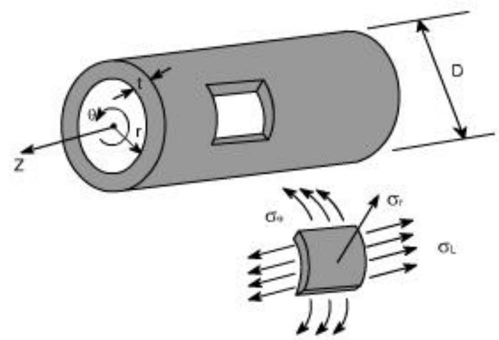
STRENGTH OF MATERIALS – SME1204 – V UNIT COURSE MATERIAL

Pressurized thin walled cylinder

Preamble Pressure vessels are exceedingly important in industry. Normally two types of pressure vessel are used in common practice such as cylindrical pressure vessel and spherical pressure vessel.

In the analysis of this walled cylinders subjected to internal pressures it is assumed that the radial plans remains radial and the wall thickness does not change due to internal pressure. Although the internal pressure acting on the wall causes a local compressive stresses (equal to pressure) but its value is neglibly small as compared to other stresses & hence the sate of stress of an element of a thin walled pressure is considered a biaxial one. Further in the analysis of thin walled cylinders, the weight of the fluid is considered neglible. Let us consider a long cylinder of circular cross - section with an internal radius of R_2 and a constant wall thickness ‘t' as showing fig.



This cylinder is subjected to a difference of hydrostatic pressure of ‘p' between its inner and outer surfaces. In many cases, ‘p' between gage pressure within the cylinder, taking outside pressure to be ambient. By thin walled cylinder we mean that the thickness ‘t' is very much smaller than the radius R_i and we may quantify this by stating than the ratio t / R_i of thickness of radius should be less than 0.1. An appropriate co-ordinate system to be used to describe such a system is the cylindrical polar one r, q, z shown, where z axis lies along the axis of the cylinder, r is radial to it and q is the angular co-ordinate about the axis. The small piece of the cylinder wall is shown in isolation, and stresses in respective direction have also been shown.

Type of failure

Such a component fails in since when subjected to an excessively high internal pressure. While it might fail by bursting along a path following the circumference of the cylinder. Under normal circumstance it fails by circumstances it fails by bursting along a path parallel to the axis. This suggests that the hoop stress is significantly higher than the axial stress.

In order to derive the expressions for various stresses we make following

Applications

Liquid storage tanks and containers, water pipes, boilers, submarine hulls, and certain air plane components are common examples of thin walled cylinders and spheres, roof domes.

ANALYSIS : In order to analyse the thin walled cylinders, let us make the following assumptions :

- There are no shear stresses acting in the wall.
- The longitudinal and hoop stresses do not vary through the wall.
- Radial stresses s_r which acts normal to the curved plane of the isolated element are neglibly small as compared to other two stresses especially when $\left[\frac{t}{R_i} < \frac{1}{20}\right]$

The state of tress for an element of a thin walled pressure vessel is considered to be biaxial, although the internal pressure acting normal to the wall causes a local compressive stress equal to the internal pressure, Actually a state of tri-axial stress exists on the inside of the vessel. However, for the walled pressure vessel the third stress is much smaller than the other two stresses and for this reason in can be neglected.

Thin Cylinders Subjected to Internal Pressure:

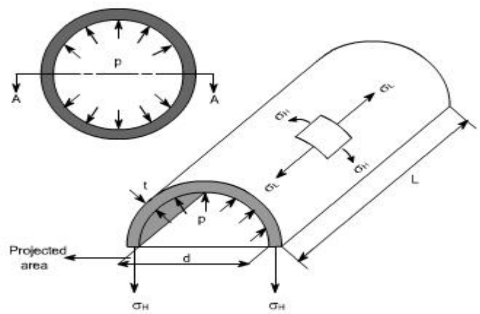
When a thin – walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, namely

- Circumferential or hoop stress
- The radial stress
- Longitudinal stress

now let us define these stresses and determine the expressions for them

Hoop or circumferential stress:

This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of the cylinder.



In the figure we have shown a one half of the cylinder. This cylinder is subjected to an internal pressure p.

i.e. p = internal pressure

d = inside diametre

L = Length of the cylinder

t = thickness of the wall

Total force on one half of the cylinder owing to the internal pressure 'p'

= p x Projected Area

= p x d x L

= **p .d. L** ------- (1)

The total resisting force owing to hoop stresses s_H set up in the cylinder walls

= **2 .s_H .L.t** ---------(2)

Because s_H.L.t. is the force in the one wall of the half cylinder.

the equations (1) & (2) we get

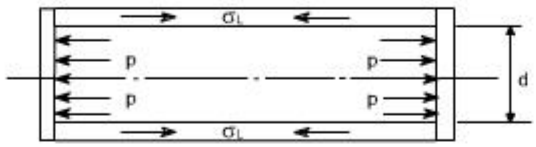
2 . s_H . L . t = p . d . L

s_H = (p . d) / 2t

Circumferential or hoop Stress (s_H) = (p .d)/ 2t

Longitudinal Stress:

Consider now again the same figure and the vessel could be considered to have closed ends and contains a fluid under a gage pressure p.Then the walls of the cylinder will have a longitudinal stress as well as a ciccumferential stress.



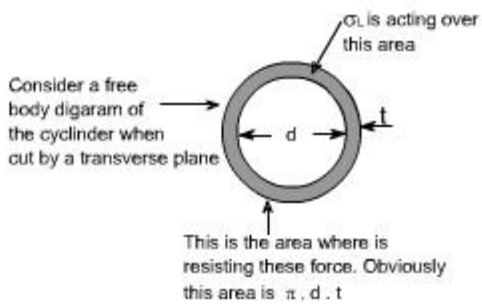
Total force on the end of the cylinder owing to internal pressure

= pressure x area

= p x p d^2 /4

Area of metal resisting this force = pd.t. (approximately)

because pd is the circumference and this is multiplied by the wall thickness



Hence the longitudnal stresses

$$\text{Set up} = \frac{\text{force}}{\text{area}} = \frac{[p \times \pi d^2/4]}{\pi d t}$$

$$= \frac{pd}{4t} \quad \text{or} \quad \sigma_L = \frac{pd}{4t}$$

or alternatively from equilibrium conditions

$$\sigma_L.(\pi d t) = p.\frac{\pi d^2}{4}$$

$$\text{Thus} \boxed{\sigma_L = \frac{pd}{4t}}$$

Change in Dimensions :

The change in length of the cylinder may be determined from the longitudinal strain.

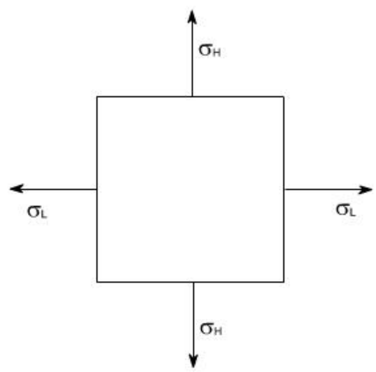
Since whenever the cylinder will elongate in axial direction or longitudinal direction, this will also get decreased in diameter or the lateral strain will also take place. Therefore we will have to also take into consideration the lateral strain as we know that the poisson's ratio (ν) is

$$\nu = \frac{-\text{lateral strain}}{\text{longitudnal strain}}$$

where the -ve sign emphasized that the change is negative

Consider an element of cylinder wall which is subjected to two mutually ^r normal stresses s_L and s_H .

Let E = Young's modulus of elasticity



Resultant Strain in longitudnal direction $= \frac{\sigma_L}{E} - \nu \frac{\sigma_H}{E} = \frac{1}{E}(\sigma_L - \nu\sigma_H)$

recalling

$$\sigma_L = \frac{pd}{4t} \qquad \sigma_H = \frac{pd}{2t}$$

$\in_1$ (longitudnal strain) $= \frac{pd}{4Et}[1-2\nu]$

or

Change in Length = Longitudalstrain x original Length

$= \in_1 . L$

Similarly the hoop Strain $\in_2 = \frac{1}{E}(\sigma_H - \nu\sigma_L) = \frac{1}{E}\left[\frac{pd}{2t} - \nu\frac{pd}{4t}\right]$

$$\in_2 = \frac{pd}{4Et}[2-\nu]$$

Infact $\in_2$ is the hoop strain if we just go by the definition then

$$\in_2 = \frac{\text{Change in diametre}}{\text{Original diametre}} = \frac{\delta d}{d}$$

where d = original diameter.

if we are interested to find out the change in diametre then

Changein diametre $= \in_2$ .Original diametre

i.e $\delta d = \in_2 . d$ substituting the value of $\in_2$ we get

$$\delta d = \frac{p.d}{4.t.E}[2-\nu].d$$

$$= \frac{p.d^2}{4.t.E}[2-\nu]$$

i.e $\boxed{\delta d = \frac{p.d^2}{4.t.E}[2-\nu]}$

Volumetric Strain or Change in the Internal Volume:

When the thin cylinder is subjected to the internal pressure as we have already calculated that there is a change in the cylinder dimensions i.e, longitudinal strain and hoop strains come into picture. As a result of which there will be change in capacity of the cylinder or there is a change in the volume of the cylinder hence it becomes imperative to determine the change in volume or the volumetric strain.

The capacity of a cylinder is defined as

V = Area X Length

= $pd^2/4$ x L

Let there be a change in dimensions occurs, when the thin cylinder is subjected to an internal pressure.

(i) The diameter **d** changes to **d + d d**

(ii) The length **L** changes to **L + d L**

Therefore, the change in volume = Final volume - Original volume

$$= \frac{\pi}{4}[d + \delta d]^2.(L + \delta L) - \frac{\pi}{4} d^2.L$$

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\frac{\pi}{4}[d + \delta d]^2.(L + \delta L) - \frac{\pi}{4} d^2.L}{\frac{\pi}{4} d^2.L}$$

$$\in_v = \frac{\left\{[d + \delta d]^2.(L + \delta L) - d^2.L\right\}}{d^2.L} = \frac{\left\{(d^2 + \delta d^2 + 2d.\delta d).(L + \delta L) - d^2.L\right\}}{d^2.L}$$

simplifying and neglecting the products and squares of small quantities, i.e. δd & δL

hence

$$= \frac{2d.\delta d.L + \delta L.d^2}{d^2 L} = \frac{\delta L}{L} + 2.\frac{\delta d}{d}$$

By definition $\frac{\delta L}{L}$ = Longitudnal strain

$\frac{\delta d}{d}$ = hoop strain, Thus

Volumetric strain = longitudnal strain + 2 x hoop strain

on substituting the value of longitudnal and hoop strains we get

$$\in_1 = \frac{pd}{4tE}[1 - 2\nu] \quad \& \quad \in_2 = \frac{pd}{4tE}[1 - 2\nu]$$

$$\text{or Volumetric} = \in_1 + 2\in_2 = \frac{pd}{4tE}[1 - 2\nu] + 2.\left(\frac{pd}{4tE}[1 - 2\nu]\right)$$

$$= \frac{pd}{4tE}\{1 - 2\nu + 4 - 2\nu\} = \frac{pd}{4tE}[5 - 4\nu]$$

$$\text{Volumetric Strain} = \frac{pd}{4tE}[5 - 4\nu] \qquad \text{or} \qquad \boxed{\in_v = \frac{pd}{4tE}[5 - 4\nu]}$$

Therefore to find but the increase in capacity or volume, multiply the volumetric strain by original volume.

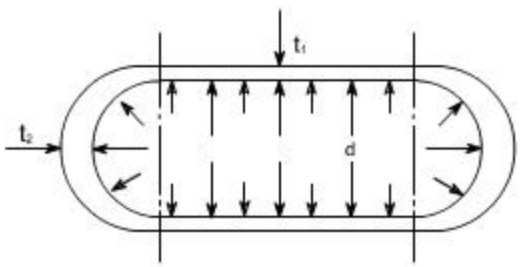
Hence

Change in Capacity / Volume or

$$\boxed{\text{Increase in volume} = \frac{pd}{4tE}[5 - 4\nu]V}$$

Cylindrical Vessel with Hemispherical Ends:

Let us now consider the vessel with hemispherical ends. The wall thickness of the cylindrical and hemispherical portion is different. While the internal diameter of both the portions is assumed to be equal. Let the cylindrical vassal is subjected to an internal pressure p.



For the Cylindrical Portion

$$\text{hoop or circumferential stress} = \sigma_{HC} \qquad \text{'c' here synifies the cylindrical portion.}$$

$$= \frac{pd}{2t_1}$$

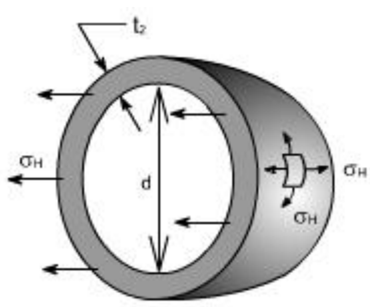
$$\text{longitudnal stress} = \sigma_{LC}$$

$$= \frac{pd}{4t_1}$$

$$\text{hoop or circumferential strain } \epsilon_2 = \frac{\sigma_{HC}}{E} - \nu\frac{\sigma_{LC}}{E} = \frac{pd}{4t_1E}[2-\nu]$$

$$\text{or} \quad \boxed{\epsilon_2 = \frac{pd}{4t_1E}[2-\nu]}$$

<u>For The Hemispherical Ends:</u>



Because of the symmetry of the sphere the stresses set up owing to internal pressure will be two mutually perpendicular hoops or circumferential stresses of equal values. Again the radial stresses are neglected in comparison to the hoop stresses as with this cylinder having thickness to diametre less than1:20.

Consider the equilibrium of the half – sphere

Force on half-sphere owing to internal pressure = pressure x projected Area

= p. pd^2/4

$$\text{Resisting force} = \sigma_H \,.\, \pi.d.t_2$$

$$\therefore \qquad p.\frac{\pi.d^2}{4} = \sigma_H \,.\, \pi\, d.t_2$$

$$\Rightarrow \ \sigma_H \,(\text{for sphere}) = \frac{pd}{4t_2}$$

$$\text{similarly the hoop strain} = \frac{1}{E}\left[\sigma_H - \nu.\sigma_H\right] = \frac{\sigma_H}{E}[1-\nu] = \frac{pd}{4t_2E}[1-\nu] \text{ or } \boxed{\epsilon_{2s} = \frac{\mathbf{pd}}{\mathbf{4t_2E}}[\mathbf{1-\nu}]}$$

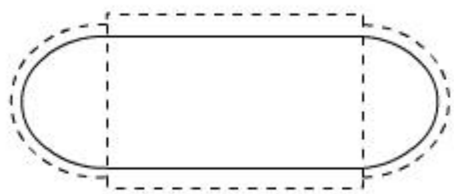


Fig – shown the (by way of dotted lines) the tendency, for the cylindrical portion and the spherical ends to expand by a different amount under the action of internal pressure. So owing to difference in stress, the two portions (i.e. cylindrical and spherical ends) expand by a different amount. This incompatibly of deformations causes a local bending and sheering stresses in the neighborhood of the joint. Since there must be physical continuity between the ends and the cylindrical portion, for this reason, properly curved ends must be used for pressure vessels.

Thus equating the two strains in order that there shall be no distortion of the junction

$$\frac{pd}{4t_1E}[2-\nu] = \frac{pd}{4t_2E}[1-\nu] \quad \text{or} \quad \frac{t_2}{t_1} = \frac{1-\nu}{2-\nu}$$

But for general steel works ν = 0.3, therefore, the thickness ratios becomes

$\mathbf{t_2 / t_1 = 0.7/1.7}$ or

$$\boxed{\mathbf{t_1 = 2.4\, t_2}}$$

i.e. the thickness of the cylinder walls must be approximately 2.4 times that of the hemispheroid ends for no distortion of the junction to occur.

SUMMARY OF THE RESULTS : Let us summarise the derived results

(A) The stresses set up in the walls of a thin cylinder owing to an internal pressure p are :

(i) Circumferential or loop stress

$\mathbf{s_H = pd/2t}$

(ii) Longitudinal or axial stress

$\mathbf{s_L = pd/4t}$

Where d is the internal diametre and t is the wall thickness of the cylinder.

then

Longitudinal strain $\mathbf{\hat{I}_L = 1 / E [s_L - \nu s_H]}$

Hoop stain $\mathbf{\hat{I}_H = 1 / E [s_H - \nu s_L]}$

(B) Change of internal volume of cylinder under pressure

$$= \frac{pd}{4tE}[5 - 4\nu]V$$

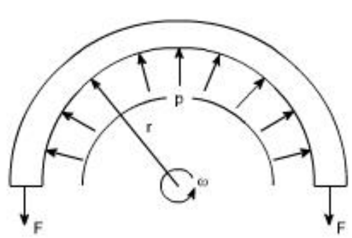
(C) Fro thin spheres circumferential or loop stress

$$\sigma_H = \frac{pd}{4t}$$

Thin rotating ring or cylinder

Consider a thin ring or cylinder as shown in Fig below subjected to a radial internal pressure p caused by the centrifugal effect of its own mass when rotating. The centrifugal effect on a unit length of the circumference is

$p = m\ w^2\ r$



Thin ring rotating with constant angular velocity w

Here the radial pressure ‘p' is acting per unit length and is caused by the centrifugal effect if its own mass when rotating.

Thus considering the equilibrium of half the ring shown in the figure,

2F = p x 2r (assuming unit length), as 2r is the projected area

F = pr

Where F is the hoop tension set up owing to rotation.

The cylinder wall is assumed to be so thin that the centrifugal effect can be assumed constant across the wall thickness.

F = mass x acceleration = $\mathbf{m\ w^2\ r\ x\ r}$

This tension is transmitted through the complete circumference and therefore is resisted by the complete cross – sectional area.

hoop stress = F/A = $\mathbf{m\ w^2\ r^2\ /\ A}$

Where A is the cross – sectional area of the ring.

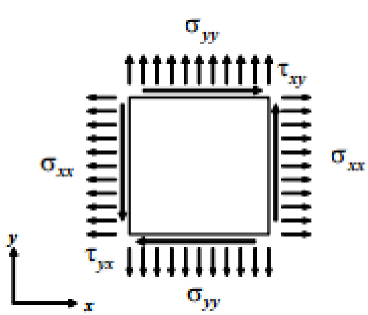
Now with unit length assumed m/A is the mass of the material per unit volume, i.e. the density r .

hoop stress = $\mathbf{r\ w^2\ r^2}$

$\mathbf{s_H = r\ .\ w^2\ .\ r^2}$

BIAXIAL STRESS SYSTEMS

A biaxial stress system has a stress state in two directions and a shear stress typically showing in Fig..



Element of a structure showing a biaxial stress system

When a Biaxial Stress state occurs in a thin metal, all the stresses are in the plane of the material. Such a stress system is called PLANE STRESS. We can see plane stress in pressure vessels, aircraft skins, car bodies, and many other structures.

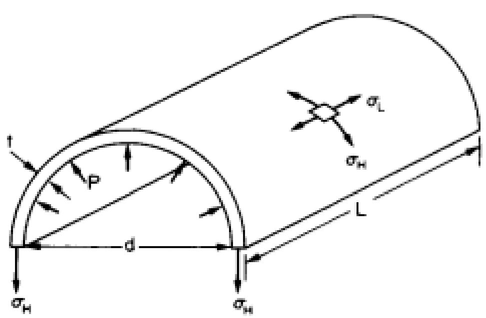
THIN CYLINDERS AND SPHERICAL SHELLS

The stresses set up in the walls of a thin cylinder owing to an internal pressure p are:
circumferential or hoop stress = pd/2t and
longitudinal or axial stress = pd/4t

DEFORMATION IN THIN CYLINDRICAL AND SPHERICAL SHELLS

Hoop or circumferential stress

This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of half of the cylinder as shown in Fig.



Half of a thin cylinder subjected to internal pressure showing the hoop and longitudinal stresses acting on any element in the cylinder surface.

Total force on half-cylinder owing to internal pressure $= p \times$ projected area $= p \times dL$

Total resisting force owing to hoop stress σ_H set up in the cylinder walls

$$= 2\sigma_H \times Lt$$

$$\therefore \quad 2\sigma_H Lt = pdL$$

$$\therefore \quad \textbf{circumferential or hoop stress } \sigma_H = \frac{pd}{2t}$$

Longitudinal stress or axial stress

Consider now the cylinder shown in the fig..

Total force on the end of the cylinder owing to internal pressure

$$= \text{pressure} \times \text{area} = p \times \frac{\pi d^2}{4}$$

Fig. 9.2. Cross-section of a thin cylinder.

Area of metal resisting this force = πdt (approximately)

$$\therefore \qquad \text{stress set up} = \frac{\text{force}}{\text{area}} = p \times \frac{\pi d^2/4}{\pi dt} = \frac{pd}{4t}$$

i.e. $$\textbf{longitudinal stress } \sigma_L = \frac{pd}{4t}$$

Problem 1

A thin cylindrical pipe of diameter 1.5 mm and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm^2. Determine:

i)Longitudinal stress developed in the pipe and

ii)Circumferential stress developed in the pipe.

Solution:

Given:

Dia of pipe d=1.5 m

Thickness, t=1.5 cm = $1.5x10^{-2}$m

Internal fluid pressure, p=1.2 N/mm^2

i) The longitudinal stress is given by

σ =pd/2t

= $(1.2x1.5)/(4x1.5x10^{-2})$

=30 N/mm^2

ii) The circumferential stress is given by

σ =pd/4t

=$(1.2x1.5) / (2x1.5x10^{-2})$

$=60\ N/mm^2$

Problem 2

A cylinder of internal diameter 2.5 m and of thickness 5cm contains a gas.If the tensile stress in the material is not to exceed 80 N/mm^2, determine the internal pressure of the gas.

