Properties of matter waves- Physical significance of wave function

Quantum Physics or Quantum mechanics is a branch of science that deals with the study and behavior of matter as well as light. The wave function in quantum mechanics can be used to illustrate the wave properties of a particle. Therefore, a particle's quantum state can be described using its wave function.

This interpretation of wave function helps define the probability of the quantum state of an element as a function of position, momentum, time, and spin. It is represented by a Greek alphabet Psi, $\boldsymbol{\Psi}$.

However, it is important to note that there is no physical significance of wave function itself. Nevertheless, its proportionate value of $\boldsymbol{\Psi}^2$ at a given time and point of space does have physical importance.

Furthermore, we will discuss the Schrodinger equation, which was introduced in 1925 to define wave function.

Schrodinger Equation

In 1925, Erwin Schrodinger introduced this partial differential equation for wave function definition as a reward to the Quantum mechanics branch. According to him, the wave function can be satisfied and solved. Here is a time-dependent equation of Schrodinger shown in the image below.

$$\text{ih}\,\frac{\partial}{\partial t}\boldsymbol{\Psi}(\overrightarrow{r},t) = \frac{-h2}{2m}\nabla 2 + V(\overrightarrow{r},t)]\boldsymbol{\Psi}(\overrightarrow{r},t)$$

In the above equations,

- m refers to the particle's mass.
- ∇ is laplacian.
- h equals to h/2$\boldsymbol{\pi}$, which is also known as the reduced Planck's constant.
- i is the imaginary unit.
- E is a constant matching energy level of a system

Properties of Wave Function

- There must be a single value for $\boldsymbol{\Psi}$, and it must be continuous.
- It is easy to compute the energy using the Schrodinger equation.
- Wave function equation is used to establish probability distribution in 3D space.
- If there is a particle, then the probability of finding it becomes 1.

- Properties which can be measured for a particle should be known.

Normalization of Wave Function

In this scenario, the probability of finding a particle becomes 1 if it exists in the system. This depicts that the exact form of wave function **Ψ** is found.

Quantum Mechanics Postulates

- It gets easier to decipher the force system wherein a particle in a conservative field resides with the help of a wave function.
- Time independent Schrodinger's equation was derived using the time-dependent equation.

The 6 Postulates of Quantum Mechanics are:

1. Associated with any particle moving in a conservative field of force is a wave function which determines everything that can be known about the system.
2. With every physical observable q there is associated an operator Q, which when operating upon the wave function associated with a definite value of that observable will yield that value times the wavefunction.
3. Any operator Q associated with a physically measurable property q will be Hermitian.
4. The set of eigenfunctions of operator Q will form a complete set of linearly independent functions.
5. Described by a given wave function for a system, the expected value of any property q can be found by performing the expectation value integral with respect to that wavefunction.
6. The time evolution of the wavefunction is given by the time dependent Schrodinger equation.

The total wavefunction must be antisymmetric with respect to the interchange of all coordinates of one fermion with those of another. Electronic spin must be included in this set of coordinates. The Pauli exclusion principle is a direct result of this antisymmetry principle.

The Physical Significance of Wave Function

There is no physical meaning of wave function as it is not a quantity which can be observed. Instead, it is complex. It is expressed as **Ψ** (x, y, z, t) = a + ib and the complex conjugate of the wave function is expressed as **Ψ**

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(x, y, z, t) = a – ib. The product of these two indicates the probability density of finding a particle in space at a time. However, Ψ^2 is the physical interpretation of wave function as it provides the probability information of locating a particle at allocation in a given time.