

* First Shifting Theorem (Frequency Shifting)

1) If $\mathcal{Z}[f(t)] = F(z)$, then $\mathcal{Z}[e^{-at} f(t)] = F[ze^{aT}]$

i.e., $\mathcal{Z}[e^{-at} f(t)] = \{F(z)\}_{z \rightarrow ze^{aT}}$

$\mathcal{Z}[e^{at} f(t)] = \{F(z)\}_{z \rightarrow ze^{-aT}}$

* Differentiation in z-Domain

$\mathcal{Z}[nf(n)] = -z \frac{d}{dz} \{F(z)\}$

* Recurrence formula:

$\mathcal{Z}[n^k] = \mathcal{Z}[n \cdot n^{k-1}] = -z \frac{d}{dz} \mathcal{Z}[n^{k-1}]$

* Damping Rule (or) Scaling in z-Domain (or) Multiplication by a^n .

$\mathcal{Z}[a^n f(n)] = F\left(\frac{z}{a}\right)$, $F(z) = \mathcal{Z}[f(n)]$

(* Second Shifting Theorem :

(* $\mathcal{Z}[f(n+1)] = zF(z) - zf(0)$

* z-Transform of Unit Impulse Function

A discrete unit Impulse function is defined by

$$\delta(k) = \begin{cases} 1, & k=0 \\ 0, & k \neq 0 \end{cases}$$

Now $\delta(n-k)$ is defined by

$$\delta(n-k) = \begin{cases} 1, & n=k \\ 0, & n \neq k. \end{cases}$$

* Unit Step Function:

A discrete unit step function is defined as

$$u(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

Initial Value Theorem:

⊛. If $\mathcal{L}[f(t)] = F(s)$, then $\lim_{s \rightarrow \infty} s F(s) = f(0)$.

ex. If $\mathcal{L}[f(t)] = F(s)$ then

$$\lim_{s \rightarrow \infty} s F(s) = \lim_{t \rightarrow 0} f(t)$$

Final Value Theorem:

⊛. If $\mathcal{L}[f(t)] = F(s)$ then

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} (s-1) F(s).$$