Solution:

Given:

Internal dia of cylinder d=2.5 cm

Thickness of cylinder $t= 5cm=5x10^{-2}m$

Maximum permissible stress $=80\ N/mm^2$

As maximum permissible stress is given, hence this should be equal to circumferential stress σ

$\sigma =80\ N/mm^2$

$\sigma =pd/2t$

$P=(2t \times \sigma)/d$

$=(2x5x10^{-2}x80) / 2.5$

$=3.2\ N/mm^2$

Efficiency of a joint

The cylindrical shells are having two types of joints namely longitudinal joint and circumferential joint.

Let η_l = efficiency of a longitudinal joint and

η_c = efficiency of a circumferential joint……

the circumferential stress(σ_1) is given by,

$\sigma_1 = (p \times d) / (2t \times \eta_l)$ and

longitudinal stress(σ_2) is given by.,

$\sigma_2 = (p \times d) / (4t \times \eta_c)$

In longitudinal joint, the circumferential stress is developed whereas in circumferential joint the longitudinal stress is developed.

Problem 3:

A boiler is subjected to an internal steam pressure of 2 N/mm^2, the thickness of a boiler plate is 2cm and permissible tensile stress is 120 N/mm^2 , find out the maximum diameter when efficiency of longitudinal joint is 90% and that of circumferential joint is 40%.

Solution:

Given

Internal steam pressure, p = 2 N/mm^2

Thickness of boiler plate, t =2cm

Permissible tensile stress = 120 N/mm^2

In case of a joint, the permissible stress may be circumferential stress or longitudinal stress.

efficiency of longitudinal joint = η_l = 90% = 0.90

efficiency of circumferential joint = η_c = 40% = 0.40

max. diameter for circumferential stress is given by,

σ_1 = (p x d) /(2t x η_l)

where σ_1 = given Permissible tensile stress = 120 N/mm^2

120 = (2 x d) / (2x 0.90 x2)

d= (120x2x0.9x2) / 2

= 216 cm.

Max.diameter for longitudinal stress is given by,

σ_2 = (p x d) / (4t x η_c)

where σ_2 = given Permissible tensile stress = 120 N/mm^2

$$120 = (2 \text{ x } d) / (4\text{x } 0.40 \text{ x}2)$$

$$d= (120\text{x}4\text{x}0.4\text{x}2) / 2$$

$$d=192 \text{ cm.}$$

the longitudinal or circumferential stresses included in the material are directly proportional to the diameter (d), and hence stress induced will be less if the value of d is less. Hence minimum value of d is taken…..so, max.diameter = 192 cm

Effect of internal pressure on the dimensions of a thin cylindrical shell

When a fluid having internal pressure (p) is stored in a thin cylindrical shell, du internal pressure of the fluid the stresses set up at any point of the material of the shell are

(i) Hoop or circumferential stress (σ_1), acting on longitudinal section.

(ii) Longitudinal stress (σ_2) acting on the circumferential section.

These stresses are principal stresses, as they are acting on principal planes. The stre the third principal plane is zero as the thickness (t) of the cylinder is very small. Actually stress in the third principal plane is radial stress which is very small for thin cylinders and be neglected.

Let p = Internal pressure of fluid
L = Length of cylindrical shell
d = Diameter of the cylindrical shell
t = Thickness of the cylindrical shell
E = Modulus of Elasticity for the material of the shell
σ_1 = Hoop stress in the material
σ_2 = Longitudinal stress in the material
μ = Poisson's ratio
δd = Change in diameter due to stresses set up in the material
δL = Change in length
δV = Change in volume.

Then, circumferential strain,

$$e_1 = (\sigma_1 / E) – (\mu \, \sigma_2 /E)$$

$$= \frac{pd}{2tE} (1- \mu/2)$$

and longitudinal strain,

$$e_2 = (\sigma_2 / E) - (\mu \sigma_1 / E)$$

$$= \frac{pd}{2tE}(1/2 - \mu)$$

Change in diameter, $\delta d/d = \frac{pd}{2tE}(1 - \mu/2)$

Change in length, $\delta L/L = \frac{pd}{2tE}(1/2 - \mu)$

Change in volume, $\delta V/V = (2e_1 + e_2)$

$$= V(2\, \delta d/d + \delta L/L)$$

Problem 4:

Calculate change in diameter, change in length and change in volume of a thin cylindrical shell 100cm diameter, 1cm thickness and 5m long when subjected to internal pressure of $3 N/mm^{2,}$ take the value of $E = 2 \times 10^5 N/mm^2$ and poisson's ratio $\mu = 0.3$

Solution:

Given: diameter of shell, d=100cm
Thickness of shell, t= 1cm
Length of shell, L= 5m= 500cm
Internal pressure, $p = 3 N/mm^2$
Young's modulus, $E = 2 \times 10^5 N/mm^2$
And Poisson's ratio $\mu = 0.3$

(*i*) **Change in diameter (δd) is given by equation**

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2} \times \mu\right)$$

$$= \frac{2.5 \times 80^2}{2 \times 1 \times 2 \times 10^5}\left[1 - \frac{1}{2} \times 0.25\right]$$

$= 0.04\ [1 - 0.125] =$ **0.035 cm.**

(*ii*) **Change in length (δL) is given by equation**

$$\delta L = \frac{pdL}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$= \frac{2.5 \times 80 \times 300}{2 \times 1 \times 2 \times 10^5}\left[\frac{1}{2} - 0.25\right] = \mathbf{0.0375\ cm.}$$

(iii) change in volume δV/V is given by,

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$= 2 \times \frac{0.035}{80} + \frac{0.0375}{300} \qquad \left(\because \begin{array}{ll} \delta d = 0.035, & \delta L = 0.0375 \\ d = 80, & L = 300 \end{array}\right)$$

$$= 0.000875 + 0.000125 = 0.001$$

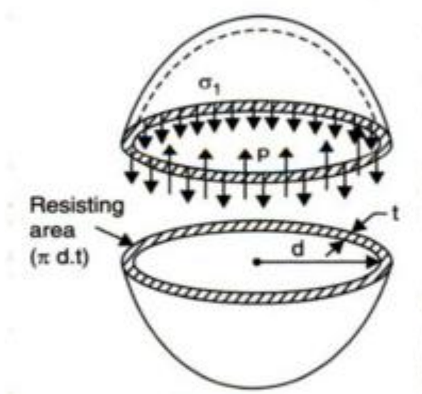
$$\therefore \qquad \delta V = 0.001 \times V$$

where volume $V = \frac{\pi}{4} d^2 \times L = \frac{\pi}{4} \times 80^2 \times 300 = 1507964.473\ cm^3$

$\therefore$ Change in volume, $\delta V = 0.001 \times 1507964.473 = \mathbf{1507.96\ cm^3}$. Ans.

Thin spherical shells

The figure shows a thin spherical shell of internal diameter d and thickness t and subjected to internal fluid pressure p , the fluid inside the shell has a tendency to split the shell into two hemispheres along x-x axis.



Circumferential or hoop stress(σ_1) is given by,

$$\sigma_1 = pd/4t$$

circumferential stress when the joint efficiency is given by,

$$\sigma_1 = pd/4t.\ \eta$$

Problem 5

A vessel in the shape of a spherical shell of 1.20m internal diameter and 12mm shell thickness is subjected to pressure of 1.6 N/mm^2, determine the stress induced in the material of the vessel.

Solution
Given.

Internal diameter , d = 1.2m = 1200mm

Shell thickness, t = 12mm and

Fluid pressure, p = 1.6 N/mm^2

The stress induced in the material of the spherical shell is given by,

$$\sigma_1 = pd/4t$$
$$= (1.6 \times 1200) / (4 \times 12)$$
$$= 40\ N/mm^2$$

Problem 6

A spherical vessel 1.5m diameter is subjected to an internal fluid pressure of 2 N/mm^2, find the thickness of the plate required if maximum stress is not to exceed 150 N/mm^2 and joint efficiency is 75%

Solution

Given

Diameter of shell, d = 1.5m = 1500mm,

Fluid pressure, p = 2 N/mm^2

Stress in the material, $\sigma_1 =$ 150 N/mm^2

Joint efficiency, η = 75% = 0.75

Let t = thickness of the plate and

Stress induced is given by,

$$\sigma_1 = pd/4t.\ \eta$$
$$t = (p \times d) / (4 \times \eta \times \sigma_1)$$
$$= (2 \times 1500) / (4 \times 0.75 \times 150)$$
$$= 6.67mm$$

Change in dimension of a thin spherical shell due to an internal pressure

Strain in any direction is also noted as $\delta d/d$ which is given by the equation

$$\delta d/d = \frac{pd}{4tE}(1-\mu)$$

and volumetric strain $\delta V/V$ is given by,

$$\delta V/V = 3 \times (\delta d/d)$$
$$= \frac{3pd}{4tE}(1-\mu)$$

Problem 7

A spherical shell of internal diameter 0.9m and of thickness 10mm is subjected to an internal pressure of 1.4 N/mm^2, determine the increase in diameter and increase in volume, take $E = 2 \times 10^5\ N/mm^2$ and $\mu = 0.33$

Solution.

Given.

Internal diameter, d = 0.9m=900mm

Thickness of the shell, t=10mm

Fluid pressure, p = 1.4 N/mm^2

And $E = 2 \times 10^5\ N/mm^2$

$\mu = 0.33$

using the relation

$\delta d/d = \frac{pd}{4tE}(1-\mu)$

$= \frac{1.4 \times 0.9 \times 1000}{4 \times 10 \times 2 \times 10000}(1-0.33)$

$= 105 \times 10^{-6}$

increase in diameter, $\delta d = 105 \times 10^{-6} \times 900$

$= 94.5 \times 10^{-3}$mm

$= 0.0945$mm.

Now,

Volumetric strain = $\delta V/V = 3 \times (\delta d/d)$

$= 3 \times 105 \times 10^{-6}$

$\delta V/V = 315 \times 10^{-6}$

increase in volume , $\delta V = 315 \times 10^{-6} \times V$

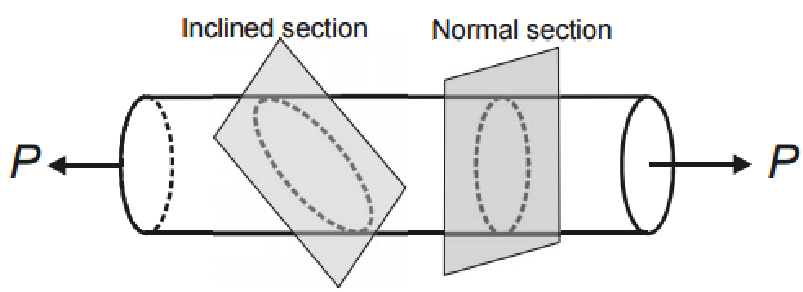
$= 315 \times 10^{-6} \times (\pi/6\ d^3)$

$= 315 \times 10^{-6} \times (\pi/6 \times 900^3)$

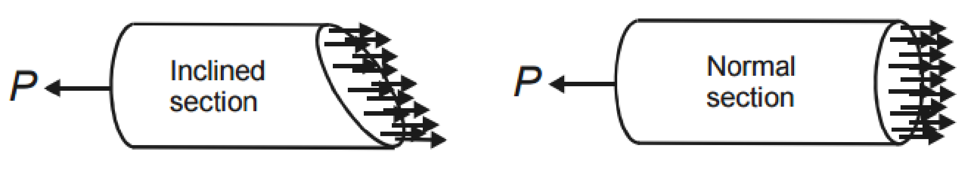
$= 12028.5\ mm^3$

Normal and shear stresses on inclined sections

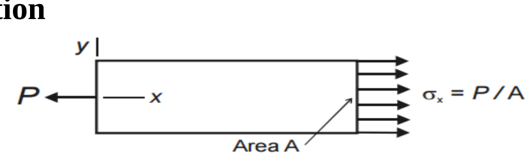
To obtain a complete picture of the stresses in a bar, we must consider the stresses acting on an "inclined" (as opposed to a "normal") section through the bar.



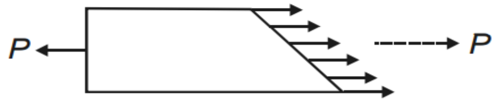
Because the stresses are the same throughout the entire bar, the stresses on the sections are uniformly distributed.



2D view of the normal section



2D view of the inclined section



CURVED BEAM

Theory of Simple Bending

Due to bending moment, tensile stress develops in one portion of section and compressive stress in the other portion across the depth. In between these two portions, there is a layer where stresses are zero. Such a layer is called neutral layer. Its trace on the cross section is called neutral axis. Again for simple bending, the bending equation is suitable for beam which is initially straight before the application of bending moment.

$M/I = \sigma/y = E/R$

Curved beam

Curved beams are the parts of machine members found in C-clamps, crane hooks, frames machines, planers etc. In straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in the case of curved beams the neutral axis of is shifted towards the centre of curvature of the beam causing a non-linear [hyperbolic] distribution of stress. The neutral axis lies between the centroidal axis and the centre of curvature and will always be present within the curved beams. It has also been found that the stresses in the fibres of a curved beam are not proportional to the distances of the fibres from the neutral surfaces, as is assumed for a straight beam.

Differences between Straight and Curved Beams

Sl. no	Straight beam	Curved beam
1	The neutral axis of beam coincides with centroidal axis.	The neutral axis is shifted towards the centre of curvature by a distance called eccentricity i.e. the neutral axis lies between centroidal axis and centre of curvature
2	The variation of normal stress due to bending is linear, tensile at the inner fibre and compressive at the outer fibre with zero value at the centroidal axis.	The variation of normal stress due to bending across section is non-linear and is hyperbolic.

Stresses in Curved Beam **(WINKLER-BATCH THEORY)**

Consider a curved beam subjected to bending moment M_b as shown in the figure. The distribution of stress in curved flexural member is determined by using the following assumptions:

i) The material of the beam is perfectly homogeneous [i.e., same material throughout] and isotropic [i.e., equal elastic properties in all directions]

ii) The cross section has an axis of symmetry in a plane along the length of the beam.

iii) The material of the beam obeys Hooke's law.

iv) The transverse sections which are plane before bending remain plane after bending also.

v) Each layer of the beam is free to expand or contract, independent of the layer above or below it.

vi) The Young's modulus is same both in tension and compression.

vii) The radial strain is negligible.

NOTATIONS AND SYMBOLS USED

$\sigma = Stress$

$\sigma_d = Direct\ stress, tensile\ or\ compressive$

$\sigma_i = Stress\ at\ inner\ fibre$

$\sigma_o = Stress\ at\ outer\ fibre$

$\sigma_{bi} = Normal\ stress\ due\ to\ bending\ at\ inner\ fibre$

$\sigma_{bo} = \ Normal\ stress\ due\ to\ bending\ at\ outer\ fibre$

$M_b = Bending\ moment\ for\ critical\ section, i.e\ moment\ about\ centroidal\ axis$

$r_c = Distance\ of\ centroidal\ axis\ from\ centre\ of\ curvature$

$r_n = Distance\ of\ neutral\ axis\ from\ centre\ of\ curvature$

$e = Eccentricity, i.e\ distance\ betweencentroidal\ axis\ \&\ neutral\ axis$

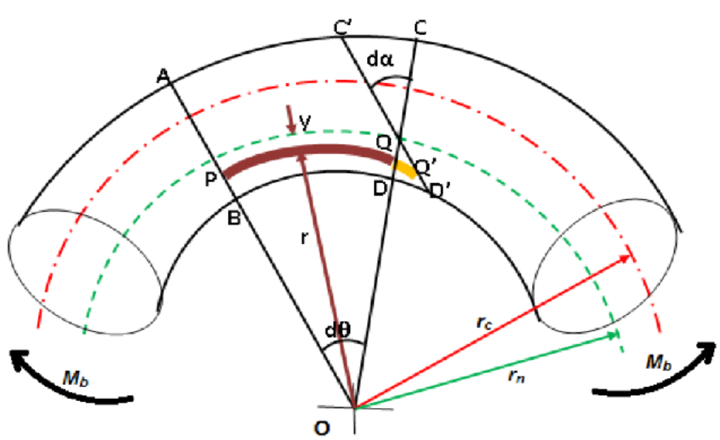
$y = Distance\ of\ fiber\ from\ netral\ axis$

$A = Area\ of\ cross - section\ of\ member, (curved\ beam),$

$P = Load\ on\ member$

$\tau_{max} = Maximum\ shear\ stress$

Derivation of Expression to Determine Stress at any Point on the Fibres of a Curved Beam



Consider a curved beam with r_c, as the radius of centroidal axis, r_n, the radius of neutral surface, r_i, the radius of inner fibre, r_o, the radius of outer fibre subjected to bending moment M_b.

Let *AB* and *CD* be the two adjacent cross-sections separated from each other by a small angle $d\theta$.

Because of M_b the section *CD* rotates through a small angle $d\alpha$ and point "C" shifted to "C' " & point D shifted to D'.

Consider a elemental fiber "PQ" at a distance "r" from center of curvature "o" or at a distance "y" from neutral axis. Due to application of moment M_b , *the point Q shifted to Q'*. The unit deformation of fibre PQ at a distance *y* from neutral surface is:

Deformation $\varepsilon = \frac{QQ\prime}{PQ}$

$$=> \varepsilon = \frac{yd\alpha}{rd\theta} = \frac{yd\alpha}{(r_n - y)d\theta} \qquad ------------------(1)$$

The unit stress on this fibre is, Stress = Strain × Young's modulus of material of beam

$$\sigma = E \times \epsilon = \frac{Eyd\alpha}{(r_n - y)d\theta} \qquad -------------- (2)$$

For equilibrium, the summation of the forces acting on the cross sectional area must be zero.

$$=> \int \sigma dA = 0$$

$$=> \int \frac{Eyd\alpha}{(r_n - y)d\theta} dA = 0$$

$$E\frac{d\alpha}{d\theta} \int \frac{ydA}{(r_n - y)} = 0 \qquad ----------------(3)$$

Also the external moment M_b applied is resisted by internal moment. i.e M_b=-M

$$=> \int y(\sigma dA) = M$$

$$=> \int y\left(\frac{Eyd\alpha}{(r_n - y)d\theta} dA\right) = M$$

$$=> \int \left(\frac{Ey^2 d\alpha}{(r_n - y)d\theta} dA\right) = M$$

$$=> E\,\frac{d\alpha}{d\theta} \int \left(\frac{y^2}{(r_n - y)} dA\right) = M$$

$$=> E\,\frac{d\alpha}{d\theta} \int \left(\frac{y^2 - yr_n + yr_n}{(r_n - y)}\right) dA = M$$

$$=> -E\,\frac{d\alpha}{d\theta} \int ydA + E\,\frac{d\alpha}{d\theta} r_n \int \frac{ydA}{(r_n - y)} = M$$

$$FROM\ equation\ (3) E\frac{d\alpha}{d\theta}\int\frac{ydA}{(r_n-y)}=0$$

$$=> -E\ \frac{d\alpha}{d\theta}\int ydA = M = -M_b$$

$$=> M_b = E\ \frac{d\alpha}{d\theta}\int ydA$$

$\int ydA =$ *The moment of cross sectional area with respect to neutral surface*

$$=> M_b = E\ \frac{d\alpha}{d\theta} \times Ae$$

Here *'e'* represents the distance between the centroidal axis and neutral axis. *i.e.*

$$e = r_c - r_n$$

Rearranging terms in equation

$$=> \frac{d\alpha}{d\theta} = \frac{M_b}{EAe}$$

$$considering\ the\ equation\ (2)\sigma = E \times \epsilon = \frac{Eyd\alpha}{(r_n-y)d\theta} = \frac{M_b}{Ae}\frac{y}{(r_n-y)}$$

$$=> \sigma = \frac{M_b}{Ae}\left[\frac{y}{r_n-y}\right]$$

$$=> \sigma = \frac{M_b}{Ae}\left[\frac{y}{r}\right]$$

Where "r" be the radius of the elemental fiber from center of curvature = $r_n - y$

On considering the equation (3)

$$E\frac{d\alpha}{d\theta}\int\frac{ydA}{(r_n-y)}=0$$

$$=> \int\frac{ydA}{(r_n-y)}=0$$

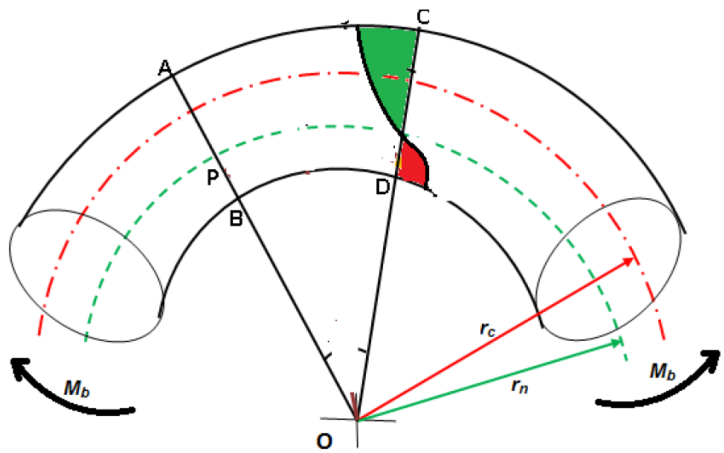
From the figure it is found that "y"= r_n – r or r_n = r + y or r = r_n – y

$$=> \int\frac{ydA}{(r_n-y)} = \int\frac{(y-r_n+r_n)dA}{(r_n-y)} = -\int dA + r_n\int\frac{dA}{r} = 0$$

$$=> r_n\int\frac{dA}{r} = \int dA = A$$

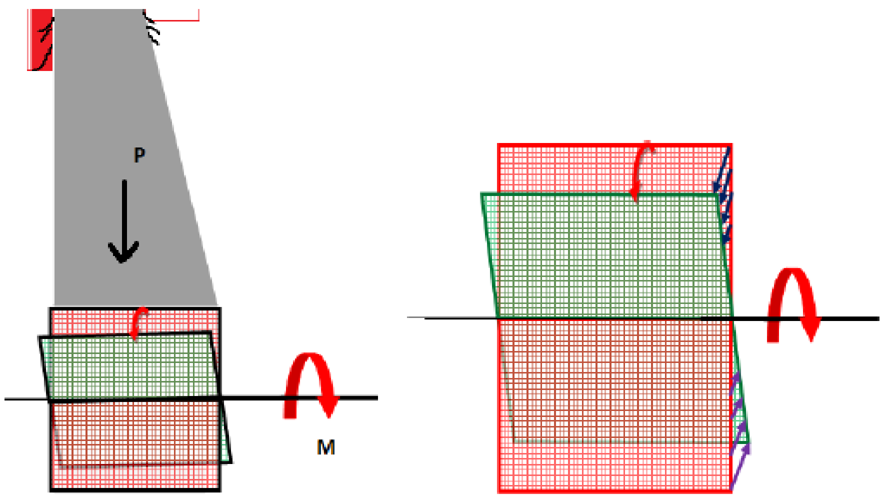
$$=> r_n = \frac{A}{\int\frac{dA}{r}}$$

The bending stress distribution diagram along the depth of the curved beam is shown as follows:



SIGN CONVENTION:-

Consider the simple bending case:-



A cantilever beam subjected to end load as shown in figure. Due to load a bending moment

A cantilever beam subjected to end load as shown in figure. Due to load a bending moment acting about the centroidal axis in clock wise direction which tends to bend the beam & produce a convex surface at the top and a concave surface at the bottom. This implies that the upper fibers / layer at the top are under tension whereas the fibers / layers at the bottom are under compression.

