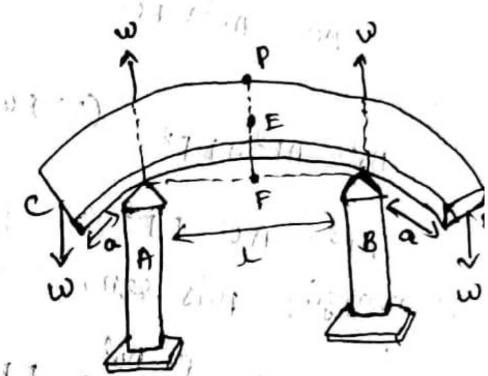
Young's modulus uniform and non-uniform bending: theory and experiment

Uniform Bending - Elevation at the centre of the beam loaded at both ends:



consider a beam CD is placed symmetrically on two knife edges A and B is horizontal level.

AB = l.

equal weights W=mg are suspended at the ends C and D.

EF = y

Now AC = a

BD = a.

Due to load applied beam bends into an arc and elevation 'y' produced from F to E.

Force W which acts vertically upwards be the reaction produced at the ~~end~~ points A and B.

consider point P on the beam.

Here the external force W at C acting vertically downwards and W at D acting vertically upwards. So these constitute a couple.

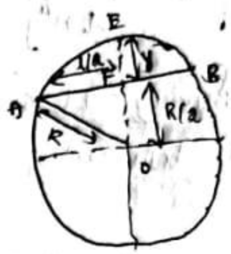
moment of the couple $= W \cdot a \rightarrow (1)$

Bending moment $= \frac{EI_g}{R} \rightarrow (2)$

at equilibrium condition,

External Bending moment = Internal Bending moment

$W.a = \frac{EI_g}{R} \rightarrow (3)$



from $\triangle$ AFO

$AO^2 = AF^2 + FO^2$

$AO^2 = AF^2 + EF^2$ $(\because FO = EF)$

$AF^2 = AO^2 - EF^2$

rearranging this eqn,

$AF^2 = FE \left[\frac{AO^2}{FE} - FE\right] \rightarrow (4)$

from figure,

$AO = R$, $EO = EF = y = R/2$, $AO = R$

(4) can be written as,

$(l/2)^2 = y\left[\frac{R^2}{R/2} - y\right]$

$l^2/4 = y(2R - y)$

$l^2/4 = 2yR - y^2$

Here the elevation 'y' is very small than the term y^2 can be neglected.

$\therefore l^2/4 = 2yR$

$y = \frac{l^2}{8R}$

$\therefore$ Radius of curvature $R = \left[l^2/8y\right] \rightarrow (5)$

Sub 'R' in eqn ③,

$$w.a = \frac{EIg}{\left[l^2/8y\right]}$$

$$w.a = \frac{8yEIg}{l^2} \rightarrow ⑥$$

The elevation $$y = \frac{wal^2}{8EIg} \rightarrow ⑦$$

Special cases:

(i) Rectangular cross sectional beam:

$$Ig = \frac{bd^3}{12}$$

sub Ig in ⑦,

$$y = \frac{wal^2}{8E\,bd^3/12}$$

$$= \frac{12\,wal^2}{8E\,bd^3}$$

$$y = \frac{3wal^2}{2Ebd^3} \rightarrow ⑧$$

(ii) circular cross section beam:

$$Ig = \frac{\pi r^4}{4}$$

Sub Ig in ⑦,

$$y = \frac{wal^2}{28E\,\frac{\pi r^4}{4}}$$

$$y = \frac{wal^2}{2E\pi r^4} \rightarrow ⑨$$

Young's Modulus - Uniform Bending

Expt. No. : Date:

AIM

To find the Young's modulus of the given material of the beam by uniform bending.

