4) Solve $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 6e^{x+y}$

5) Solve $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{x-y} + e^{2x+y}$.

4) $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = e^{x+y}$

Soln: $D^2 - 5DD' + 6D'^2 = e^{x+y}$

Replace $D \to m$ & $D' \to 1$

C.F: The auxiliary eqn is

$m^2 - 5m + 6 = 0$

$(m-3)(m-2) = 0$

$m_1 = 3$; $m_2 = 2$

$C.F = f_1(y+3x) + f_2(y+2x)$

Particular Integral

$$P.I = \frac{1}{f(D,D')} F(x,y)$$

$$= \frac{1}{(D^2 - 5DD' + 6D'^2)} e^{x+y}$$

Here $a=1$; $b=1$ $D \to a=1$; $D' \to b=1$

$$= \frac{1}{1-5+6} e^{x+y} = \frac{1}{2} e^{x+y}$$

$$PI = \frac{e^{x+y}}{2}$$

$$z = CF + P.I$$
$$= f_1(y+3x) + f_2(y+2x) + \frac{e^{x+y}}{2}$$

5) $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{x-y} + e^{2x+y}$

Soln: $D^2 - 4DD' + 4D'^2 = e^{x-y} + e^{2x+y}$

C.F: The auxiliary eqn is

$D \to m$; $D' \to 1$

$m^2 - 4m + 4 = 0$

$(m-2)(m-2) = 0$

$m_1 = m_2 = 2$

The roots are real and equal

$CF = f_1(y+2x) + x f_2(y+2x)$

Particular Integral

$$PI = \frac{1}{f(D,D')} F(x,y)$$

$$= \frac{1}{D^2 - 4DD' + 4D'^2} e^{x-y} + e^{2x+y}$$

$$= \frac{1}{D^2 - 4DD' + 4D'^2} e^{x-y} + \frac{1}{D^2 - 4DD' + 4D'^2} e^{2x+y}$$

$$= PI_1 + PI_2$$

$$PI_1 = \frac{1}{D^2-4DD'+4D'^2} e^{x-y}$$

Here $a=1$; $b=-1$

$D \rightarrow a=1$; $D' \rightarrow b=-1$

$$PI_1 = \frac{1}{1+4+4} e^{x-y} = \frac{1}{9} e^{x-y}$$

$$PI_1 = \frac{1}{9} e^{x-y}.$$

$$PI_2 = \frac{1}{D^2-4DD'+4D'^2} e^{2x+y}$$

Here $a=2$; $b=1$

$D \rightarrow a=2$; $D' \rightarrow b=1$

$$PI_2 = \frac{1}{4-8+4} e^{2x+y} = \frac{1}{0} e^{2x+y}$$

diff Dr w.r.to x and put an x in Nr.

$$PI_2 = \frac{x}{2D-4D'} e^{2x+y} = \frac{x}{4-4} e^{2x+y}$$

$$= \frac{x^2}{2} e^{2x+y}$$

$\therefore$ $PI_2 = e^{2x+y}$

$\therefore$ The general solution is

$$y = CF + PI_1 + PI_2$$

$$= f_1(y+2x) + x f_2(y+2x) + \frac{1}{9} e^{x-y} + \frac{x^2}{2} e^{2x+y}$$