Let consider the cross-section of the beam (plane hatched in red colour), due to moment "M" the cross-section plane (plane hatched in red colour) tends to rotate in clock wise direction (moment direction), i.e the plane hatched in green colour above the neutral axis comes out of the cross-section plane whereas bellow the neutral axis it goes into the cross-section plane. The side of the neutral axis where plane hatched in green colour comes out, that side of the neutral axis the bending stress is tensile in nature and vice-versa.

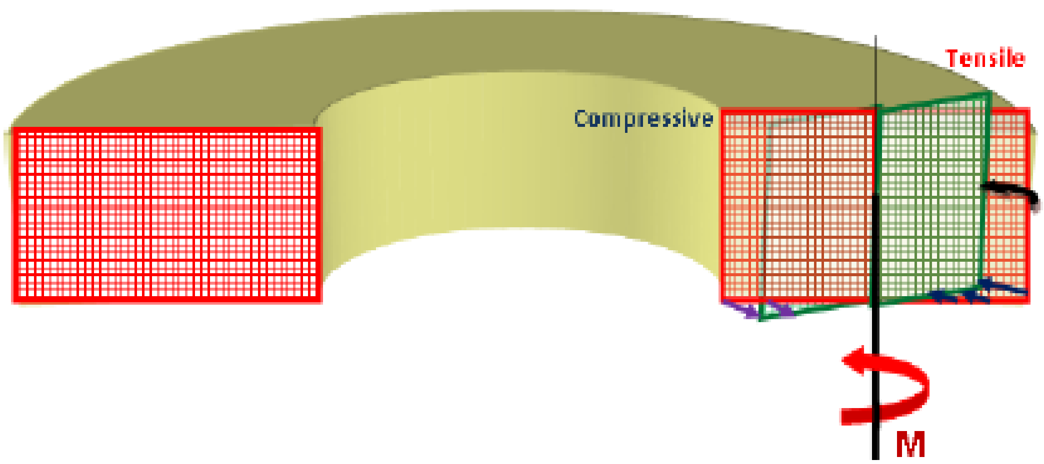
SIGN CONVENTION FOR BENDING MOMENT "M"

CONSIDER THE CASE FOR CURVED BEAM

Case-1

The applied moment trying to close the curve beam or trying to bend more or trying to reduce the radius of curvature.

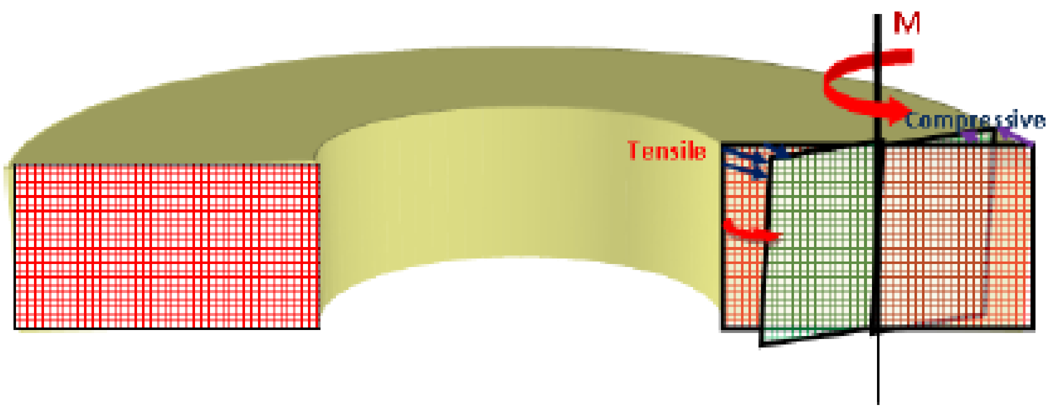
When the applied moment trying to reduce the radius of curvature, the outer surface is under tension and the inner surface is under compression. The bending moment "M" is considered as negative.



Case-2

The applied moment trying to open the curve beam or trying to straight the beam or trying to increase the radius of curvature.

When the applied moment trying to increase the radius of curvature, the outer surface is under compression and the inner surface is under tension. The bending moment "M" is considered as positive.

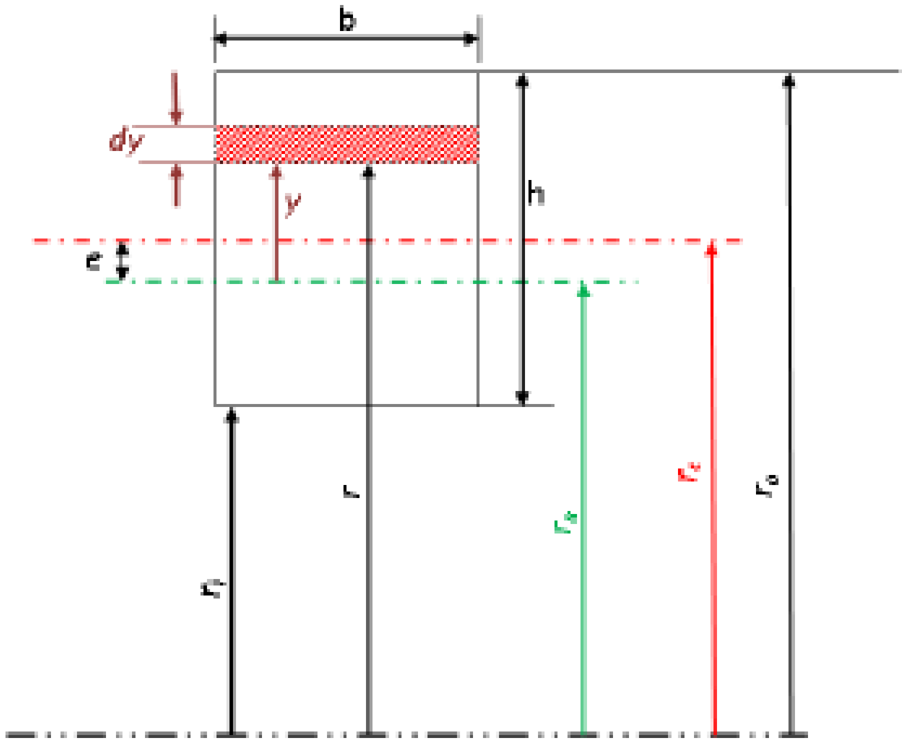


SIGN CONVENTION FOR "Y"

"y" considered as positive when it is measure in the direction towards the center of curvature & negative when it is measure in the direction outwards from the center of curvature.

LOCATION OF THE NEUTRAL AXIS

A. ***Rectangular section:-***



Axis Passing Through Center Of Curvature

$$\text{Radius of neutral axis } "r_n" = \frac{A}{\int \frac{dA}{r}}$$

Area of the rectangular section "A"= bxh

Consider a elemental section at distance "y" from neutral axis of thickness "dy"

Area of the elemental section "dA"= bxdy

$$=> \int \frac{dA}{r} = \int \frac{b \times dy}{r}$$

As "r" = $r_n + y$

Differentiating both the side w.r.t "y" $\frac{dr}{dy} = 1$ $\quad => dr = dy$

=> $\int \frac{dA}{r} = \int \frac{b \times dr}{r} = b\int_{r_i}^{r_o} \frac{dr}{r} = b[\ln r]_{r_i}^{r_o} = b[\ln r_o - \ln r_i] = b\ln\left(\frac{r_o}{r_i}\right)$

Hence r_n :

$$r_n = \frac{A}{\int \frac{dA}{r}} = \frac{b \times h}{b\ln\left(\frac{r_o}{r_i}\right)} = \frac{h}{\ln\left(\frac{r_o}{r_i}\right)}$$

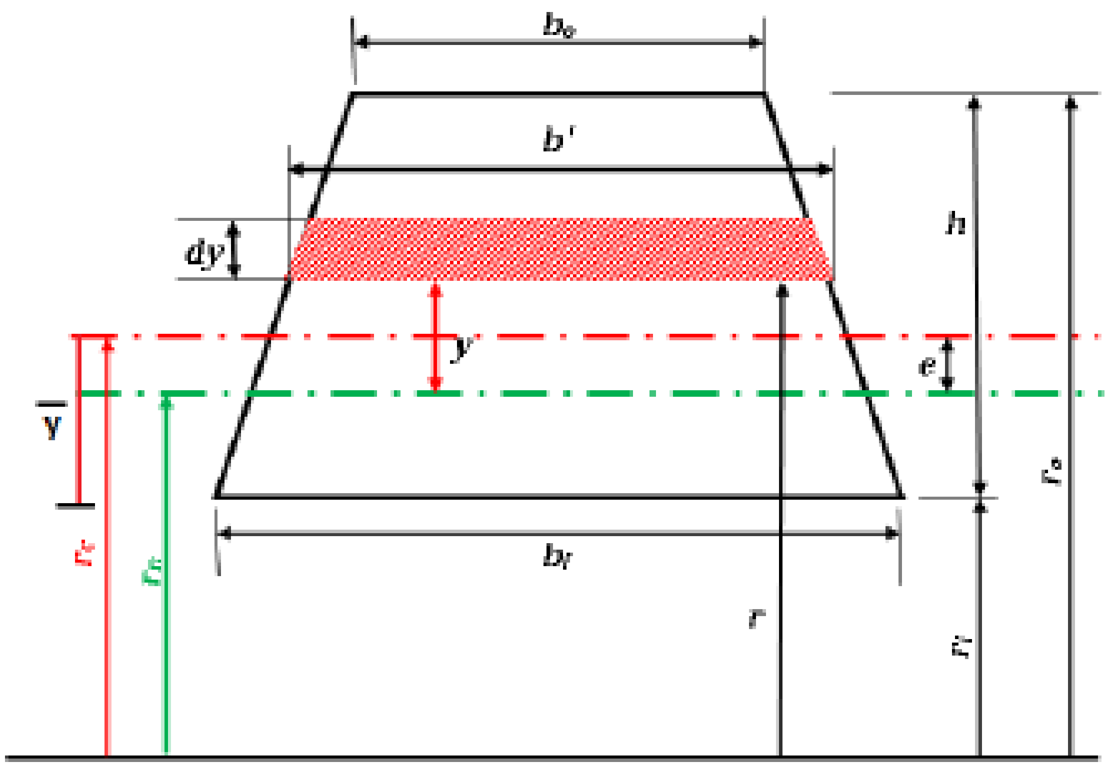
Determination of C.G. of the section:-

C.G of the section situated at a distance $\bar{y}$ from the bottom of the section.

$\bar{y}$ *from the bottom of the section* $= \frac{h}{2}$

Radius of curvature of centridal axis $r_c = r_i + \frac{h}{2}$

B. **Trapezoidal Section:-**



AXIS PASSING THROUGH CENTER OF CURVATURE

Consider an elemental section at a distance "y" from neutral axis of thickness "dy".

Let width of the elemental section "b' ", where :

$$b' = b_o + \left(\frac{b_i - b_o}{h}\right) \times (r_o - r)$$

Area of the elemental section "dA"

$$dA = b' \times dy = \left[b_o + \left(\frac{b_i - b_o}{h}\right) \times (r_o - r)\right] dy$$

$$As\ r = r_n + y$$

$$\frac{dr}{dy} = 1 \qquad => \ dr = dy$$

$$Hence\ \ dA = b' \times dy = b' \times dr = \left[b_o + \left(\frac{b_i - b_o}{h}\right) \times (r_o - r)\right] dr$$

Where "h" $= r_o - r_i$

$$Total\ area\ A = \int dA = \int b' \times dr = \int_{r_i}^{r_o} \left[b_o + \left(\frac{b_i - b_o}{h}\right) \times (r_o - r)\right] dr$$

$$= \int_{r_i}^{r_o} b_o\ dr + \left(\frac{b_i - b_o}{h}\right) \int_{r_i}^{r_o} (r_o - r)\ \ dr$$

$$= b_o[r]_{r_i}^{r_o} + \left(\frac{b_i - b_o}{h}\right) \left\{r_o[r]_{r_i}^{r_o} - \left[\frac{r^2}{2}\right]_{r_i}^{r_o}\right\}$$

$$= b_o[r_o - r_i] + \left(\frac{b_i - b_o}{h}\right) \left\{r_o[r_o - r_i] - \left[\frac{r_o{}^2}{2} - \frac{r_i{}^2}{2}\right]\right\}$$

$$= b_o[h] + \left(\frac{b_i - b_o}{h}\right) \left\{r_o[h] - \frac{1}{2}[(r_o - r_i)(r_o + r_i)]\right\}$$

$$= b_o[h] + \left(\frac{b_i - b_o}{h}\right) \left\{r_o[h] - \frac{1}{2}[h(r_o + r_i)]\right\}$$

$$= b_o[h] + \left(\frac{b_i - b_o}{h}\right) \left\{\frac{1}{2}[h(r_o - r_i)]\right\}$$

$$= b_o[h] + \left(\frac{b_i - b_o}{h}\right) \left\{\frac{1}{2}[h(h)]\right\}$$

$$= b_o[h] + (b_i - b_o) \ \left\{\frac{1}{2}h\right\} = \frac{1}{2}(b_i + b_o)h$$

$$Radius\ of\ neutral\ axis\ "r_n" = \frac{A}{\int \frac{dA}{r}}$$

$$=> \ \int \frac{dA}{r} = \int \frac{b' \times dr}{r} = \int \frac{\left[b_o + \left(\frac{b_i - b_o}{h}\right) \times (r_o - r)\right] \times dr}{r}$$

$$=> \ \int \frac{dA}{r} = \int_{r_i}^{r_o} \frac{\left[b_o + \left(\frac{b_i - b_o}{h}\right) \times (r_o - r)\right]}{r} dr = \int_{r_i}^{r_o} \frac{b_o}{r}\ dr + \left(\frac{b_i - b_o}{h}\right) \left\{\int_{r_i}^{r_o} \frac{r_o}{r}\ dr - \int_{r_i}^{r_o} dr\right\}$$

$$= b_o[lnr]_{r_i}^{r_o} + \left(\frac{b_i - b_o}{h}\right) \ [r_o lnr - r]_{r_i}^{r_o}$$

$$= b_o\ ln\left(\frac{r_o}{r_i}\right) + \ \left(\frac{b_i - b_o}{h}\right) \left[r_o\ ln\left(\frac{r_o}{r_i}\right) - \ (r_o - r_i)\right]$$

$$= b_o \ln\left(\frac{r_o}{r_i}\right) + \left(\frac{b_i - b_o}{h}\right)\left[r_o \ln\left(\frac{r_o}{r_i}\right) - h\right]$$

$$\textbf{Hence}\ Radius\ of\ neutral\ axis\ "r_n" = \frac{A}{\int \frac{dA}{r}}$$

$$r_n = \frac{\frac{1}{2}(b_i + b_o)h}{b_o \ln\left(\frac{r_o}{r_i}\right) + \left(\frac{b_i - b_o}{h}\right)\left[r_o \ln\left(\frac{r_o}{r_i}\right) - h\right]}$$

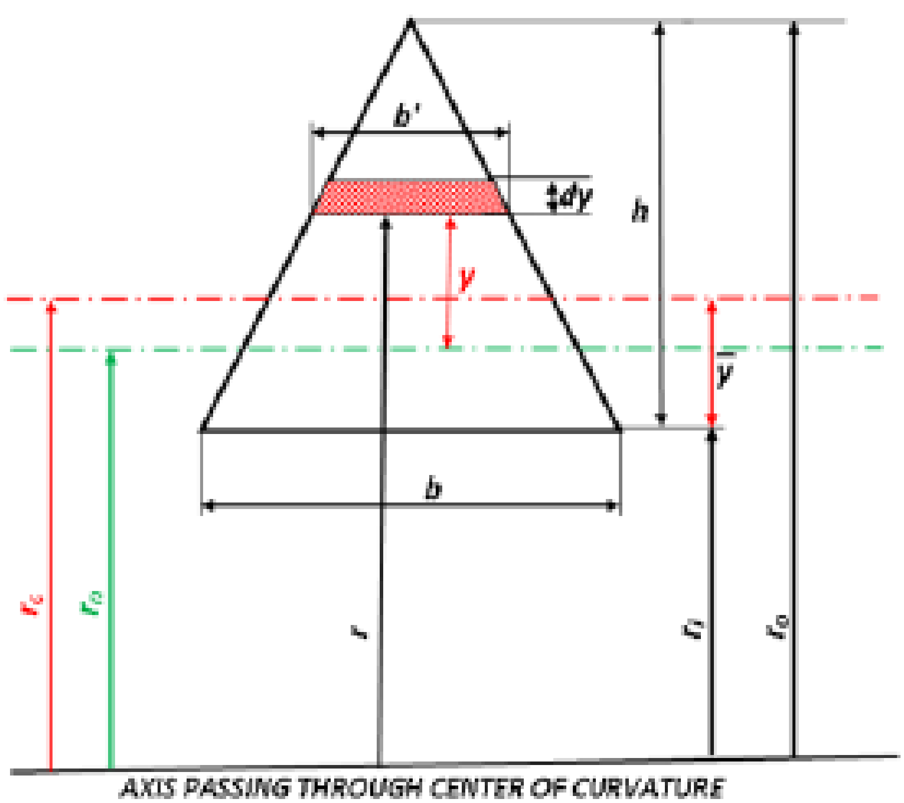
Determination of C.G. of the section:-

C.G of the section situated at a distance $\bar{y}$ from the bottom of the section.

$$\bar{y}\ from\ the\ bottom\ of\ the\ section = \frac{h}{3}\left[\frac{b_i + 2b_o}{b_i + b_o}\right]$$

$$Radius\ of\ curvature\ of\ centridal\ axis\ r_c = r_i + \bar{y}$$

C. <u>Triangular Section:</u>



On putting the boundary condition in formulation of trapezoidal section, the radius of curvature of neutral axis may be calculated.

$$b_i = b \qquad \& \qquad b_o = 0$$

$$r_n = \frac{\frac{1}{2}(b_i + b_o)h}{b_o \ln\left(\frac{r_o}{r_i}\right) + \left(\frac{b_i - b_o}{h}\right)\left[r_o \ln\left(\frac{r_o}{r_i}\right) - h\right]} = \frac{\frac{1}{2}(b)h}{\left(\frac{b}{h}\right)\left[r_o \ln\left(\frac{r_o}{r_i}\right) - h\right]} = \frac{h^2}{2\left[r_o \ln\left(\frac{r_o}{r_i}\right) - h\right]}$$