GENERAL OBJECTIVE

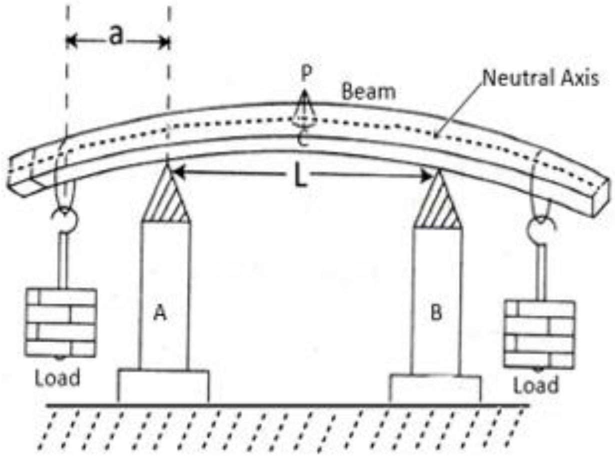
To evaluate the elastic behavior of the given wooden beam by pin and microscope experimental method and to find its Young's modulus

SPECIFIC OBJECTIVES

1. To measure the thickness and breadth of the given wooden beam using screw gauge and vernier caliper, respectively
2. To determine the elevation of the given wooden beam loaded on both ends by uniform bending method
3. To find the slope from the graph drawn between the load versus elevation
4. To calculate the Young's modulus of the wooden beam from the mean elevation and slope obtained from table and graph, respectively
5. To analyze the elastic behavior of the given wooden beam from the results obtained

APPARATUS REQUIRED

- Wooden beam
- Weight hanger with slotted weights
- Knife edges
- Travelling microscope
- Vernier caliper
- Screw gauge
- Metre scale



A, B – Knife edges
C – Midpoint
P – Pin
L – Distance between the two knife edges
a – Distance between the knife edge and loading point

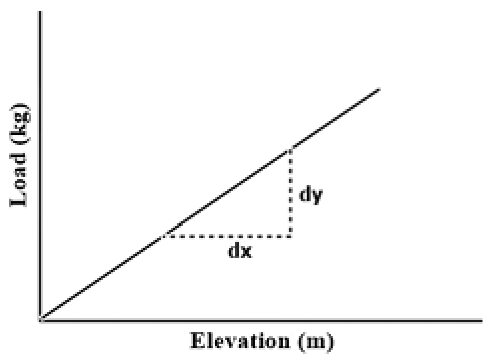


Figure 2.2 Model Graph

LEAST COUNT FOR SCREW GAUGE

$$\text{Least Count (LC)} = \frac{\text{Pitch}}{\text{Number of head scale divisions}}$$

$$\text{Pitch} = \frac{\text{Distance moved}}{\text{Number of rotations given}} = \frac{5\text{ mm}}{5} = 1\text{mm}$$

$$\text{LC} = \frac{1\text{ mm}}{100} = 0.01\text{mm}$$

TABLE – I

To determine the thickness (d) of the beam using screw gauge

Zero Error (ZE) : .... divisions Zero Correction (ZC) : ....mm

S. No.	Pitch Scale Reading PSR 10^{-3} m	Head Scale Coincidence HSC divisions	Observed Reading OR = PSR + (HSC ×LC) (10^{-3} m)	Correct Reading CR = OR ±ZC (10^{-3} m)
1				
2				
3				
4				
5				

Mean (d) = ... **x10^{-3} m**

FORMULA

Young's modulus of the material of the beam

$$Y = \frac{3MgaL^2}{2sbd^3} \quad (N/m^2)$$

Symbol	Explanation	Unit
Y	Young's modulus of the material of the beam	N/m^2
M	Load applied	kg
L	Distance between the knife edges	m
a	Distance between the load and the nearest knife edge	m
g	Acceleration due to gravity	m/s^2
b	Breadth of the beam	m
d	Thickness of the beam	m
s	Elevation produced for 'M' kg load	m

