Lagrange's Linear Equations:

General form $Pp + Qq = R$

To solve, $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

(i) Method of Grouping:

*) A.E $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

*) Consider 1st two, eliminate z ; $u(x,y) = c_1$

*) Consider 2nd two, eliminate x ; $v(x,y) = c_2$

*) Solution is $\phi(u(x,y), v(x,y)) = 0$.

1) Solve $x^2p + y^2q = z^2$.

Soln: The equation is of the form $Pp + Qq = R$. (L.F)

where $P = x^2$; $Q = y^2$; $R = z^2$

the auxiliary eqn is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \Rightarrow \frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$$

Consider, $\frac{dx}{x^2} = \frac{dy}{y^2}$

Integrating

$$\int \frac{dx}{x^2} = \int \frac{dy}{y^2}$$

$$\frac{x^{-2+1}}{-2+1} = \frac{y^{-2+1}}{-2+1} + C_1$$

$$\frac{x^{-1}}{-1} = \frac{y^{-1}}{-1} + C_1$$

$$C_1 = \frac{1}{x} - \frac{1}{y}$$

Consider $\frac{dy}{y^2} = \frac{dz}{z^2}$

Integrating

$$\int \frac{dy}{y^2} = \int \frac{dz}{z^2}$$

$$\frac{y^{-2+1}}{-2+1} = \frac{z^{-2+1}}{-2+1} + C_2$$

$$\frac{y^{-1}}{-1} = \frac{z^{-1}}{-1}$$

$$C_2 = \frac{1}{y} - \frac{1}{z}.$$

Soln is $\phi\left(\frac{1}{x} - \frac{1}{y}, \frac{1}{y} - \frac{1}{z}\right) = 0$

2) Solve $xp + yq = 0$ —— (1)

Soln: Gn: is of the form $Pp + Qq = R$ (L.F)

$$P = x \,;\, Q = y \,;\, R = 0.$$

The auxiliary eqn is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \Rightarrow \frac{dx}{x} = \frac{dy}{y} = \frac{dz}{0}$$

Consider,

$$\frac{dx}{x} = \frac{dy}{y}$$

$$\int \frac{dx}{x} = \int \frac{dy}{y}$$

$$\log x = \log y + \log c_1$$

$$\log\left(\frac{x}{y}\right) = \log c_1$$

$$\Rightarrow c_1 = \frac{x}{y}$$

Consider

$$\frac{dy}{y} = \frac{dz}{0}$$

$$\int \frac{dy}{y} = \int \frac{dz}{0}$$

~~logy=elle~~:

$$\int dy = 0$$

$$y = c_2$$

$\therefore$ The solution is $\phi(c_1, c_2) = 0$.

$$\phi\left(\frac{x}{y}, y\right) = 0.$$

3) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$ (L.F

Soln:- The eqn is of the form $Pp + Qq = R$.

where $P = \sqrt{x} \,;\, Q = \sqrt{y} \,;\, R = \sqrt{z}$.

The auxiliary eqn is given by,

$$\frac{dx}{\sqrt{x}} = \frac{dy}{\sqrt{y}} = \frac{dz}{\sqrt{z}}$$

Consider,

$$\frac{dx}{\sqrt{x}} = \frac{dy}{\sqrt{y}}$$

$$\int \frac{dx}{\sqrt{x}} = \int \frac{dy}{\sqrt{y}}$$

$$\frac{-1}{2} x^{1/2} = \frac{-1}{2} y^{1/2} + c_1$$

$$2\sqrt{x} = 2\sqrt{y} + c_1$$

Consider,

$$\frac{dy}{\sqrt{y}} = \frac{dz}{\sqrt{z}}$$

$$\int \frac{dy}{\sqrt{y}} = \int \frac{dz}{\sqrt{z}}$$

$$\frac{-1}{2} y^{1/2} = \frac{-1}{2} z^{1/2} + c_2$$

$$2\sqrt{y} = 2\sqrt{z} + c_2$$

Sol^n is $\phi\left(\sqrt{x} - \sqrt{y}, \sqrt{y} - \sqrt{z}\right) = 0$

4) Solve $p \cdot \tan x + q \tan y = \tan z$.

Soln: $Pp + Qq = R$.

The A.E is $Pp + Qq = R$ (L.E)

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$$

consider

$$\frac{dx}{\tan x} = \frac{dy}{\tan y}$$

$$\cot x\, dx = \cot y\, dy$$

$$\int \cot x\, dx = \int \cot y\, dy$$

$$\log(\sin x) = \log(\sin y) + \log c_1$$

$$\log\left(\frac{\sin x}{\sin y}\right) = \log c_1$$

$$c_1 = \frac{\sin x}{\sin y}$$

$$\frac{dy}{\tan y} = \frac{dz}{\tan z}$$

$$\cot y\, dy = \cot z\, dz.$$

$$\int \cot y\, dy = \int \cot z\, dz.$$

$$\log(\sin y) = \log(\sin z) + \log c_2$$

$$\log\left(\frac{\sin y}{\sin z}\right) = \log c_2$$

$$c_2 = \frac{\sin y}{\sin z}$$

The soln is $\phi\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$

5) Solve $\frac{y^2 z}{x} p + xzq = y^2$ — (1).

Soln: The eqn is of the form $Pp + Qq = R$ (L.E) where $P = \frac{y^2 z}{x}$; $Q = xz$; $R = y^2$.

The A.E is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{(y^2 z/x)} = \frac{dy}{xz} = \frac{dz}{y^2}.$$

Consider, $\frac{dx}{\left(\frac{y^2 z}{x}\right)} = \frac{dy}{xz}$

$x^2 dx = y^2 dy$

$\int x^2 dx = \int y^2 dy$

$\frac{x^3}{3} = \frac{y^3}{3} + c_1$

$c_1 = x^3 - y^3$

$\frac{dy}{\left(\frac{y^2 z}{x}\right)} = \frac{dz}{y^2}$

$x\,dx = z\,dz$

$\int x\,dx = \int z\,dz$

$\frac{x^2}{2} = \frac{y^2}{2} + c_2$

$c_2 = x^2 - y^2$

The soln of the p.d.e is $\phi(c_1, c_2) = 0$

$\phi(x^3 - y^3, x^2 - y^2) = 0$.

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