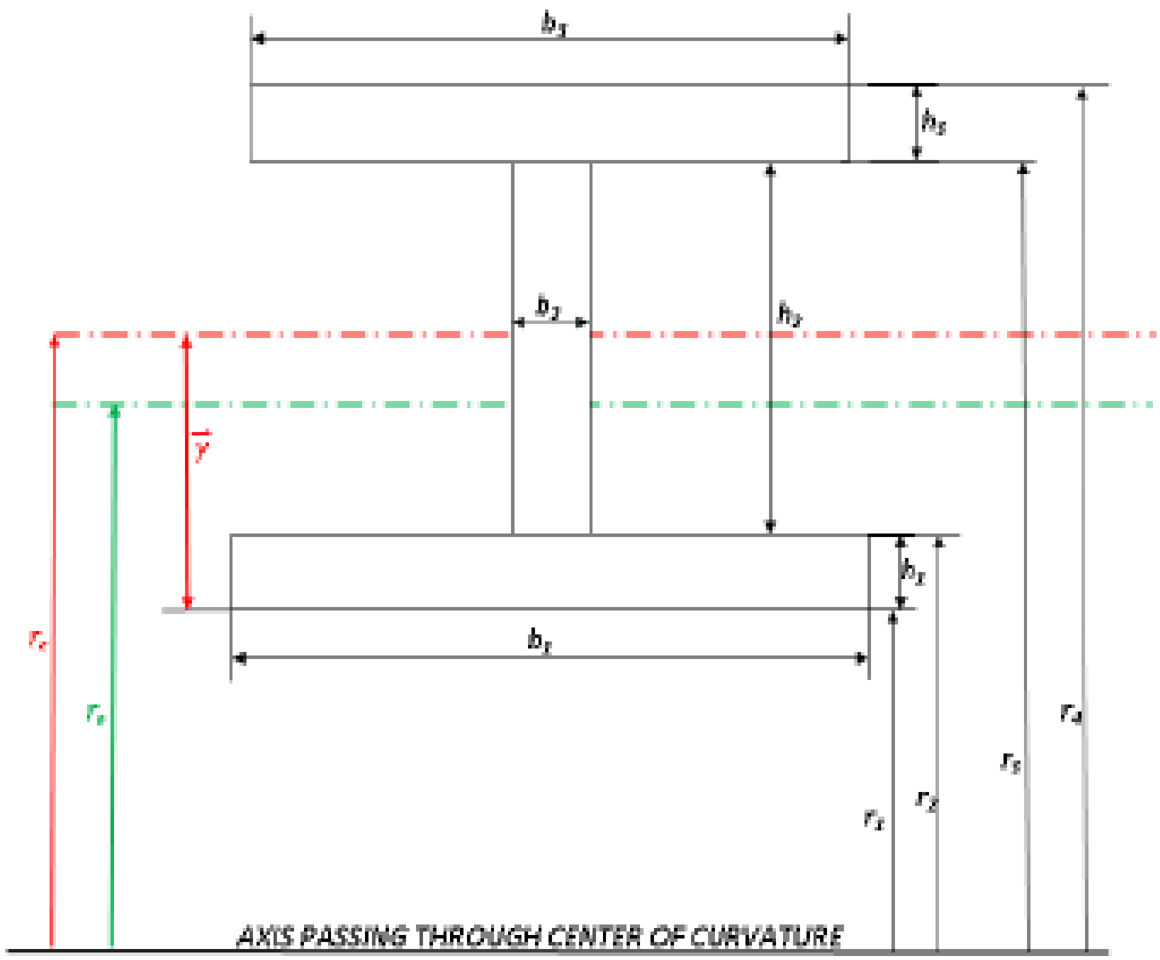
Determination of C.G. of the section:-

C.G of the section situated at a distance $\bar{y}$ from the bottom of the section.

$$\bar{y} \text{ from the bottom of the section} = \frac{h}{3}\left[\frac{b_i + 2b_o}{b_i + b_o}\right] = \frac{h}{3}\left[\frac{b + 0}{b + 0}\right] = \frac{h}{3}$$

Radius of curvature of centridal axis "r_c" = $r_i + \bar{y}$

A. **<u>Composite section</u>: (I-section)**



Total area of the I-section is "A"

$$A = \sum_{i=1}^{3} A_i = b_1 \times h_1 + b_2 \times h_2 + b_3 \times h_3$$

$$\int \frac{dA}{r} = \sum_{i=1}^{3} \frac{dA_i}{r_i} = b_1 ln\left(\frac{r_2}{r_1}\right) + b_2 ln\left(\frac{r_3}{r_2}\right) + b_3 ln\left(\frac{r_4}{r_3}\right)$$

$$\text{Radius of neutral axis " } r_n \text{" } = \frac{A}{\int \frac{dA}{r}} = \frac{b_1 \times h_1 + b_2 \times h_2 + b_3 \times h_3}{b_1 ln\left(\frac{r_2}{r_1}\right) + b_2 ln\left(\frac{r_3}{r_2}\right) + b_3 ln\left(\frac{r_4}{r_3}\right)}$$

Where :

$$r_2 = r_1 + h_1$$

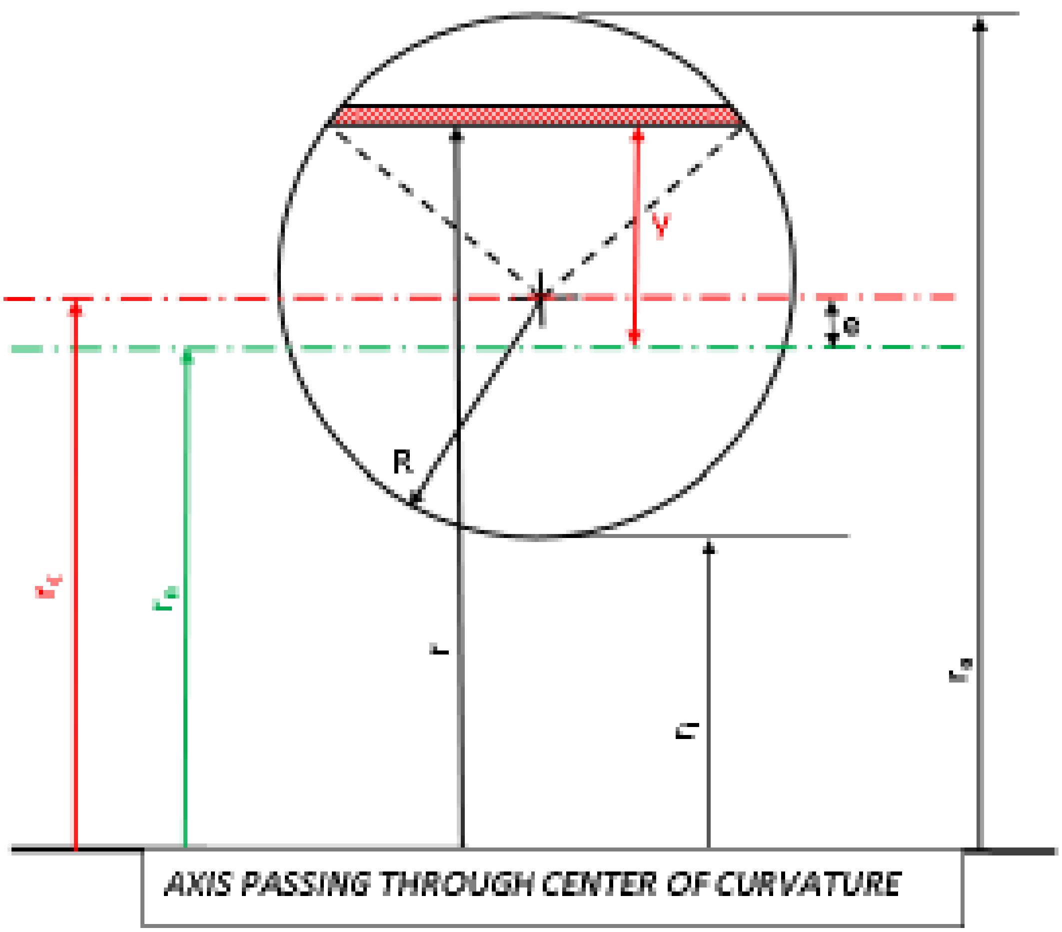
$$r_4 = r_3 + h_3 = r_1 + h_1 + h_2 + h_3$$

Determination of C.G. of the section:-

C.G of the section situated at a distance $\bar{y}$ from the bottom of the section.

Radius of curvature of centridal axis "r_c" $= r_i + \bar{y}$

B. **Circular section:-**



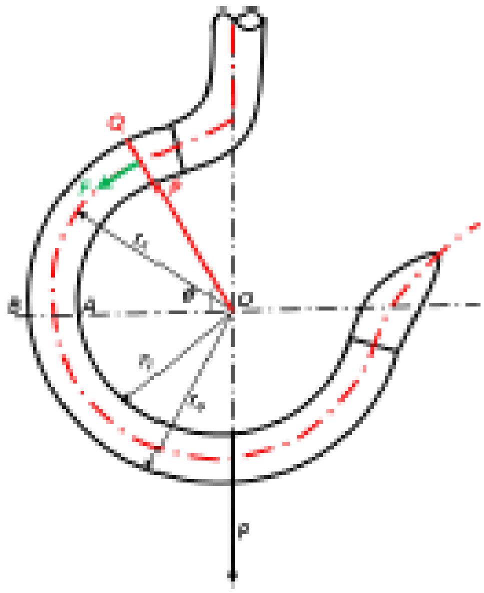
$$\text{Radius of neutral axis "} r_n \text{"} = \frac{A}{\int \frac{dA}{r}} = \frac{\left[\sqrt{r_o} + \sqrt{r_i}\right]^2}{4}$$

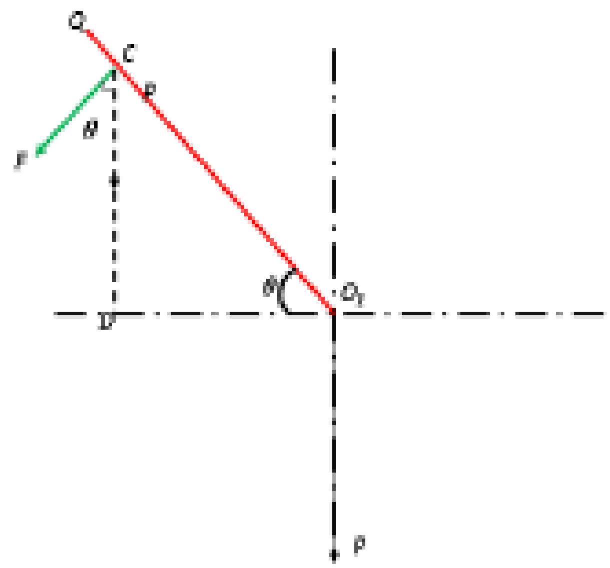
Determination of C.G. of the section:-

C.G of the section situated at a distance $\bar{y}$ *from the bottom of the section.*

Radius of curvature of centridal axis $r_c = r_i + \bar{y} = r_i + R$

CRANE HOOK PROBLEM:-





Consider a section "PQ" at an angel θ from horizontal.

Bending moment about the centroid "C" at the section "PQ" is :

$$M_b = P \times O_1D = P \times O_1C \cos\theta = P \times r_c \cos\theta$$

Normal load acting on the cross- section "PQ" is :

$$F = P \cos\theta$$

Direct-stress acting at the section is :

$$\sigma_d = \frac{F}{A} = \frac{P \cos\theta}{A}$$

Where "A" is the cross-sectional area of the beam.

Note:- when θ=0, the bending moment and direct stress are maximum & plane is known as critical plane.

Q.1. a crane hook is of trapezoidal cross-section having inner side 80 mm, outer side 30 mm, and depth 120 mm. the radius of curvature of the inner side is 80 mm. If the load 100 kN is applied in following two condition, find the maximum tensile and compressive stress across critical cross-section.

a). when the load pass through center of curvature.

b). when the load shifted by 10 mm towards inside surface from center of curvature.

c). when the load shifted by 10 mm away from center of curvature.

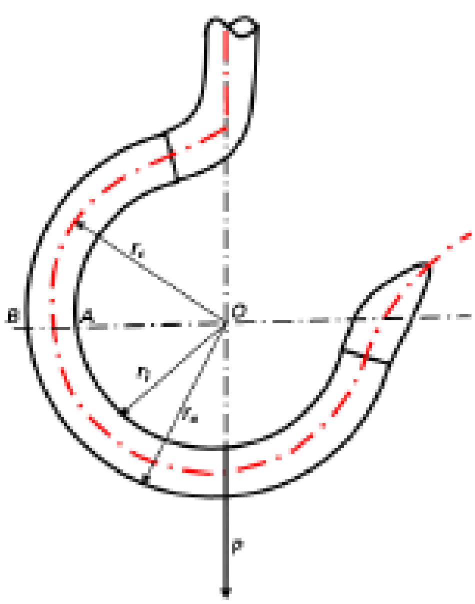
Solution:-

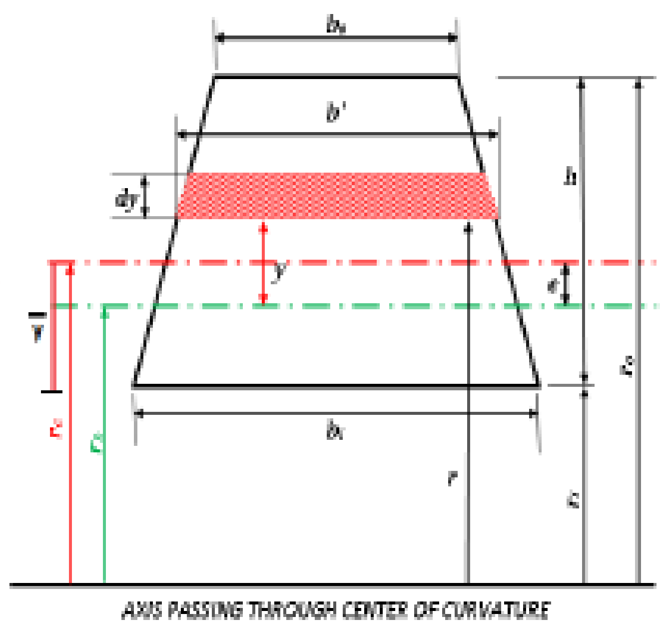
Given data:-

Point "O" be the center of curvature

$r_i = 80mm,\ \ h = 120mm,\ b_i = 80mm,\ \ b_o = 30mm,\ P = 100kN = 10 \times 10^4\ N$

$$r_o = r_i + h = 200mm$$





Determination of C.G. of the section:-

$C.G$ *of the section situated at a distance* $\bar{y}$ *from the bottom of the section.*

$$\bar{y}\ \text{from the bottom of the section} = \frac{h}{3}\left[\frac{b_i + 2b_o}{b_i + b_o}\right] = \frac{120}{3}\left[\frac{80 + 2\times 30}{80 + 30}\right] = 50.91mm$$

$$\text{Radius of curvature of centridal axis } r_c = r_i + \bar{y} = 80 + 50.91 = 130.91mm$$

$$\textbf{Hence}\ \text{Radius of neutral axis " } r_n \text{" } = \frac{A}{\int \frac{dA}{r}}$$

$$r_n = \frac{\frac{1}{2}(b_i + b_o)h}{b_o\, ln\left(\frac{r_o}{r_i}\right) + \left(\frac{b_i - b_o}{h}\right)\left[r_o\, ln\left(\frac{r_o}{r_i}\right) - h\right]} = \frac{\frac{1}{2}(80+30)120}{30\, ln\left(\frac{200}{80}\right) + \left(\frac{80-30}{120}\right)\left[200\, ln\left(\frac{200}{80}\right) - 120\right]}$$

$$r_n = \frac{6600}{27.488 + 0.417\,[183.258 - 120]} = \frac{6600}{27.488 + 0.417\,[63.258]} = \frac{6600}{53.866} = 122.525mm$$

$$\text{Area A} = \frac{1}{2}(b_i + b_o)h = 6600\ mm^2$$

'e' represents the distance between the centroidal axis and neutral axis. *i.e.*

$$e = r_c - r_n = 130.91 - 122.525 = 8.385mm$$

$$\text{we know that } => \sigma = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right]$$

A) <u>*For case (a) when load passing through center of curvature.*</u>

$$M_b = \text{bending moment about centroid of the section} = P \times r_c = 13.091 \times 10^6 Nmm$$

By applying sign convention:

M_b *is positive, as the* Applied moment trying to increase the radius of curvature, the outer surface is under compression and the inner surface is under tension. The bending moment "M" is considered as positive. => $M_b = 13.091 \times 10^6 Nmm$

"y" considered as positive when it is measure in the direction towards the center of curvature & negative when it is measure in the direction outwards from the center of curvature.

For point "A" , fiber at inner surface $y_A = is\ positive$

$$y_A = r_n - r_i = 122.525 - 80 = 42.525mm$$

$$=> r_A = r_n - y_A = r_i = 80mm$$

$Stress\ due\ to\ bending\ at$ A $= \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_n - y_A}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_A}\right] = \frac{13.091 \times 10^6}{6600 \times 8.385}\left[\frac{42.525}{80}\right]$
$= 125.742\ N/mm^2 (tensile)$

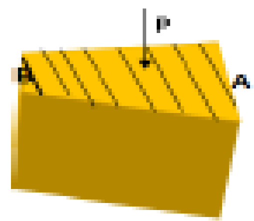
For point "B" , fiber at outer surface $y_B = is\ negative$

$$y_B = r_O - r_N = 200 - 122.525 = 77.475mm$$

$$=> r_B = r_n - (-y_{B)} = r_n + y_B = r_O = 200mm$$

$$Stress\ due\ to\ bending\ at\ \text{A} = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{-y_B}{r_n - (-y_B)}\right] = -\frac{M_b}{Ae}\left[\frac{y_B}{r_B}\right]$$
$$= -\frac{13.091 \times 10^6}{6600 \times 8.385}\left[\frac{77.475}{2080}\right] = -91.634\ N/mm^2 (compressive)$$

Direct stress acting at the section "A-B" :-



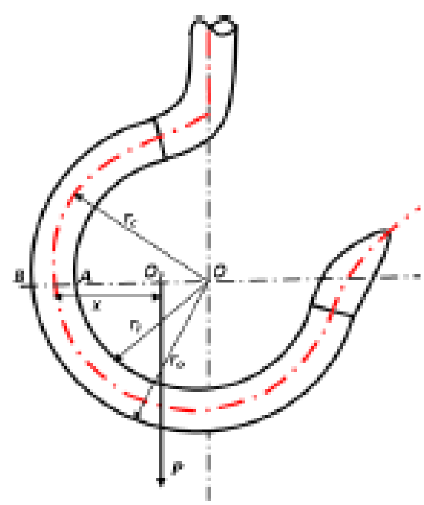
Direct stress at section "A-B"

$\sigma_{dA} = \sigma_{dB} = \sigma_d = \frac{P}{A} = \frac{100000}{6600} = 15.152\ N/mm^2 (tensile)$

Resultant stress:-

$\sigma_A = \sigma_{bA} + \sigma_{dA} = 140.894\ N/mm^2 (tensile)$

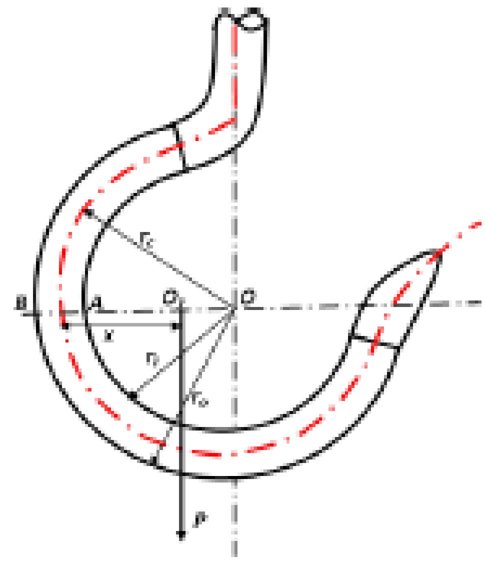
$\sigma_b = \sigma_{bB} + \sigma_{dB} = -76.482\ N/mm^2 (compressive)$



B. For case (b) when the load shifted by 10 mm towards inside surface from center of curvature.

$M_b = bending\ moment\ about\ centroid\ of\ the\ section$
$x = r_c - OO_1 = 130.91 - 10 = 120.91MM$
$M_b = P \times x = 12.091 \times 10^6 Nmm$

$$Stress\ due\ to\ bending\ at\ A = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right]$$
$$= \frac{M_b}{Ae}\left[\frac{y_A}{r_n - y_A}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_A}\right]$$
$$= \frac{12.091 \times 10^6}{6600 \times 8.385}\left[\frac{42.525}{80}\right]$$
$$= 116.137\ N/mm^2 (tensile)$$
$$Stress\ due\ to\ bending\ at\ A = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right]$$
$$= \frac{M_b}{Ae}\left[\frac{-y_B}{r_n - (-y_B)}\right] = -\frac{M_b}{Ae}\left[\frac{y_B}{r_B}\right]$$
$$= -\frac{12.091 \times 10^6}{6600 \times 8.385}\left[\frac{77.475}{2080}\right]$$
$$= -84.634\ N/mm^2 (compressive)$$



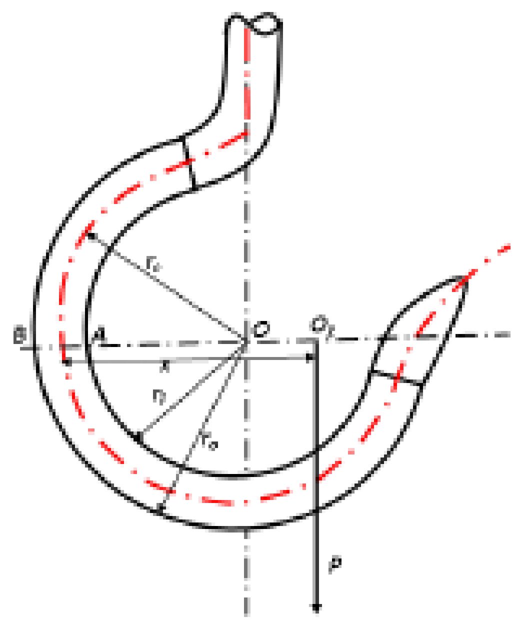
Direct stress at section "A-B"
$\sigma_{dA} = \sigma_{dB} = \sigma_d = \frac{P}{A} = \frac{100000}{6600} = 15.152\ N/mm^2 (tensile)$

Resultant stress:-
$\sigma_A = \sigma_{bA} + \sigma_{dA} = 131.289\ N/mm^2 (tensile)$
$\sigma_b = \sigma_{bB} + \sigma_{dB} = -69.482\ N/mm^2 (compressive)$

C. For case (b) when the load shifted by 10 mm away from center of curvature.

$M_b = bending\ moment\ about\ centroid\ of\ the\ section$
$x = r_c + OO_1 = 130.91 + 10 = 140.91MM$
$M_b = P \times x = 12.091 \times 10^6 Nmm$

$$Stress\ due\ to\ bending\ at\ A = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right]$$
$$= \frac{M_b}{Ae}\left[\frac{y_A}{r_n - y_A}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_A}\right]$$
$$= \frac{14.091 \times 10^6}{6600 \times 8.385}\left[\frac{42.525}{80}\right]$$
$$= 135.347\ N/mm^2 (tensile)$$
$$Stress\ due\ to\ bending\ at\ A = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right]$$
$$= \frac{M_b}{Ae}\left[\frac{-y_B}{r_n - (-y_B)}\right] = -\frac{M_b}{Ae}\left[\frac{y_B}{r_B}\right]$$
$$= -\frac{14.091 \times 10^6}{6600 \times 8.385}\left[\frac{77.475}{2080}\right]$$
$$= -98.634\ N/mm^2 (compressive)$$



Direct stress at section "A-B"

$\sigma_{dA} = \sigma_{dB} = \sigma_d = \frac{P}{A} = \frac{100000}{6600} = 15.152\ N/mm^2 (tensile)$

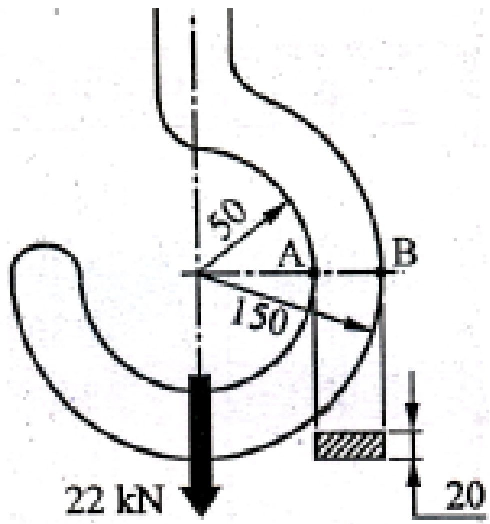
Resultant stress:-

$\sigma_A = \sigma_{bA} + \sigma_{dA} = 150.499\ N/mm^2 (tensile)$

$\sigma_b = \sigma_{bB} + \sigma_{dB} = -83.482\ N/mm^2 (compressive)$

WORK SHEET

Q.2. Plot the stress distribution about section A-B of the hook as shown in figure.



