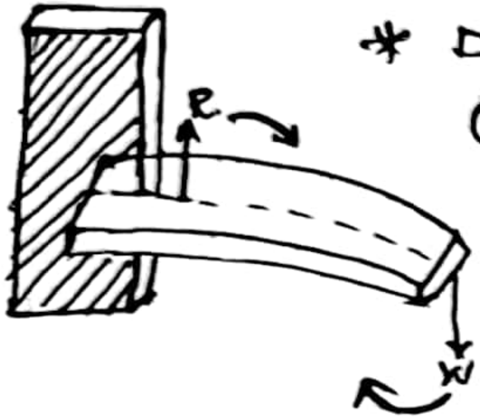
x ——— x

Cantilever: A cantilever is a light beam fixed horizontally at one end and loaded at the other end.



* Due to load applied at the free end a couple is created b/w the two forces

(i) Force (load W) applied at the free end towards ↓ward direction.

(ii) Reaction (R) acting in the ↑ward direction at the fixed end.

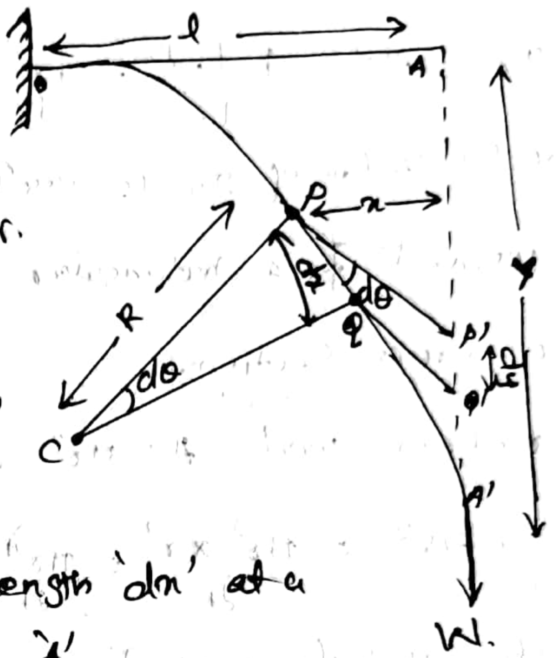
* External bending couple tends to bend the beam in the clockwise direction.

* One end of the beam is fixed and the beam cannot rotate, ∴ External bending couple must be balanced by another equal & opposite couple created due to the elastic nature of the body. (i.e) called Int. bending moment

Under Equilibrium condition:

Ext. Bending moment = Int. Bending moment

Depression of Cantilever - Loaded at it's ends:-



- l - length of the cantilever OA fixed at 'O'.
- W - Weight suspended at the loaded end 'A' of the cantilever.
- Due to the load applied the cantilever moves to a new position OA'.
- Consider small element PQ - length 'dx' at a distance 'x' from the free end 'A'.
- C - Center of curvature of the element PQ.
- R - Radius of curvature.
- Due to load applied at the free end of the cantilever an ext. couple is created b/w the load W at 'A' & the force of reaction at 'P'.

∴ Ext. bending moment (or) mo. of the force about the element PQ $\Big\} = W.x \longrightarrow (1)$

WKT, the Int. bending moment $= \dfrac{EI_g}{R} \longrightarrow (2)$

For Equilibrium: Ext. Bending moment = Int. Bending moment.

$$\therefore W.x = \frac{EI_g}{R}$$

$$\therefore R = \frac{EI_g}{W.x} \longrightarrow (3)$$

- Radius of ~~the~~ curvature of the neutral axis varies from point to point of the cantilever, the bending is said to be non-uniform.

- Now, two tangents are draw at the points P & Q wich meet the verticle line AA' at P' & Q'.

- Smallest depression produced from P' to Q' = dy.
- Angle b/w the two tangents = $d\theta$.
- Then we can write the angle b/w CP and CQ is also $d\theta$.

ie $\angle PCQ = d\theta$ & the arc length $PQ = R d\theta = dx$.

$$\therefore d\theta = \frac{dx}{R} \rightarrow (4)$$

Sub equ (3) in eqn (4) we get.

$$d\theta = \frac{dx}{(EI_g/Wx)}$$

$$\therefore d\theta = \frac{Wx.dx}{EI_g} \rightarrow (5)$$

$$P'Q' = x.d\theta$$

$$dy = x.d\theta \rightarrow (6)$$

Sub equ (5) in eqn (6) we get the small depression produced by the element PQ.

$$dy = x.\frac{Wx.dx}{EI_g} \rightarrow (7)$$

$\therefore$ Total depression at the free end of the cantilever can be derived by $\int$n the equ (7) within the limits "$0 - l$"

$$\therefore y = \frac{W}{EI_g}\int_0^l x^2 dx \qquad \left[\int \frac{x^{n+1}}{n+1} = \int x^n dx\right]$$

$$= \frac{W}{EI_g}\left[\frac{x^3}{3}\right]_0^l$$

$$= W/EI_g \left[\frac{l^3}{3} - 0\right]$$

∴ The total depression of the Cantilever at the free end

$$y = \frac{Wl^3}{3EI_g} \rightarrow (2)$$

∴ Young's modulus of the material of the cantilever

$$E = \frac{Wl^3}{3yI_g}$$

Special Cases:

(i) **Rectangular Cross Sectional Beam.**

- b – Breadth
- d – Thickness

$$\therefore I_g = \frac{bd^3}{12}$$

$$y = \frac{Wl^3}{3E(bd^3/12)}$$

$$\boxed{y = \frac{4Wl^3}{Ebd^3}}$$

(ii) **Circular Cross sectional Beam:**

- r – Radius of the circular section of the beam

$$\therefore I_g = \frac{\pi r^4}{4}$$

$$\therefore y = \frac{Wl^3}{3E(\pi r^4/4)}$$

$$\boxed{y = \frac{4Wl^3}{3\pi r^4 E}}$$

x — a