Unit	Equivalent Units	
N/m^2	$kg\ m^{-1}\ s^{-2}$	1 Pa

PREREQUISITE KNOWLEDGE

1. **Elastic materials**

   Materials which can completely regain their original condition of shape and size on removal of deforming forces are said to be elastic

2. **Plastic materials**

   Materials which retain the deformed nature even after the removal of deforming forces are said to be plastic

3. **Hooke's law**

   Within the elastic limit, the stress is directly proportional to the strain

LEAST COUNT FOR VERNIER CALIPER

Least Count (LC)	= Value of 1 Main Scale Division (MSD)/ Number of divisions in the vernier
10 MSD	= 1 cm
Value of 1 MSD	= 1/10 cm = 0.1 cm
Number of divisions in the vernier	= 10
LC	= 0.1/ 10 = 0.01 cm

TABLE - II

To determine the breadth (b) of the beam using vernier caliper

LC = **0.01** cm Zero Error (ZE): **..........** Zero Correction (ZC): **..........**

S. No.	Main Scale Reading MSR (10^{-2} m)	Vernier Scale Coincidence VSC (divisions)	Observed Reading OR = MSR + (VSC × LC) (10^{-2} m)	Correct Reading CR = OR ± ZC (10^{-2} m)
1				
2				
3				
4				
5				

PROCEDURE

1. The given beam is supported on two knife edges separated by a distance 'L'. A pin is fixed vertically at the mid-point.
2. Two weight hangers are suspended, one each on either side of the knife edges so that their distances from the nearer knife edge are equal. The beam is brought to the elastic mood by loading and unloading it several times.
3. With the dead load 'W', the pin is focused through microscope. The microscope is adjusted so that the horizontal crosswire coincides with the tip of the pin. The microscope reading is taken.
4. The load is changed in steps of 0.05 kg and in each case the microscope reading is taken during loading and unloading. The readings are tabulated. The elevation at the mid-point for 'M' kg is calculated.
5. The distance between the knife edges (L) is measured using a metre scale. The breadth (b) and thickness (d) of the beam are found using vernier caliper and screw gauge, respectively.

LEAST COUNT FOR TRAVELLING MICROSCOPE

Least Count (LC)	= Value of 1 Main Scale Division (MSD)/ Number of divisions in the vernier
20 MSD	= 1 cm
Value of 1 MSD	= 1/20cm = 0.05 cm
Number of divisions in the vernier	= 50
LC	= 0.05/50= 0.001 cm

TABLE -III

To find elevation's'

LC = **0.001 cm**

*TR= MSR + (VSC × LC)

Load M (10^{-3} kg)	Microscope reading							Elevation 's' for M kg (10^{-2} m)
	Loading			Unloading			Mean (10^{-2} m)	
	MSR (10^{-2} m)	VSC (div)	TR (10^{-2} m)	MSR (10^{-2} m)	VSC (div)	TR (10^{-2} m)		
W								
W + 50								
W + 100								
W + 150								
W+ 200								

Mean (s) = **………. × 10^{-2} m**

OBSERVATION

Mass for the elevation	M	=	……….	$\times 10^{-3}$ kg
Distance between two knife edges	L	=	……….	$\times 10^{-2}$ m
Acceleration due to gravity	g	=	……….	m / s^2
Breadth of the beam	b	=	……….	$\times 10^{-2}$ m
Thickness of the beam	d	=	……….	$\times 10^{-3}$ m
Elevation produced for 'M' kg of load	s	=	……….	$\times 10^{-2}$ m
Distance between one of the knife edges and the adjacent weight hanger	a	=	……….	$\times 10^{-2}$ m

CALCULATION

Young's modulus of the material of the beam

$$Y = \frac{3MgaL^2}{2sbd^3} \quad (\text{N/m}^2)$$

RESULT

The Young's modulus of the material of the given beam **Y = ………. × 10^{10} N/m^2**

APPLICATIONS

AFM probe, wings of air craft, helicopter rotator, marine fittings, designing of bridges, bicycle frames and wind mill turbine blades