Given data:
r_i = 50mm, r_o = 150mm, P = $22X10^3$N, b = 20mm, h = 150-50 = 100mm
A = bh = 20X100 = 2000mm^2

Determination of C.G. of the section:-

$C.G$ *of the section situated at a distance* $\bar{y}$ *from the bottom of the section* $= \frac{h}{2}$

$$= \frac{100}{2} = 50mm$$

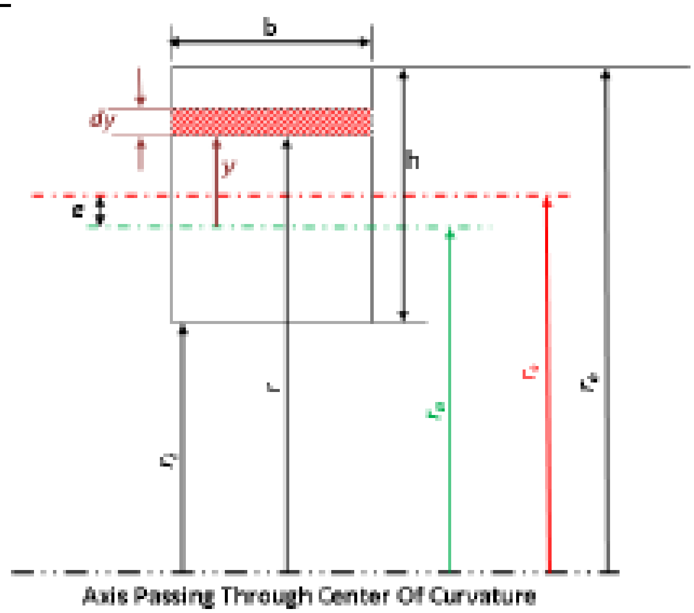
Radius of curvature of centridal axis r_c

$$= r_i + \frac{h}{2} = 50 + 50 = 100mm$$

$$r_n = \frac{A}{\int \frac{dA}{r}} = \frac{b \times h}{b\, ln\left(\frac{r_o}{r_i}\right)} = \frac{h}{ln\left(\frac{r_o}{r_i}\right)}$$

$$= \frac{100}{ln\left(\frac{150}{50}\right)} = 91.024mm$$

$$e = r_c - r_n = 100 - 91.024 = 8.976mm$$



M_b = *bending moment about centroid of the section AB* $= P \times r_c = 22 \times 10^3 \times 100 = 2.2 \times 10^6 Nmm$

M_b *is positive, as the* **Applied moment trying to increase the radius of curvature, the outer surface is under compression and the inner surface is under tension. The bending moment "M" is considered as positive.**

Direct stress at section "A-B"

$\sigma_{dA} = \sigma_{dB} = \sigma_d = \frac{F}{A} = \frac{22000}{2000} = 11\ N/mm^2 (tensile)$

For point "A" , fiber at inner surface $y_A = is\ positive$

$$y_A = r_n - r_i = 91.024 - 50 = 41.024mm$$

$$=> r_A = r_n - y_A = r_i = 50mm$$

$$Stress\ due\ to\ bending\ at\ \text{A} = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_n - y_A}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_A}\right] = \frac{2.2 \times 10^6}{2000 \times 8.976}\left[\frac{41.024}{50}\right]$$

$$= 100.55\ N/mm^2 (tensile)$$

For point "B" , fiber at outer surface $y_B = is\ negative$

$$y_B = r_O - r_N = 150 - 91.024 = 58.976mm$$

$$as\ y_B = is\ negative, \quad y_B = -58.976mm$$

$$=> r_B = r_n - (-y_{B)}) = r_n + y_B = r_O = 150mm$$

$$Stress\ due\ to\ bending\ at\ \text{A} = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{-y_B}{r_n - (-y_B)}\right] = -\frac{M_b}{Ae}\left[\frac{y_B}{r_B}\right]$$

$$= -\frac{2.2 \times 10^6}{2000 \times 8.976}\left[\frac{58.976}{150}\right] = -48.183\ N/mm^2 (compressive)$$

$Resultant\ stresses\ are:$

$\sigma_A = \sigma_{bA} + \sigma_{dA} = 111.55\ N/mm^2 (tensile)$

$\sigma_B = \sigma_{bB} + \sigma_{dB} = -37.183\ N/mm^2 (compressive)$

Bending stress at any radius "r" is $\sigma_b = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right]$ *where* $y = r_n - r$

Bending stress at neutral axis i.e at "y" =0, $\sigma_{bn} = o$

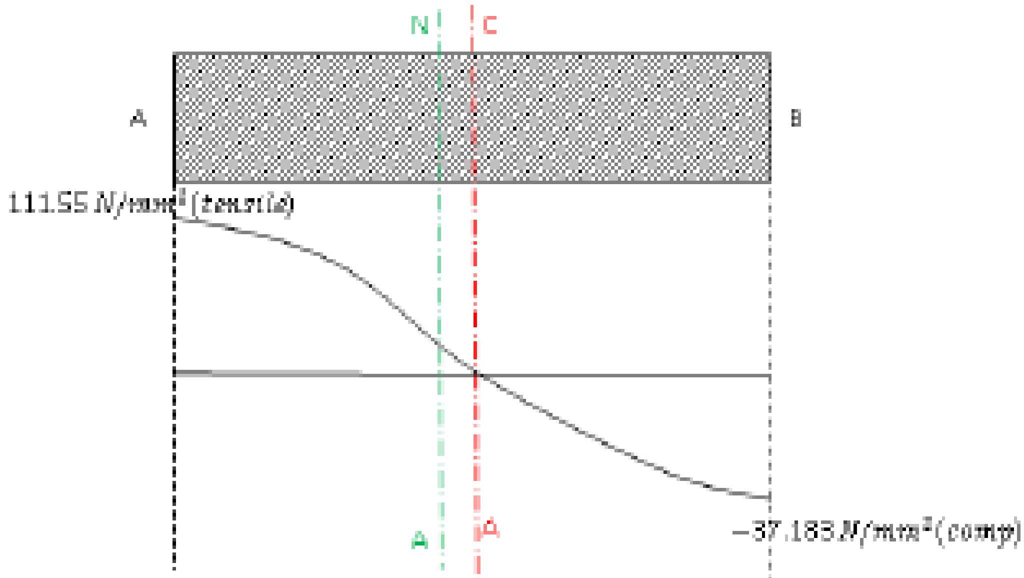
But Resultant stress at neutral axis=$\sigma_N = \sigma_{bn} + \sigma_d$ *=11* $N/mm^2 (tensile)$

Bending stress at centroidal axis i.e at "y" = -e = -8.976mm,

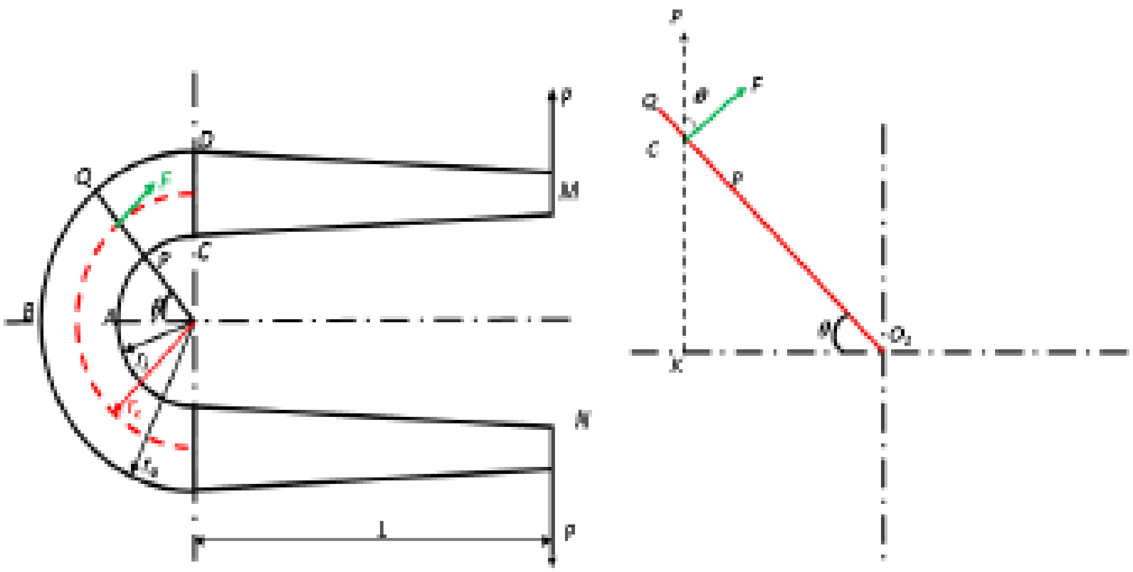
$\sigma_{bc} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = -\frac{11N}{mm^2} (compressive)$

But Resultant stress at centroidal axis =$\sigma_c = \sigma_{bc} + \sigma_d = 0$

Stress distribution across the cross-section "AB"



Problem on "U" fork



Consider a section "PQ" at an angel θ from horizontal.

Bending moment about the centroid "C" at the section "PQ" is :

$$M_b = P \times (MD + O_1K) = P \times (MD + O_1C\ \cos\theta) = P \times (L + r_c \cos\theta)$$

Normal load acting on the cross- section "PQ" is :

$$F = P\ \cos\theta$$

Direct-stress acting at the section is :

$$\sigma_d = \frac{F}{A} = \frac{P\ \cos\theta}{A}$$

Where "A" is the cross-sectional area of the beam.

The "U" section consists of two portions:-

A. Straight portion
B. Curve portion

__Straight portion__

For calculating stress in straight portion (MD) general pure bending formula can be applied:

$$\textbf{Bending stress } \sigma_{b(straight)} = \frac{M_{straight}}{I} \times y$$

$$\textbf{Wher } M_{straight} = P \times x$$

$x = distance\ of\ the\ section\ from\ load\ P$

$Where\ maximum\ bending\ moment\ M_{straight(max)} = P \times L$ & $it\ is\ occurs\ at\ the\ section\ CD$

$On\ putting\ the\ value\ \theta = 90\ M_{CD} = P \times (L + r_c \cos 90) = P \times L$

$$Direct\ stress\ \sigma_{d(straight)} = \frac{F}{A} = \frac{P\ \cos 90}{A} = 0$$

$Hence\ resultant\ stress\ \sigma_{(straight)} = \sigma_{b(straight)} \pm \sigma_{d(straight)} = \sigma_{b(straight)}$

<u>Curve portion</u>

For calculating stress in curved portion Winkler-batch theory can be applied:

Bending moment about the centroid "C" at the section "PQ" is :

$M_b = P \times (MD + O_1K) = P \times (MD + O_1C\ \cos\theta) = P \times (L + r_c \cos\theta)$

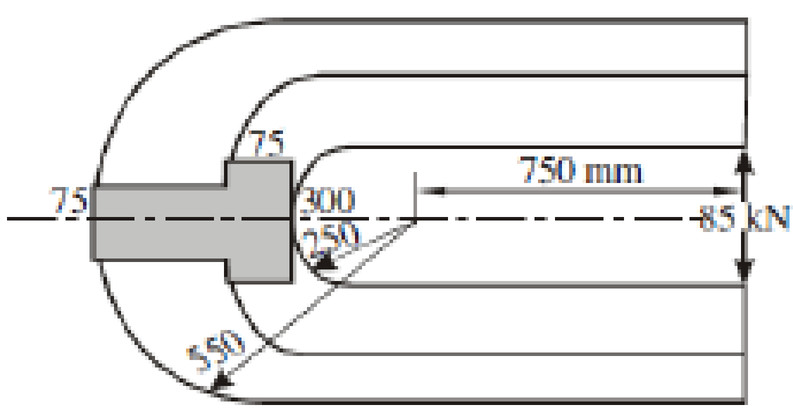
$$\sigma_{b(curved)} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right]$$

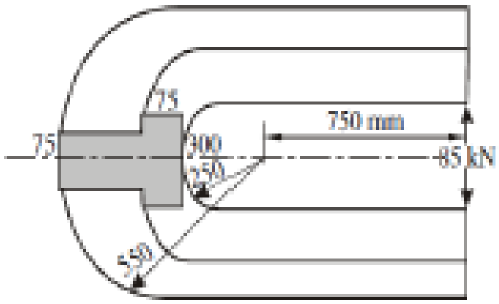
Direct-stress acting at the section is :

$$\sigma_{d(curved)} = \frac{F}{A} = \frac{P\ \cos\theta}{A}$$

$Hence\ resultant\ stress\ \sigma_{(curved)} = \sigma_{b(curved)} \pm \sigma_{d(curved)}$

Q.3. Figure shows a frame of a punching machine and its various dimensions. Determine the maximum stress in the frame, if it has to resist a force of 85kN.





Given data:-

L=750mm,

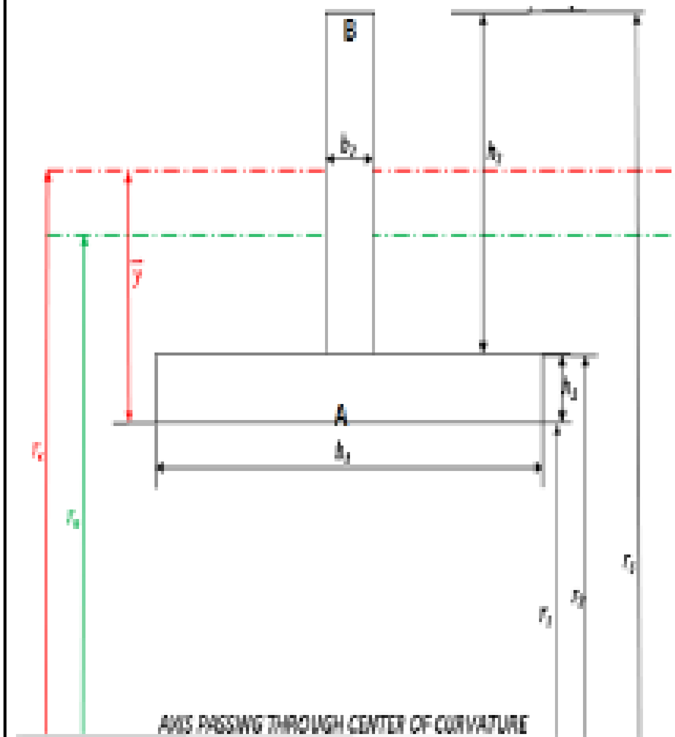
r_i = 250mm, r_o = 550mm, P = 85X10^{3}N

$b_1 = 300mm, b_2 = 75mm, h_1 = 75mm,$

$h_2 = 225mm, r_1 = 250mm,$

$r_2 = 250 + 75 = 325mm,$

$r_3 = 325 + 225 = 550mm$



$A = (300 \times 75) + (225 \times 75) = 39375\ mm^2$

$$\bar{y} = \frac{(300 \times 75) \times \left(75/_2\right) + (225 \times 75) \times \left(75 + 225/_2\right)}{(300 \times 75) + (225 \times 75)} = 101.785mm$$

$r_c = r_1 + \bar{y} = 250 + 101.785 = 351.785mm$

$$Radius\ of\ neutral\ axis\ "r_n" = \frac{A}{\int \frac{dA}{r}} = \frac{b_1 \times h_1 + b_2 \times h_2}{b_1 ln\left(\frac{r_2}{r_1}\right) + b_2 ln\left(\frac{r_3}{r_2}\right)}$$

$$r_n = \frac{(300 \times 75) + (225 \times 75)}{300 ln\left(\frac{325}{250}\right) + 75 ln\left(\frac{550}{325}\right)} = 333.217mm$$

Distance of neutral axis to centroidal axis "e" = r_c- r_n = 18.568mm

Bending moment about the centroid "C" at the section is :

$M_b = P \times (L + r_c \cos\theta) when\ \theta = 0,$

The bending moment is maximum & the section is at horizontal in the curved portion

$M_b = P \times (L + r_c \cos 0) = 85 \times 10^3 \times 1101.785 = 93.652 \times 10^6 Nmm$

M_b *is positive, as the* Applied moment trying to increase the radius of curvature, the outer surface is under compression and the inner surface is under tension. The bending moment "M" is considered as positive.

Direct-stress acting at the section is :

$$\sigma_d = \frac{F}{A} = \frac{P \cos\theta}{A} = \frac{85 \times 10^3 \cos 0}{39375} = 2.16\ ^N/_{mm^2}\ (tensile)$$

Bending stress at "A"

For point "A" , fiber at inner surface $y_A = is\ positive$

$$y_A = r_n - r_i = 333.217 - 250 = 83.217mm$$

$$=> r_A = r_n - y_A = r_i = 250mm$$

$$Stress\ due\ to\ bending\ at\ \text{A} = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_n - y_A}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_A}\right] = \frac{93.652 \times 10^6}{39375 \times 18.568}\left[\frac{83.217}{250}\right]$$

$$= 42.64\ N/mm^2 (tensile)$$

For point "B" , fiber at outer surface $y_B = is\ negative$

$$y_B = r_O - r_N = 550 - 333.217 = 216.783mm$$

$$as\ y_B = is\ negative, \quad y_B = -216.783mm$$

$$=> r_B = r_n - (-y_{B)} = r_n + y_B = r_O = 550mm$$

$$Stress\ due\ to\ bending\ at\ \text{A} = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{-y_B}{r_n - (-y_B)}\right] = -\frac{M_b}{Ae}\left[\frac{y_B}{r_B}\right]$$

$$= -\frac{93.652 \times 10^6}{39375 \times 18.568}\left[\frac{216.783}{550}\right] = -50.49\ N/mm^2 (compressive)$$

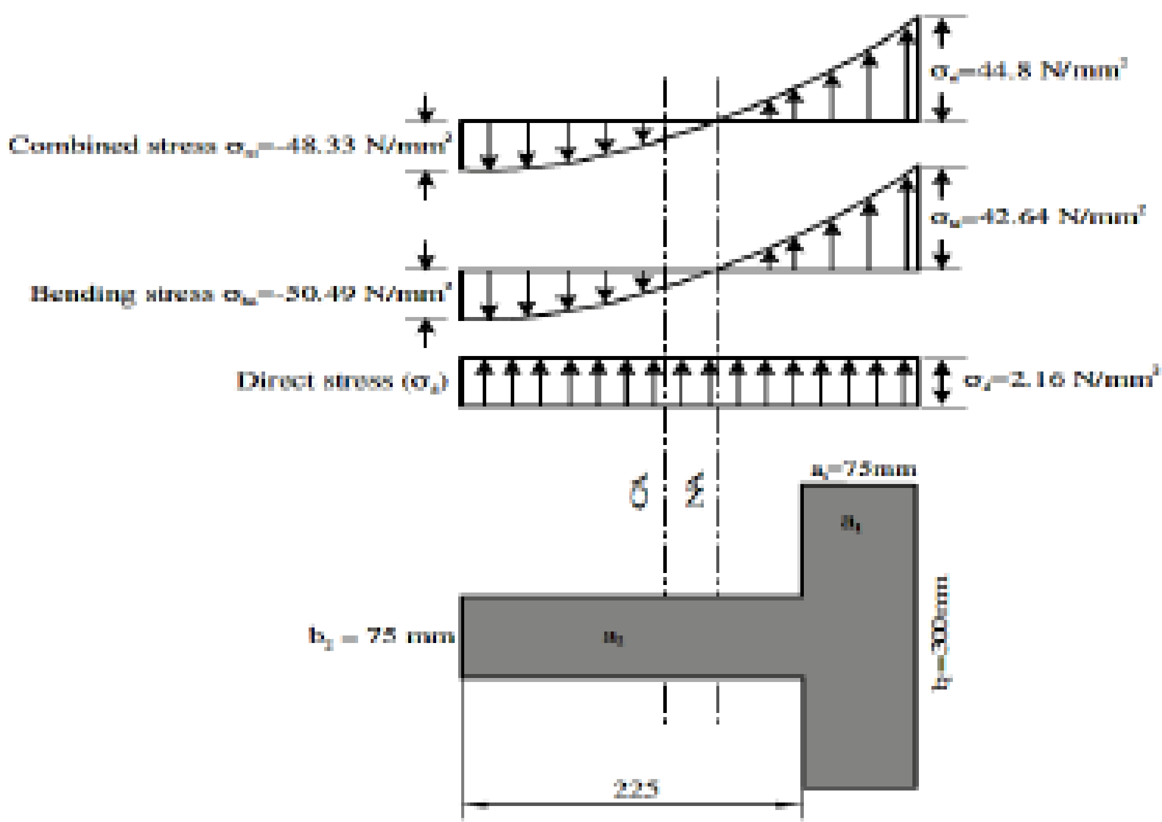
Resultant stresses are :

$\sigma_A = \sigma_{bA} + \sigma_{dA} = 44.8\ N/mm^2 (tensile)$

$\sigma_B = \sigma_{bB} + \sigma_{dB} = -48.33\ N/mm^2 (compressive)$

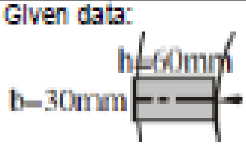
$$Maximum\ shear\ stress\ \tau_{max} = 0.5\ \sigma_{max} = -24.165\ N/mm^2$$

The below figure shows the stress distribution.



