{2\pi NT}{60000} \Rightarrow 20 = \frac{2\pi \times 3000T}{60000} \Rightarrow \text{T= 63.66 Nm}$$

$$\tau_{\max} = \frac{T}{Z} \Rightarrow \frac{63.66\times 10^3}{0.1841\,d_0^3} = 30 \Rightarrow d_0^3 = 11.53\times 10^3\,,\ d_0 = 22.59\,\text{mm}$$

17. A rigid horizontal rod of length $2L$ is fixed to a circular cylinder of radius R as shown in the figure. Vertical forces of magnitude P are applied at the two ends as shown in the figure. The shear modulus for the cylinder is G and the Young's Modulus is E. GATE ME2 2016



The vertical deflection at point **A** is

a. $\frac{PL^3}{\pi R^4 G}$ b. $\frac{PL^3}{\pi R^4 E}$ c. $\frac{2PL^3}{\pi R^4 E}$ d. $\frac{4PL^3}{\pi R^4 G}$

17. D

Radius of circular cylinder = R

Torque produced on cylinder, $T = 2PL$

Polar moment of inertial, $J = \dfrac{\pi R^4}{2}$

Angle of twist, $\theta = \dfrac{TL}{GJ}$

$$= \frac{2PL.L}{G.\dfrac{\pi R^4}{2}} = \frac{4PL^2}{\pi R^4 G}$$

Deflection at point A, $\delta_A = \theta.L$

$$= \frac{4PL^2}{\pi R^4 G}.L = \frac{4PL^3}{\pi R^4 G}$$

21. A shaft with a circular cross-section is subjected to pure twisting moment. The ratio of the maximum shear stress to the largest principal stress is

GATE ME2 2016

a. 2.0 b. 1.0 c. 0.5 d. 0

21. B

When the shaft is subjected to pure twisting moment, only shear stress induced in it. Largest principal stress is equal to the maximum shear stress.

$$\frac{\tau_{max}}{\sigma_1} = \frac{\tau}{\tau} = 1.0$$

22. Two circular shafts made of same material, one solid (S) and one hollow (H), have the same length and polar moment of inertia. Both are subjected to same torque. Here, θ_s is the twist and τ_s is the maximum shear stress in the solid shaft, whereas θ_H is the twist and τ_H is the maximum shear stress in the hollow shaft. Which one of the following is TRUE? GATE ME3 2016

a. $\theta_s = \theta_H$ and $\tau_s = \tau_H$

b. $\theta_s > \theta_H$ and $\tau_s > \tau_H$

c. $\theta_s < \theta_H$ and $\tau_s < \tau_H$

d. $\theta_s = \theta_H$ and $\tau_s < \tau_H$

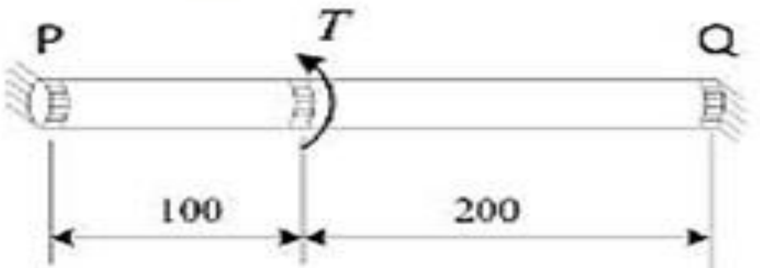
22. d

For solid and hollow circular shaft have the same modulus of rigidity (G), length (L) polar moment of inertia (J) and subjected to same torque (T).

Angle of twist, $\theta = \dfrac{TL}{GJ} \Rightarrow \theta_3 = \theta_H$

Maximum shear stress, $\tau = \dfrac{T}{Z} = \dfrac{T}{\frac{J}{R}} = \dfrac{T.R}{J} \Rightarrow T \alpha R \Rightarrow \tau_S < \tau_H$

23. A bar of circular cross section is clamped at ends P and Q as shown in the figure. A torsional moment $T = 150$ Nm is applied at a distance of 100 mm from end P. The torsional reactions (T_P, T_Q) in Nm at the ends P and Q respectively are (All dimensions are in mm) GATE ME2 2018



a. (50, 100) b. (75, 75) c. (100, 50) d. (120, 30)

23. c

$$T_p = \frac{T.b}{L} = \frac{150 \times 200}{300} = 100 \text{ kNm}$$

$$T_Q = \frac{T.a}{L} = \frac{150 \times 100}{300} = 50 \text{ kNm}$$

20. A cylindrical rod of diameter 10 mm and length 1.0 m is fixed at one end. The other end is twisted by an angle of 10° by applying a torque. If the maximum shear strain in the rod is $p \times 10^{-3}$, then p is equal to.... (round off to two decimal places) GATE ME 2019

20. **0.873**

Length of cylindrical rod, L =1.0m

Diameter of rod, D =10mm, R =5mm

Angle of twist, $\theta = 10^0$

Maximum shear strain in the rod, $\phi_{max} = ?$

$$\phi_{max} = P \times 10^{-3}$$

Torsion equation is $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau_{max}}{R}$

$$\frac{G\theta}{L} = \frac{\tau_{max}}{R} \Rightarrow \frac{\tau_{max}}{G} = \frac{R\theta}{L} \Rightarrow \phi_{max} = \frac{R\theta}{L}$$

$$\phi_{max} = \frac{5 \times 10 \times \frac{\pi}{180}}{1 \times 10^3} = 0.8727 \times 10^{-3}$$

P =0.8727


Thank You