Q.4. The section of a crane hook is rectangular in shape whose width is 30mm and depth is 60mm. The centre of curvature of the section is at distance of 125mm from the inside section and the load line is 100mm from the same point. Find the capacity of hook if the allowable stress in tension is 75N/mm².

Given data:



$$\sigma_{max} = 75\frac{N}{mm^2} \quad (tension)$$

Point "O" be the center of curvature

$r_i = 125mm,\ h = 60mm,$ b$= 30mm,\ \ P = ?$

$$r_o = r_i + h = 185mm$$

Determination of C.G. of the section:-

$C.G\ of\ the\ section\ situated\ at\ a\ distance\ \bar{y}\ from\ the\ b$

$$\bar{y}\ from\ the\ bottom\ of\ the\ section = \frac{h}{2} = \frac{60}{2} = 30mm$$

$$Radius\ of\ curvature\ of\ centridal\ axis\ r_c = r_i + \frac{h}{2}$$

$$= 125 + 30 = 155mm$$

$$r_n = \frac{A}{\int \frac{dA}{r}} = \frac{b \times h}{b\ ln\left(\frac{r_o}{r_i}\right)} = \frac{h}{ln\left(\frac{r_o}{r_i}\right)} = \frac{60}{ln\left(\frac{185}{125}\right)}$$

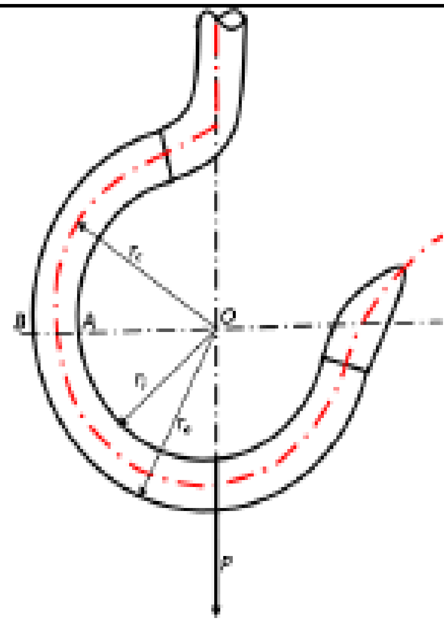
$$= 153.045mm$$

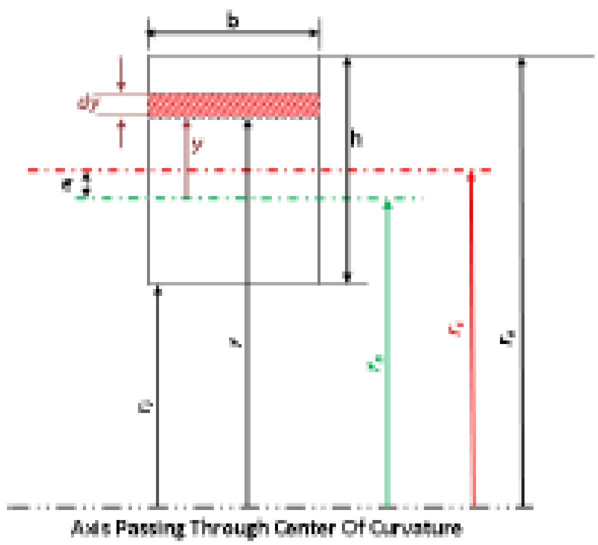
$$e = r_c - r_n = 155 - 153.045 = 1.955mm$$

$$A = 30 \times 60 = 1800\ mm^2$$

M_b
$= bending\ moment\ about\ centroid\ of\ the\ section\ AB$
$= P \times r_c = P \times 155 = 155P\ Nmm$

$M_b is\ positive, as\ the$ **Applied moment trying to increase the radius of curvature, the outer surface is under compression and the inner surface is under tension. The bending moment "M" is considered as positive.**





Direct stress at section "A-B"

$$\sigma_{dA} = \sigma_{dB} = \sigma_d = \frac{P}{A} = \frac{P}{1800} = 0.555 \times 10^{-3}P\ N/mm^2(tensile)$$

For point "A" , fiber at inner surface $y_A = is\ positive$

$$y_A = r_n - r_i = 153.045 - 125 = 28.045mm$$

$$=> r_A = r_n - y_A = r_i = 125mm$$

$$\textit{Stress due to bending at A} = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_n - y_A}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_A}\right] = \frac{155P}{1800 \times 1.955}\left[\frac{28.045}{125}\right]$$

$$= 9.882 \times 10^{-3} P\ N/mm^2 (tensile)$$

For point "B" , fiber at outer surface $y_B = is\ negative$

$$y_B = r_O - r_N = 185 - 153.045 = 31.955mm$$

$$as\ y_B = is\ negative, \quad y_B = -31.955mm$$

$$=> r_B = r_n - (-y_{B)} = r_n + y_B = r_O = 185mm$$

$$\textit{Stress due to bending at A} = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{-y_B}{r_n - (-y_B)}\right] = -\frac{M_b}{Ae}\left[\frac{y_B}{r_B}\right] = -\frac{155P}{1800 \times 1.955}\left[\frac{31.955}{185}\right]$$

$$= -7.608 \times 10^{-3} P\ N/mm^2 (compressive)$$

Resultant stresses are :

$\sigma_A = \sigma_{bA} + \sigma_{dA} = 10.437 \times 10^{-3} P\ N/mm^2 (tensile)$

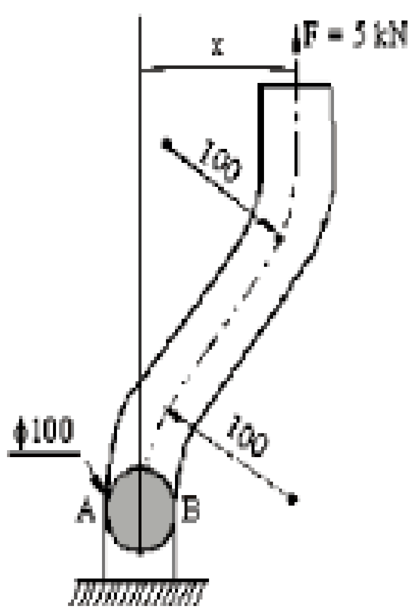
$\sigma_B = \sigma_{bB} + \sigma_{dB} = -7.053 \times 10^{-3} P\ N/mm^2 (compressive)$

Maximum tensile stress is :

$$\sigma_{max} = 75\frac{N}{mm^2} \quad => 75 = 10.437 \times 10^{-3} P$$

- P=7185.972 N

Q.5. The figure shows a loaded offset bar. What is the maximum off-set distance 'x' if the allowable stress in tension is limited to $50 N/mm^2$.



Solution:-

Given data:-

$$r_c = 100mm$$

$r_i = r_c - R = 50mm,\ R = 50mm,\quad P = 5 \times 10^3 N$

$$r_o = r_c + R = 150mm$$

Maximum tensile stress is :

$$\sigma_{max} = 50\frac{N}{mm^2}$$

Determination of C.G. of the section:-

$\bar{y} = R = 50mm$

$$Radius\ of\ neutral\ axis\ "r_n" = \frac{A}{\int \frac{dA}{r}} = \frac{\left[\sqrt{r_o} + \sqrt{r_i}\right]^2}{4}$$

$$= \frac{\left[\sqrt{150} + \sqrt{50}\right]^2}{4} = 93.301mm$$

$$A = \frac{\pi}{4}D^2 = \pi R^2 = 7853.982mm^2$$

$$e = r_c - r_n = 100 - 93.301 = 6.699mm$$

M_b

$= bending\ moment\ about\ centroid\ of\ the\ section$ AB

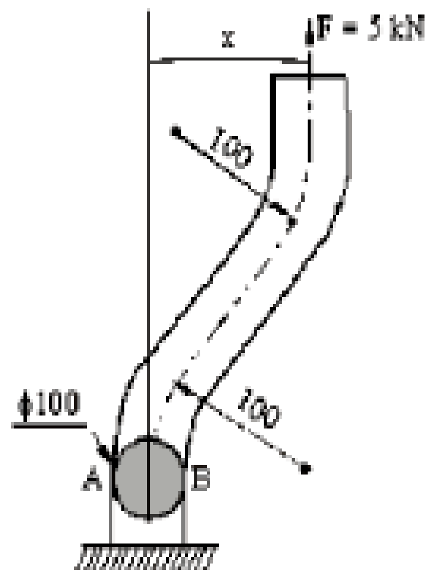
$= P \times x = 5 \times 10^3 \times x = 5 \times 10^3 x\ Nmm$

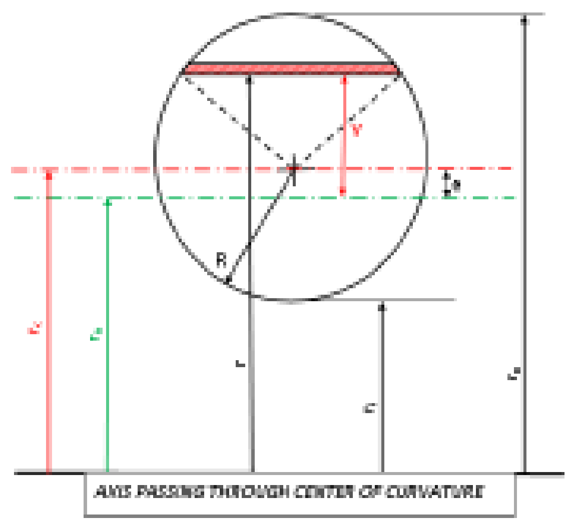
$M_b is\ positive, as\ the$ Applied moment trying to increase the radius of curvature, the outer surface is under compression and the inner surface is under tension. The bending moment "M_b" is considered as positive.

Direct stress at section "A-B"

$$\sigma_{dA} = \sigma_{dB} = \sigma_d = \frac{P}{A} = \frac{5 \times 10^3}{7853.982}$$

$= 0.637 N/mm^2 (compressive)$





For point "A" , fiber at inner surface $y_A = is\ positive$

$$y_A = r_n - r_i = 93.301 - 50 = 43.301mm$$

$$=> r_A = r_n - y_A = r_i = 50mm$$

$$Stress\ due\ to\ bending\ at\ A = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_n - y_A}\right] = \frac{M_b}{Ae}\left[\frac{y_A}{r_A}\right]$$

$$= \frac{5 \times 10^3 x}{7853.982 \times 6.699}\left[\frac{43.301}{50}\right] = 0.0823x\ N/mm^2 (tensile)$$

For point "B" , fiber at outer surface $y_B = is\ negative$

$$y_B = r_O - r_N = 150 - 93.301 = 56.699mm$$

$$as\ y_B = is\ negative, \quad y_B = -56.699mm$$

$$=> r_B = r_n - (-y_{B)}) = r_n + y_B = r_O = 150mm$$

$$Stress\ due\ to\ bending\ at\ \text{A} = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{-y_B}{r_n - (-y_B)}\right] = -\frac{M_b}{Ae}\left[\frac{y_B}{r_B}\right]$$

$$= -\frac{5\times10^3 x}{7853.982\times6.699}\left[\frac{56.699}{150}\right] = -0.036x\ N/mm^2(compressive)$$

$Resultant\ stresses\ are:$

$\sigma_A = \sigma_{bA} - \sigma_{dA} = (0.0823x - 0.637)N/mm^2(tensile)$

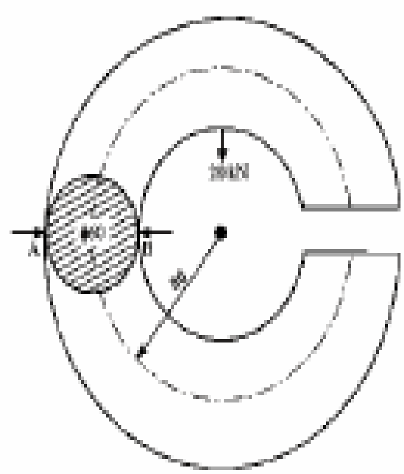
$\sigma_B = \sigma_{bB} + \sigma_{dB} = -(0.036x + 0.637)\ N/mm^2(compressive)$

Maximum tensile stress is :

$$\sigma_{max} = 50\frac{N}{mm^2} \quad => 50 = 0.0823x - 0.637$$

- x=615.273 mm

Q.6. Determine the stresses at point A and B of the split ring shown in figure.



Solution:-

Given data:-

$$r_c = 80mm$$

$r_i = r_c - R = 50mm,\ \ R = 30mm,\quad P = 20 \times 10^3 N$

$$r_o = r_c + R = 110mm$$

Determination of C.G. of the section:-

$\bar{y} = R = 30mm$

$$Radius\ of\ neutral\ axis\ "r_n" = \frac{A}{\int \frac{dA}{r}}$$

$$= \frac{\left[\sqrt{r_o} + \sqrt{r_i}\right]^2}{4}$$

$$= \frac{\left[\sqrt{110} + \sqrt{50}\right]^2}{4} = 77.081mm$$

$$A = \frac{\pi}{4} D^2 = \pi R^2 = 2827.433mm^2$$

$$e = r_c - r_n = 80 - 77.081 = 2.919mm$$

M_b

$= bending\ moment\ about\ centroid\ of\ the\ section$

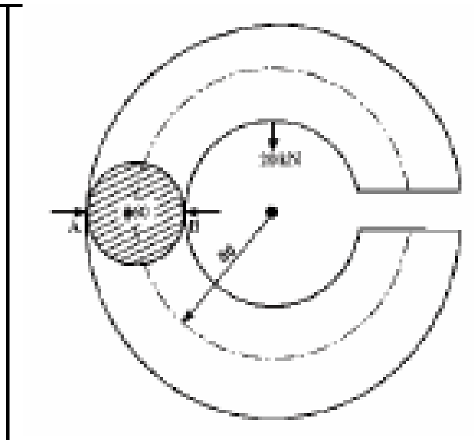
$= P \times r_c = 20 \times 10^3 \times 80 = 16 \times 10^5\ Nmm$

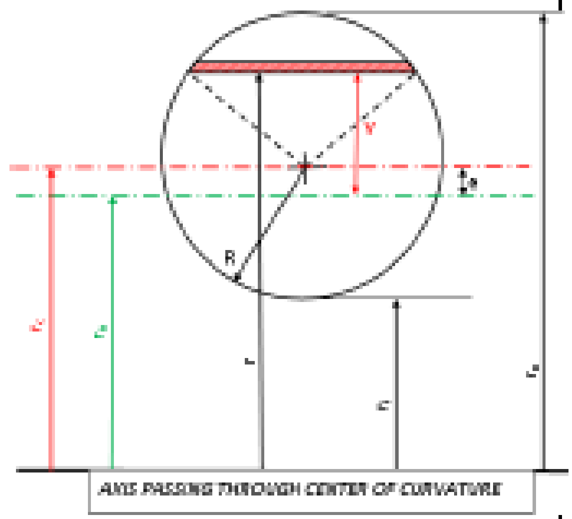
$M_b\ is\ negative, as\ the$ Applied moment trying to decrease the radius of curvature, the outer surface is under tension and the inner surface is under compression. The bending moment "M_b" is considered as negative.

Direct stress at section "A-B"

$$\sigma_{dA} = \sigma_{dB} = \sigma_d = \frac{P}{A} = \frac{20 \times 10^3}{2827.433}$$

$= 7.074\ N/mm^2 (compressive)$





For point "B" , fiber at inner surface $y_B = is\ positive$

$$y_B = r_n - r_i = 77.081 - 50 = 27.081mm$$

$$=> r_B = r_n - y_A = r_i = 50mm$$

$$Stress\ due\ to\ bending\ at\ B = \sigma_{bB} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{y_B}{r_n - y_B}\right] = \frac{M_b}{Ae}\left[\frac{y_B}{r_B}\right]$$

$$= \frac{-16 \times 10^5}{2827.433 \times 2.919}\left[\frac{27.081}{50}\right] = -105\ N/mm^2 (compressive)$$

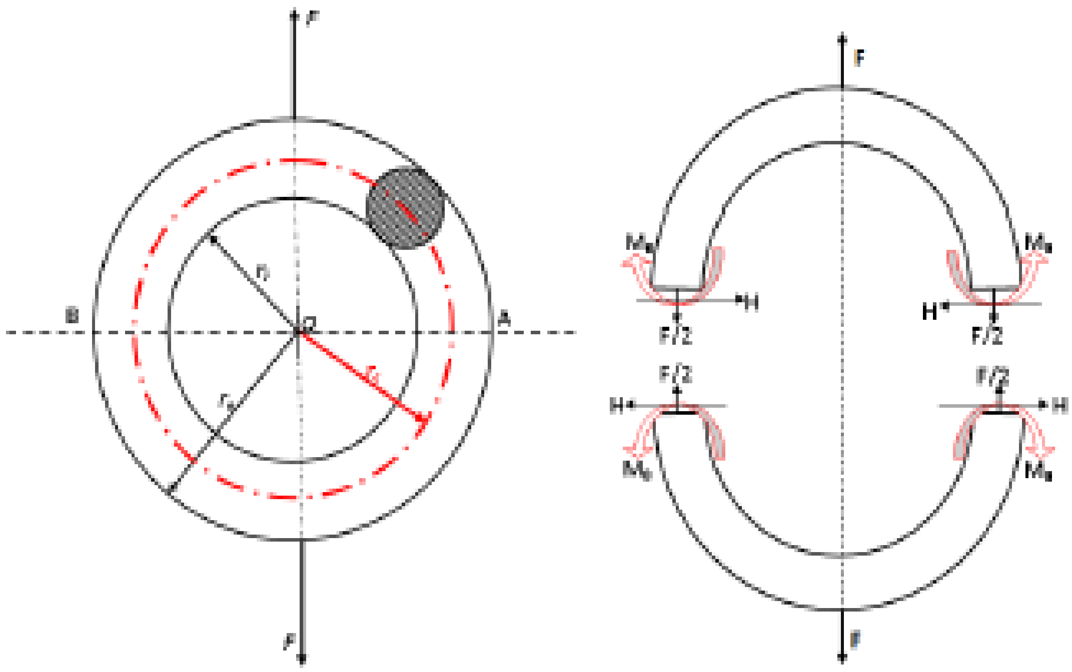
For point "A" , fiber at outer surface $y_A = is\ negative$

$$y_A = r_O - r_N = 110 - 77.081 = 32.919\ mm$$

$as\ y_A = is\ negative, \qquad y_A = -32.919mm$ $=> r_A = r_n - (-y_{A)} = r_n + y_B = r_O = 110mm$ $Stress\ due\ to\ bending\ at\ A = \sigma_{bA} = \frac{M_b}{Ae}\left[\frac{y}{r_n - y}\right] = \frac{M_b}{Ae}\left[\frac{-y_A}{r_n - (-y_A)}\right] = -\frac{M_b}{Ae}\left[\frac{y_A}{r_A}\right]$ $= -\frac{-16\times 10^5}{2827.433\times 2.919}\left[\frac{32.919}{110}\right] = 58.016\ N/mm^2(TENSILE)$
$Resultant\ stresses\ are:$ $\sigma_A = \sigma_{bA} - \sigma_{dA} = 50.942\ N/mm^2(tensile)$ $\sigma_B = \sigma_{bB} + \sigma_{dB} = -112.074\ N/mm^2(compressive)$

STRESSES IN CLOSED RING

Consider a thin circular ring subjected to symmetrical load F as shown in the figure.



The ring is symmetrical and is loaded symmetrically in vertical directions. Consider the horizontal section as shown in the A and B, the vertical forces would be F/2.

No horizontal forces would be there at A and B i.e H=0. this argument can be proved by understanding that since the ring and the external forces are symmetrical, the reactions too must be symmetrical.

Assume that two horizontal inward forces H, act at A and B in the upper half, as shown in the figure. In this case, the lower half must have forces H acting outwards as shown to maintain equilibrium.

This however, results in violation of symmetry and hence H must be zero. Besides the forces, moments of equal magnitude M_0 act at A and B. It should be noted that these moments do not violate the condition of symmetry. Thus loads on the section can be treated as that shown in the figure. The unknown quantity is M_0. Again Considering symmetry, we conclude that the tangents at A and B must be vertical and must remain so after deflection or M_0 does not rotate.

By Castigliano's theorem, the partial derivative of the strain energy with respect to the load gives the displacement of the load. In this would be zero.

$$\frac{\partial U}{\partial M_0} = 0 \qquad ------------(1)$$

The bending moment at any point C, located at angle "θ", as shown in figure. Will be

$$M_b = M_0 - \frac{F}{2}(R - R\cos\theta)$$

$$M_b = M_0 - \frac{FR}{2}(1 - \cos\theta) ------ (2)$$

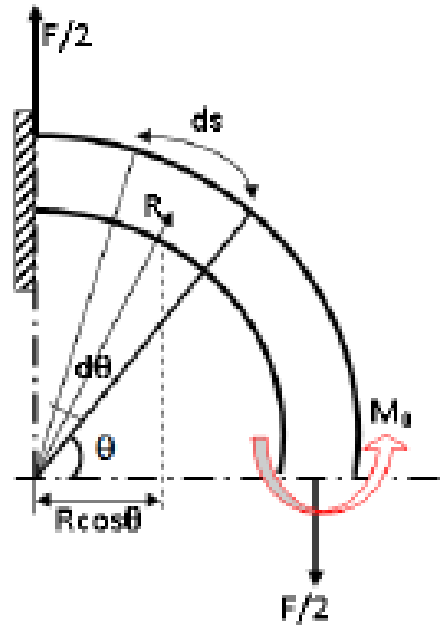
As per Castigliano's theorem

$$\frac{\partial U}{\partial M_0} = 0$$

$$=> \frac{\partial U}{\partial M_0} = \int_0^L \frac{M_b}{EI}\left(\frac{\partial M_b}{\partial M_0}\right) ds = 0$$

$$\frac{\partial M_b}{\partial M_0} = \frac{\partial\left[M_0 - \frac{FR}{2}(1 - \cos\theta)\right]}{\partial M_0} = 1$$

$$\& \; ds = R d\theta$$



$$=> \frac{\partial U}{\partial M_0} = \int_0^L \frac{M_b}{EI}\left(\frac{\partial M_b}{\partial M_0}\right) ds = 4\int_0^{\pi/2} \frac{\left[M_0 - \frac{FR}{2}(1 - \cos\theta)\right]}{EI} \times 1 \times R d\theta$$

$$= \frac{4R}{EI}\int_0^{\pi/2}\left[M_0 - \frac{FR}{2}(1 - \cos\theta)\right] d\theta = 0$$

$$=> \int_0^{\pi/2}\left[M_0 - \frac{FR}{2} + \frac{FR}{2}\cos\theta\right] d\theta = 0$$

$$=> \left[M_0\theta - \frac{FR}{2}\theta + \frac{FR}{2}\sin\theta\right]_0^{\pi/2} = 0$$

$$=> M_0\frac{\pi}{2} - \frac{FR}{2}\frac{\pi}{2} + \frac{FR}{2} = 0$$

$$=> M_0 = \frac{FR}{\pi}\left(\frac{\pi}{2} - 1\right) \qquad ---------(3)$$

As this quantity (M_0) is positive the direction assumed for M_0 is correct and it produces tension in the inner fibers and compression on the outer.

It should be noted that these equations are valid in the region, θ = 0 to θ = 90^0. The bending moment M_b at any angle θ from equation (2) will be:-

$$M_b = \frac{FR}{\pi}\left(\frac{\pi}{2} - 1\right) - \frac{FR}{2}(1 - \cos\theta) = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right) \qquad -------- (4)$$

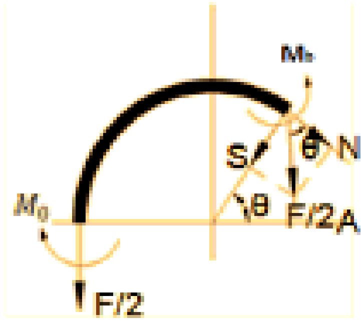
Bending moment will be Zero when,

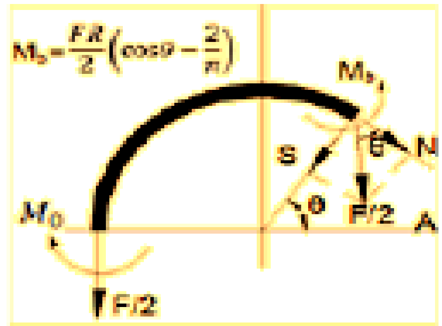
$$\cos\theta = \frac{2}{\pi} \qquad => \; \theta = 50.46^0 \qquad ---------- (5)$$

At load point, i.e. at = $\pi/2$, the bending moment is maximum :

$$M_{b-max} = \frac{FR}{2}\left(-\frac{2}{\pi}\right) = -\frac{FR}{\pi} \qquad -------------- (6)$$

It is seen that numerically, M_{b-max} is greater than M_0. The stress at any angle θ can be found out by considering the forces as shown in the figure.





The vertical force F/2 can be resolved into two components (creates normal direct stresses) (creates shear stresses).

$$N = \frac{F}{2}\cos\theta$$

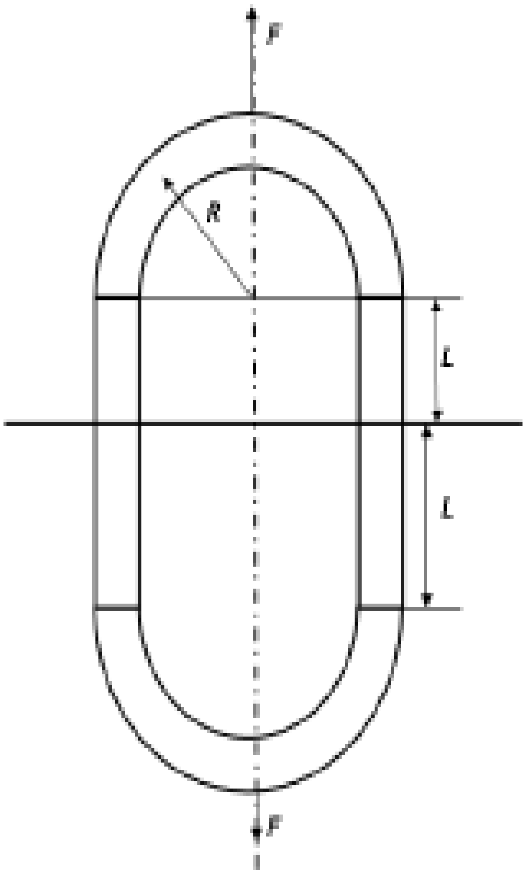
$$S = \frac{F}{2}\sin\theta$$

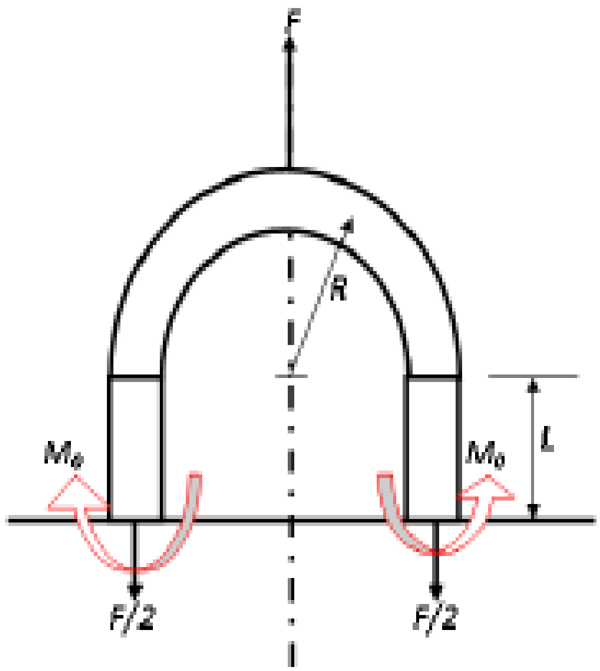
The combined normal stress across any section will be: $\sigma = \pm\frac{M_b y}{Ae(r_n \pm y)} + \frac{F}{2A}\cos\theta$

It should be noted that in calculating the bending stresses, it is assumed that the radius is large compared to the depth, or the beam is almost a straight beam.

THIN EXTENDED CLOSED LINK

Consider a thin closed ring subjected to symmetrical load F as shown in the figure. At the two ends C and D, the vertical forces would be F/2.





No horizontal forces would be there at C and D, as discussed earlier ring. The unknown quantity is M_0.

Again considering symmetry, we conclude that the tangents at C and D must be vertical and must remain so after deflection or M_0 does not rotate.

There are two regions to be considered in this case:

- The straight portion, $(0 < y < L)$ where $M_b = M_0$
- The curved portion, where bending moment about point "C" as shown in figure is:

$$M_b = M_0 - \frac{F}{2}(R - R\cos\theta)$$

As per Castigliano's theorem

$$\frac{\partial U}{\partial M_0} = 0 \quad => \quad \frac{\partial U_{straight}}{\partial M_0} + \frac{\partial U_{curved}}{\partial M_0} = 0$$

$$\frac{\partial U}{\partial M_0} = 4\int_0^L \frac{M_0}{EI}\left(\frac{\partial M_0}{\partial M_0}\right) ds + 4\int_0^{\pi/2} \frac{M_b}{EI}\left(\frac{\partial M_b}{\partial M_0}\right) ds = 0$$

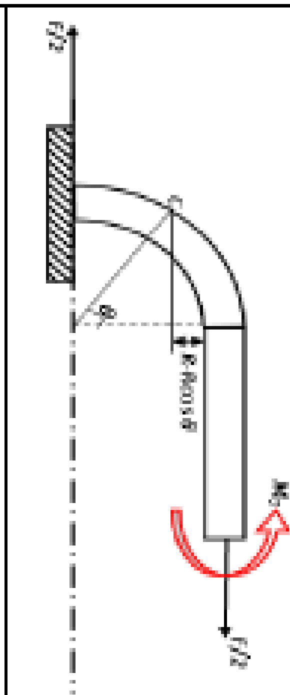
$$=> 4\int_0^L \frac{M_0}{EI}\, ds + 4\int_0^{\pi/2} \frac{M_b}{EI}(1)\, ds = 0$$

$$=> \frac{4M_0L}{EI} + \frac{4R}{EI}\int_0^{\pi/2}\left[M_0 - \frac{F}{2}(R - R\cos\theta)\right] d\theta = 0 \quad as\ ds = Rd\theta$$

$$=> \frac{M_0L}{EI} + \frac{M_0\pi R}{2EI} - \frac{F\pi R^2}{4EI} + \frac{FR^2}{2EI} = 0$$

$$=> M_0L + \frac{M_0\pi R}{2} - \frac{F\pi R^2}{4} + \frac{FR^2}{2} = 0$$

$$=> M_0\left[\frac{2L + \pi R}{2}\right] = \frac{FR^2}{2}\left[\frac{\pi - 2}{2}\right]$$



It can be observed that at L = 0 equation reduces to the same expression as obtained for a circular ring. i.e

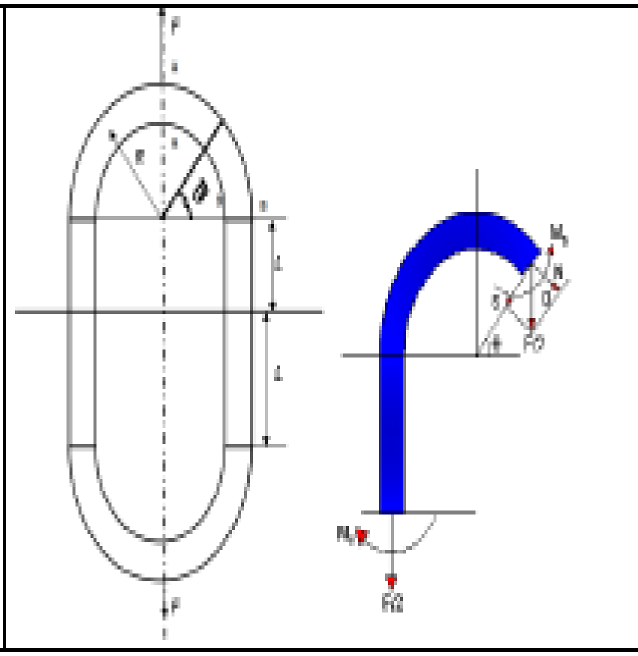
$$=> M_0 = \frac{FR}{\pi}\left(\frac{\pi}{2} - 1\right)$$

The M_0 produces tension in inner fiber and Compression on the outer.

The bending moment M_b at any angle θ will be:

$$M_b = M_0 - \frac{F}{2}(R - R\cos\theta) = \frac{FR^2}{2}\left[\frac{\pi - 2}{2L + \pi R}\right] - \frac{FR}{2}(1 - \cos\theta)$$

Noting that the above equation is valid in the region, θ = 0 to θ = π/2

At θ = 0, the bending moment at section BB $$M_b = M_0 = \frac{FR^2}{2}\left[\frac{\pi - 2}{2L + \pi R}\right]$$ At θ = π/2, the bending moment at section AA $$M_b = \frac{FR^2}{2}\left[\frac{\pi - 2}{2L + \pi R}\right] - \frac{FR}{2}$$	

The stress at any angle θ can be found by considering the force as shown in the above figure. The vertical force F/2 can be resolved in two components (creates normal direct stresses) and S (creates shear stresses).

Normal force "N"

$$N = \frac{F}{2}\cos\theta$$

& shear stress "S"

$$S = \frac{F}{2}\sin\theta$$

The combined normal stress across any section will be

$$\sigma = \pm\frac{M_b y}{Ae(r_n \pm y)} + \frac{F}{2A}\cos\theta$$

Q.7. Determine the stress induced in a circular ring of circular cross section of 25 mm diameter subjected to a tensile load 6500N. The inner diameter of the ring is 60 mm.

Solution:

Given data:-

$D_i = 60mm,\ d = 25mm, r = 12.5mm \quad F = 6500N,$

$r_i = 30mm, r_o = r_i + d = 55mm$

Determination of C.G. of the section:-

$\bar{y} = r = 12.5mm$

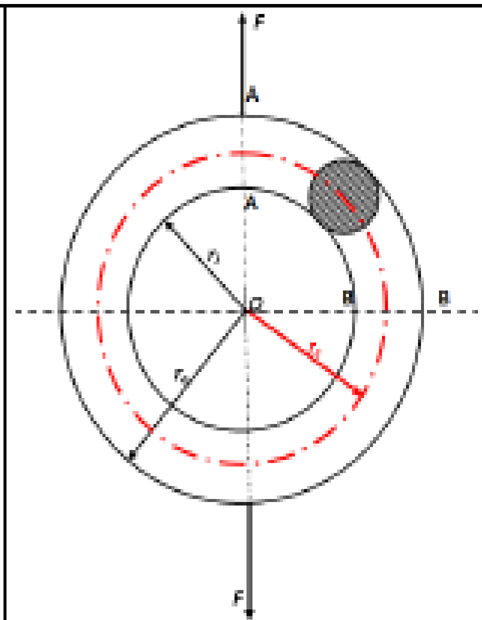
$r_c = R = r_i + r = 42.5mm$

$$\text{Radius of neutral axis } "r_n" = \frac{A}{\int \frac{dA}{r}} = \frac{\left[\sqrt{r_o}+\sqrt{r_i}\right]^2}{4}$$

$$= \frac{\left[\sqrt{55}+\sqrt{30}\right]^2}{4} = 41.56mm$$

$$A = \frac{\pi}{4}d^2 = \pi r^2 = 490.874\ mm^2$$

$$e = r_c - r_n = 42.5 - 41.56 = 0.94\ mm$$



The bending moment M_b at any angle θ

$$M_b = \frac{FR}{\pi}\left(\frac{\pi}{2}-1\right) - \frac{FR}{2}(1-\cos\theta) = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

At section B-B, θ = 0

$$M_b = M_{bB-B} = \frac{FR}{\pi}\left(\frac{\pi}{2}-1\right) - \frac{FR}{2}(1-\cos 0)$$

$$M_{bB-B} = \frac{FR}{2}\left(\cos 0 - \frac{2}{\pi}\right) = \frac{FR}{2}\left(1-\frac{2}{\pi}\right)$$

$$= \frac{6500 \times 42.5}{2}\left(1-\frac{2}{\pi}\right) = 50277.5Nmm$$

At inner fiber

$r_{Bi} = r_i = 30mm, \quad y_{Bi} = r_n - r_i = 11.56mm(+ve)$

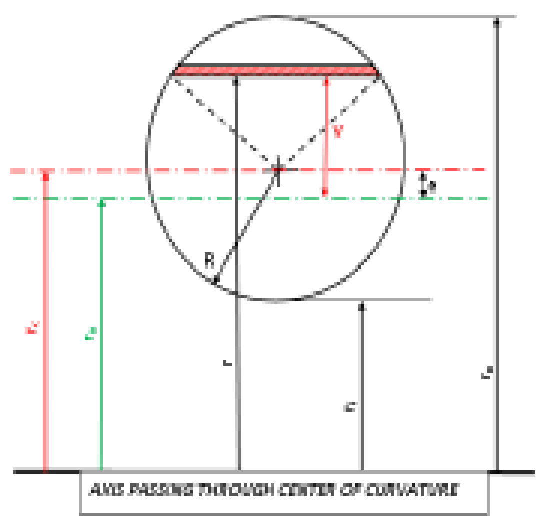
$$\sigma_{bBi} = \frac{M_{bB-B} \times y_{Bi}}{Ae(r_n - y_{Bi})} = \frac{M_{bB-B} \times y_{Bi}}{Ae\, r_{Bi}}$$

$$= \frac{50277.5 \times 11.56}{490.874 \times 0.94 \times 30}$$

$$= 41.987\frac{N}{mm^2}\ (Tensile)$$

At outer fiber

$r_{Bo} = r_o = 55mm, \quad y_{Bo} = r_o - r_n = 13.44mm(-ve)$



$$\sigma_{bBo} = \frac{M_{bB-B} \times y_{Bo}}{Ae(r_n - (-y_{Bo}))} = \frac{M_{bB-B} \times y_{Bo}}{Ae\, r_{Bo}}$$
$$= \frac{50277.5 \times (-13.44)}{\mathbf{490.874} \times \mathbf{0.94} \times 55}$$
$$= -26.626 \frac{N}{mm^2}\ (compressive)$$

Direct stress at section B-B:

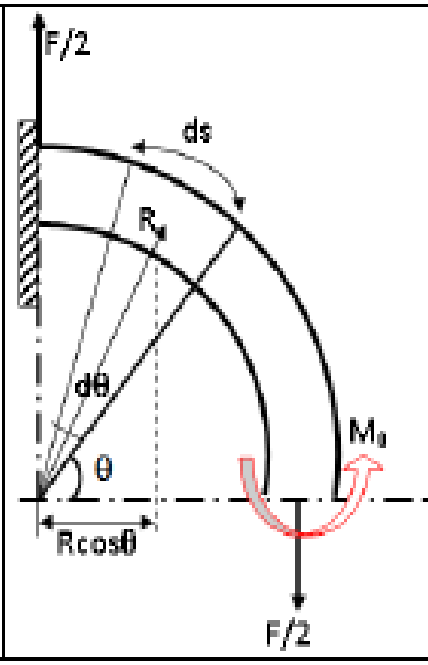
At section B-B, θ = 0

$$\sigma_{dB-B} = \frac{F}{2A}\cos\theta = \frac{6500}{2 \times \mathbf{490.874}}\cos 0$$
$$= 6.621\ N/mm^2 (Tensile)$$

Resultant stress at section *B-B*

$$\sigma_{Bi} = \sigma_{bBi} + \sigma_{dB-B} = 48.608\ N/mm^2 (Tensile)$$

$$\sigma_{Bo} = \sigma_{bBo} + \sigma_{dB-B} = -20.005\ \frac{N}{mm^2}\ (compressive)$$



At section A-A, θ = π/2

$$M_b = M_{bA-A} = \frac{FR}{\pi}\left(\frac{\pi}{2} - 1\right) - \frac{FR}{2}(1 - \cos 90) = \frac{FR}{2}\left(\cos 90 - \frac{2}{\pi}\right) = \frac{6500 \times 42.5}{2}\left(\cos 90 - \frac{2}{\pi}\right)$$
$$= -87933.106\ Nmm$$

At inner fiber

$$r_{Ai} = r_i = 30mm, \qquad y_{Ai} = r_n - r_i = 11.56mm(+ve)$$

$$\sigma_{bAi} = \frac{M_{bA-A} \times y_{Ai}}{Ae(r_n - y_{Ai})} = \frac{M_{bA-A} \times y_{Ai}}{Ae\, r_{Ai}} = \frac{-87933.106 \times 11.56}{\mathbf{490.874} \times \mathbf{0.94} \times \mathbf{30}} = -73.433 \frac{N}{mm^2}\ (compressive)$$

At outer fiber

$$r_{Ao} = r_o = 55mm, \qquad y_{AO} = r_0 - r_n = 13.44mm(-ve)$$

$$\sigma_{bAo} = \frac{M_{bA-A} \times y_{Ao}}{Ae(r_n - (-y_{Ai}))} = \frac{M_{bA-A} \times y_{Ao}}{Ae\, r_{Ao}} = \frac{-87933.106 \times (-13.44)}{\mathbf{490.874} \times \mathbf{0.94} \times \mathbf{55}} = 46.568 \frac{N}{mm^2}\ (TENSILE)$$

Direct stress at section A-A:

At section A-A, θ = π/2

$$\sigma_{dA-A} = \frac{F}{2A}\cos\theta = \frac{6500}{2 \times \mathbf{490.874}}\cos\pi/2 = 0$$

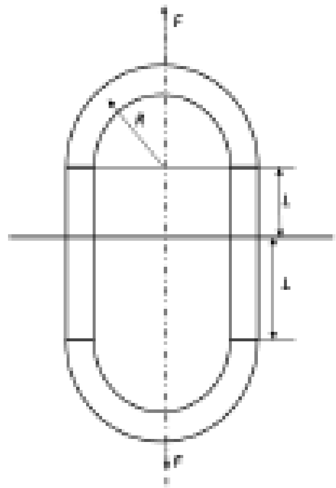
Resultant stress at section *A-A*

$$\sigma_{Ai} = \sigma_{bAi} + \sigma_{dA-A} == -73.433 \frac{N}{mm^2}\ (compressive)$$

$$\sigma_{Ao} = \sigma_{bAo} + \sigma_{dA-A} = 46.568 \frac{N}{mm^2}\ (TENSILE)$$

Q.8. A chain link is made of 40 mm diameter rod is circular at each end, the mean diameter of which is 80mm. The straight sides of the link are also 80mm. If the link carries a load of 90kN; estimate the tensile and compressive stress along the section of load line. Also find the stress at a section 90^0 from the load line.

Solution:



Solution:-

Given data:-

$D_c = 80mm,\ r_c = 40mm \quad d = 40mm,\ r = 20mm$

$F = 90000N,\ r_i = 20mm,\ r_o = r_i + d = 60mm$

$$2L = 80mm$$

Determination of C.G. of the section:-

$\bar{y} = r = 20mm$

$r_c = R = 40mm$

$$Radius\ of\ neutral\ axis\ "r_n" = \frac{A}{\int \frac{dA}{r}} = \frac{\left[\sqrt{r_o} + \sqrt{r_i}\right]^2}{4}$$

$$= \frac{\left[\sqrt{60} + \sqrt{20}\right]^2}{4} = 37.32mm$$

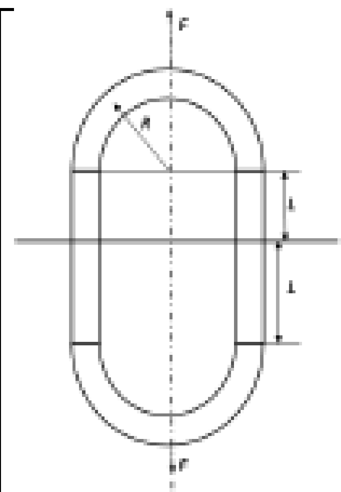
$$A = \frac{\pi}{4} d^2 = \pi r^2 = 1256.637\ mm^2$$

$$e = r_c - r_n = 40 - 37.32 = 2.68\ mm$$

The bending moment M_b at any angle θ will be:

$$M_b = M_0 - \frac{F}{2}(R - R\cos\theta)$$

$$= \frac{FR^2}{2}\left[\frac{\pi - 2}{2L + \pi R}\right] - \frac{FR}{2}(1 - \cos\theta)$$



At section B-B, θ = 0

$$M_b = M_{bB-B} = \frac{FR^2}{2}\left[\frac{\pi-2}{2L+\pi R}\right] - \frac{FR}{2}(1-\cos 0)$$

$$M_{bB-B} = \frac{FR^2}{2}\left[\frac{\pi-2}{2L+\pi R}\right]$$

$$= \frac{90000\times 40^2}{2}\left(\frac{\pi-2}{2\times 40+\pi 40}\right) = 399655.7\ Nmm$$

At inner fiber

$r_{Bi} = r_i = 20mm, \qquad y_{Bi} = r_n - r_i = 17.32\ mm(+ve)$

$$\sigma_{bBi} = \frac{M_{bB-B}\times y_{Bi}}{Ae(r_n - y_{Bi})} = \frac{M_{bB-B}\times y_{Bi}}{Ae\ r_{Bi}} = \frac{399655.7\times 17.32}{\mathbf{1256.637\times 2.68\times 20}} = 102.768\ \frac{N}{mm^2}\ (Tensile)$$

At outer fiber

$r_{Bo} = r_o = 60\ mm, \qquad y_{Bo} = r_o - r_n = 22.68\ mm(-ve)$

$$\sigma_{bBo} = \frac{M_{bB-B}\times y_{Bo}}{Ae(r_n-(-y_{Bo}))} = \frac{M_{bB-B}\times (y_{Bo})}{Ae\ r_{Bo}} = \frac{399655.7\times(-22.68)}{\mathbf{1256.637\times 2.68\times 60}} = -44.857\frac{N}{mm^2}\ (compressive)$$

Direct stress at section B-B:

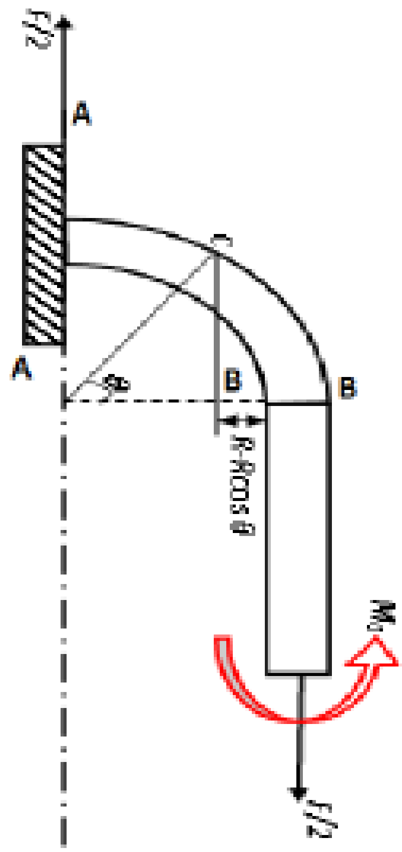
At section B-B, θ = 0

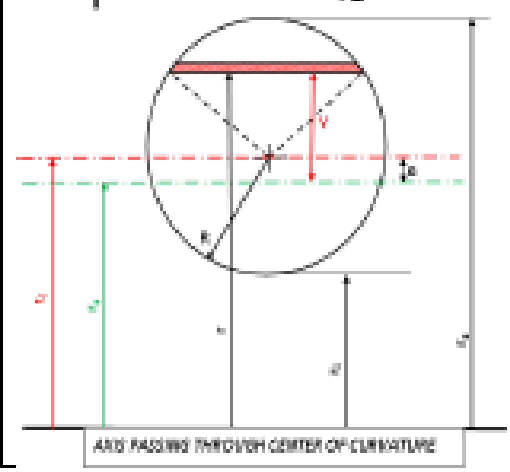
$$\sigma_{dB-B} = \frac{F}{2A}\cos\theta = \frac{90000}{2\times\mathbf{1256.637}}\cos 0 = 35.81\ N/mm^2\ (Tensile)$$

Resultant stress at section B-B

$$\sigma_{Bi} = \sigma_{bBi} + \sigma_{dB-B} = 138.578\ N/mm^2\ (Tensile)$$

$$\sigma_{Bo} = \sigma_{bBo} + \sigma_{dB-B} = -9.047\ \frac{N}{mm^2}\ (compressive)$$





At section A-A, θ = π/2

$$M_b = M_{bA-A} = \frac{FR^2}{2}\left[\frac{\pi-2}{2L+\pi R}\right] - \frac{FR}{2}(1-\cos 90) = \frac{\mathbf{FR^2}}{\mathbf{2}}\left[\frac{\boldsymbol{\pi}-\mathbf{2}}{\mathbf{2L+\pi R}}\right] - \frac{\mathbf{FR}}{\mathbf{2}}$$

$$= \frac{90000\times 40^2}{2}\left(\frac{\pi-2}{2\times 40+\pi\times 40}\right) - \frac{\mathbf{90000\times 40}}{\mathbf{2}} = -1.4\times 10^6\ Nmm$$

At inner fiber

$r_{Ai} = r_i = 20mm, \qquad y_{Ai} = r_n - r_i = 17.32\ mm(+ve)$

$$\sigma_{bAi} = \frac{M_{bA-A}\times y_{Ai}}{Ae(r_n - y_{Ai})} = \frac{M_{bA-A}\times y_{Ai}}{Ae\ r_{Ai}} = \frac{-1.4\times 10^6\times 17.32}{\mathbf{1256.637\times 2.68\times 20}} = -360\frac{N}{mm^2}\ (compressive)$$

At outer fiber

$$r_{Ao} = r_o = 60\ mm, \qquad y_{AO} = r_0 - r_n = 13.44mm(-ve)$$

$$\sigma_{bAo} = \frac{M_{bA-A} \times y_{Ao}}{Ae(r_n - (-y_{Ai}))} = \frac{M_{bA-A} \times y_{Ao}}{Ae\, r_{Ao}} = \frac{-1.4 \times 10^6 \times (-22.68)}{\mathbf{1256.637 \times 2.68 \times 60}} = 157.14\frac{N}{mm^2}\ (TENSILE)$$

Direct stress at section A-A:

At section A-A, θ = *π/2*

$$\sigma_{dA-A} = \frac{F}{2A}\cos\theta = \frac{90000}{2 \times \mathbf{1256.637}}\cos\pi/2 = 0$$

Resultant stress at section *A-A*

$$\sigma_{Ai} = \sigma_{bAi} + \sigma_{dA-A} == -360\frac{N}{mm^2}\ (compressive)$$

$$\sigma_{Ao} = \sigma_{bAo} + \sigma_{dA-A} = 157.14\frac{N}{mm^2}\ (TENSILE)$$

Q.9. A steel ring of rectangular section 8 mm width by 5 mm thickness has a mean diameter of 30 cm. A narrow saw cut was made and tangential separating force of 0.5 kg each are applied at the cut in the plane of the ring. Find the additional separation due to these forces.(E=$2x10^6$ kg/cm^2.)

Solution:-

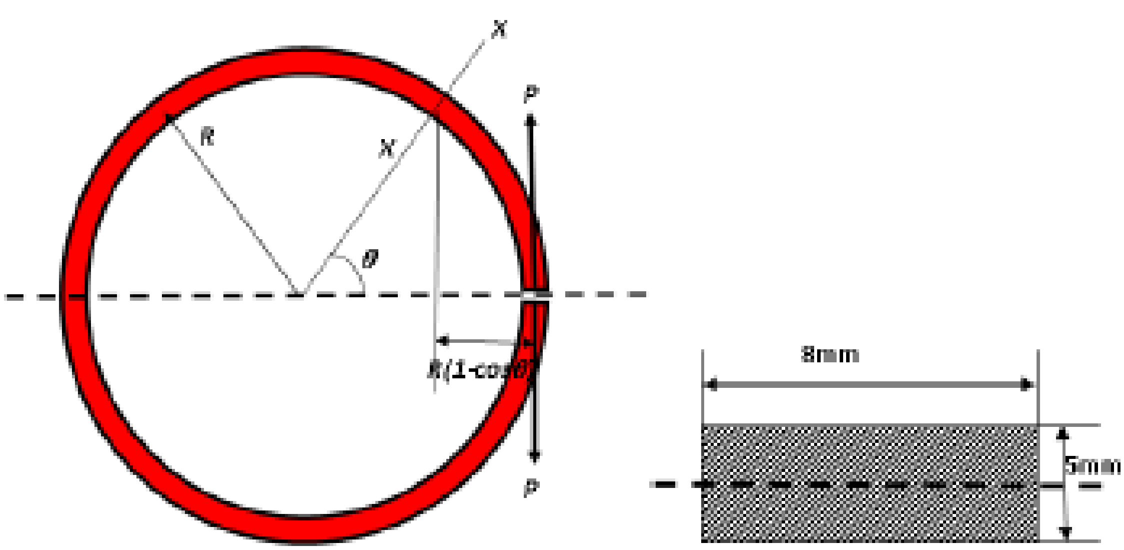
Given data:-

R= 15 cm =150mm

b= 8 mm, t= 5mm

P=0.5 kg

E=$2x10^6$ kg/cm^2



$$I = \frac{1}{12} b\, t^3 = 83.333\ mm^4$$

Bending moment at any section X-X at any angle θ is "M":

$$M = PR(1 - \cos\theta)$$

As per castgiliano's theorem :

$$\delta = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\pi} M \left(\frac{\partial M}{\partial P}\right) R d\theta$$

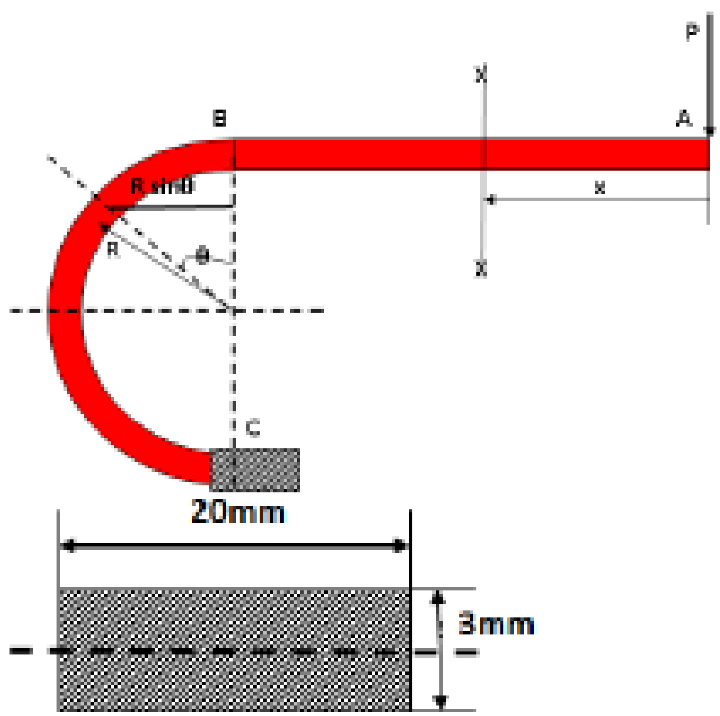
$$AS\ \frac{\partial M}{\partial P} = R(1 - \cos\theta)$$

$$\delta = \frac{\partial U}{\partial P} = \frac{PR^2}{EI}\int_0^{\pi} (1 - \cos\theta)(1 - \cos\theta)\, d\theta$$

$$= \frac{PR^2}{EI}\int_0^{\pi} (1 - \cos\theta)^2\, d\theta = \frac{PR^2}{EI}\int_0^{\pi} (1 + \cos^2\theta - 2\cos\theta) d\theta$$

$$= \frac{PR^2}{EI}\left[\theta - 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi} = \frac{3\pi PR^3}{EI} = 9.54\ mm$$

Q.10. A steel spring ABC of radius R=60mm & AB of length 120mm is firmly fixed at point "c" as shown in figure. Find the vertical deflection at point "A" neglecting the effect of shear, where take E=2x10[6] kg/cm[2].



Solution:-

Given data:-

R=60mm,   L=120mm, E=2x10^6 kg/cm^2.

$I = \frac{1}{12} b\, t^3 = \frac{1}{12} 20 \times 3^3 = 45\ mm^4 = 45 \times 10^{-4}\ cm^4$

$U_{AC} = U_{AB} + U_{BC}$

$Vertical\ deflection\ at\ "A"\ \ is\ \delta_A$

$\delta_A = \frac{\partial U_{AC}}{\partial P} = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P}$

FOR the portion "AB": (0<x<L)

Bending moment at any distance "x" from point A is:

$M_{AB} = Px$

$\frac{\partial M_{AB}}{\partial P} = x$

$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI} \int_0^L M_{AB} \left(\frac{\partial M_{AB}}{\partial P}\right) dx = \frac{1}{EI} \int_0^L Px \times x\, dx = \frac{PL^3}{3EI}$

FOR the portion "BC": (0<θ<π)

Bending moment at any distance "x" from point A is:

$M_{BC} = P(L + Rsin\theta)$

$\frac{\partial M_{BC}}{\partial P} = (L + Rsin\theta)$

$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI} \int_0^S M_{BC} \left(\frac{\partial M_{BC}}{\partial P}\right) ds = \frac{1}{EI} \int_0^{\pi} P(L + Rsin\theta) \times (L + Rsin\theta) R\, d\theta \quad AS\ ds = Rd\theta$

$= \frac{PR}{EI} \int_0^{\pi} (L^2 + R^2 sin^2\theta + 2RLsin\theta)\ d\theta = \frac{PR}{EI} \left[L^2\theta - 2RLcos\theta + \left(\frac{R^2\theta - \frac{R^2 sin2\theta}{2}}{2} \right) \right]_0^{\pi}$

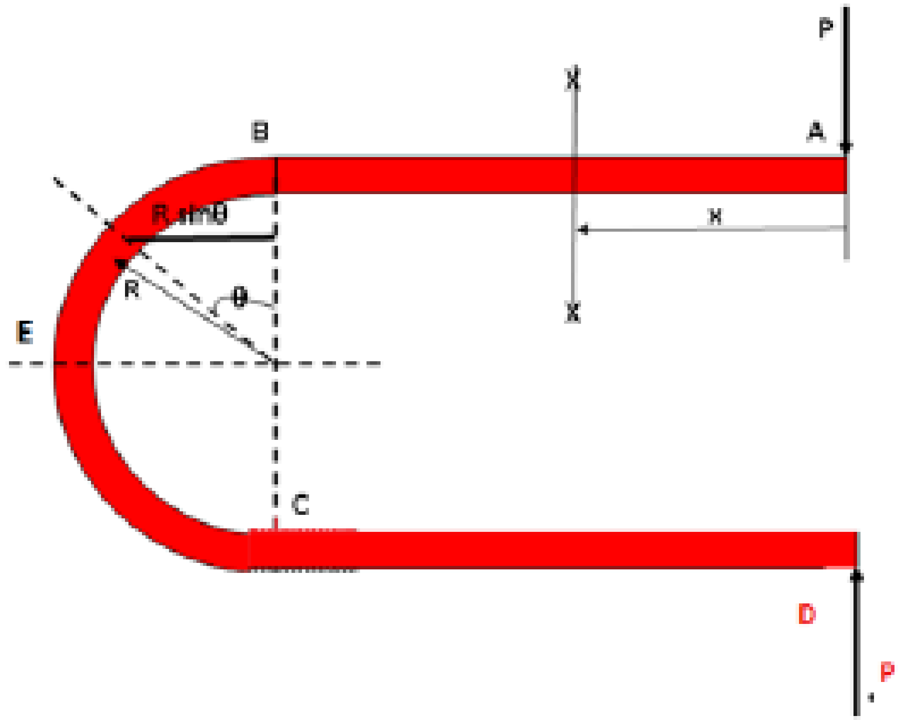
$= \frac{PR}{EI} \left[L^2\pi + 4RL + \frac{R^2\pi}{2} \right]$

$\delta_A = \frac{PL^3}{3EI} + \frac{PR}{EI} \left[L^2\pi + 4RL + \frac{R^2\pi}{2} \right] = \frac{1}{EI} \left[\frac{PL^3}{3} + PR(L^2\pi + 4RL + \frac{R^2\pi}{2}) \right]$

$= \frac{1}{EI} [1152 + 12(452.389 + 288 + 56.548)] = \frac{1}{EI} [10715.244] = \frac{10715.244}{2 \times 10^6 \times 45 \times 10^{-4}}$

$= 1.1906cm$

Q.11.   A steel spring ABCD made of from a rod of diameter "d". the semi-circular portion B to C of radius R & straight portion AB and CD of length L is loaded by end loads P as shown in figure. Find the increase in distance A&D due to loads.



Solution:-

Vertical deflection/separation between AD is δ_{AD}

Due to symmetric

$\delta_{AD} = 2\delta_A$ *where* δ_A *is the verical deflection of the point A for the beam from A toE*

$$\delta_{AD} = \frac{\partial U_{AD}}{\partial P} = 2 \times \frac{\partial U_{AE}}{\partial P} = 2\left[\frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BE}}{\partial P}\right] = 2\delta_A$$

$$\delta_A = \left[\frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BE}}{\partial P}\right]$$

FOR the portion "AB": (0<x<L)

Bending moment at any distance "x" from point A is:

$$M_{AB} = Px$$

$$\frac{\partial M_{AB}}{\partial P} = x$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^L M_{AB}\left(\frac{\partial M_{AB}}{\partial P}\right) dx = \frac{1}{EI}\int_0^L Px \times x\, dx = \frac{PL^3}{3EI}$$

FOR the portion "BC": (0<θ<π/2)

Bending moment at any distance "x" from point A is:

$$M_{BC} = P(L + R\sin\theta)$$

$$\frac{\partial M_{BC}}{\partial P} = (L + R\sin\theta)$$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^S M_{BC}\left(\frac{\partial M_{BC}}{\partial P}\right) ds = \frac{1}{EI}\int_0^{\pi/2} P(L + R\sin\theta) \times (L + R\sin\theta) R\, d\theta \quad AS\ ds = Rd\theta$$

$$= \frac{PR}{EI}\int_0^{\pi/2}(L^2 + R^2 sin^2\theta + 2RLsin\theta)\ d\theta = \frac{PR}{EI}\left[L^2\theta - 2RLcos\theta + \left(\frac{R^2\theta - \frac{R^2 sin2\theta}{2}}{2}\right)\right]_0^{\pi/2}$$

$$= \frac{PR}{EI}\left[L^2\frac{\pi}{2} + 2RL + \frac{R^2\pi}{4}\right]$$

$$\delta_A = \frac{PL^3}{3EI} + \frac{PR}{EI}\left[L^2\frac{\pi}{2} + 2RL + \frac{R^2\pi}{4}\right] = \frac{1}{12EI}\left[4PL^3 + PR(6L^2\pi + 24RL + 3R^2\pi)\right]$$

$$\delta_{AD} = 2\delta_A = \frac{1}{6EI}\left[4PL^3 + PR(6L^2\pi + 24RL + 3R^2\pi)\right]$$