Density of holes in valence band:-

$$n_h = \int_{-\infty}^{E_v} Z(E)\,[1 - F(E)]\cdot dE \qquad -⑧$$

$F(E) \rightarrow$ probability of filled state

$1 - F(E) \Rightarrow$ Probability of unfilled state

we know, $$Z(E)\cdot dE = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} (E_v - E)^{1/2}\cdot dE \qquad -⑨$$

$$1 - F(E) = 1 - \frac{1}{1 + e^{(E-E_F)/k_BT}}$$

$$= \frac{e^{(E-E_F)/k_BT}}{1 + e^{(E-E_F)/k_BT}}$$

Here, $E - E_F << k_BT$; $\frac{E-E_F}{k_BT} << 1$; $e^{\frac{(E-E_F)}{k_BT}} << 1$

So, $$1 + e^{(E-E_F)/k_BT} \approx 1$$

$$\therefore\ 1 - F(E) = e^{(E-E_F)/k_BT} \qquad -⑩$$

substitute equations ⑨ and ⑩ in ⑧

$$n_h = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} \int_{-\infty}^{E_v} (E_v - E)^{1/2}\, e^{(E-E_F)/k_BT}\cdot dE \qquad -⑪$$

Let $E_v - E = x k_BT$

$E = E_v - x k_BT \Rightarrow$ when $E = -\infty$, $x = \infty$ | $E = E_v$, $x = 0$

$dE = -dx\, k_BT$

Limits ~~are~~

So Limits are ~~xx~~ ∞ to 0,

So equation ⑪ becomes,

$$n_h = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} \int_{\infty}^{0} (x k_BT)^{1/2}\, e^{\frac{(E_v - x k_BT - E_F)}{k_BT}} \cdot (-dx)\, k_BT$$

$$= \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} \int_{0}^{\infty} x^{1/2}\,(k_BT)^{3/2}\, e^{(E_v - E_F)/k_BT}\, e^{-x}\cdot dx$$

$$n_h = \frac{\pi}{2}\left(\frac{8m_h^* k_BT}{h^2}\right)^{3/2}\cdot e^{(E_v-E_F)/k_BT} \underbrace{\int_0^{\infty} x^{1/2}\, e^{-x}\cdot dx}_{\sqrt{\pi}/2}$$

$$n_h = \frac{\pi}{2}\left(\frac{8m_h^* k_BT}{h^2}\right)^{3/2} \cdot e^{(E_v-E_F)/k_BT} \cdot \frac{\sqrt{\pi}}{2}$$

$$= \frac{1}{4}\left(\frac{8\pi m_h^* k_BT}{h^2}\right)^{3/2} \cdot e^{(E_v-E_F)/k_BT}$$

$8^{3/2} = 8\sqrt{8} = 8\times 2\sqrt{2}$
$8\times(2^{3/2})$

$$n_h = \frac{8}{4}\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} \cdot e^{(E_v-E_F)/k_BT}$$

$$\boxed{n_h = 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} e^{(E_v-E_F)/k_BT}} \quad \text{--- (12)}$$

equation (12) is the density of holes in valance-band.

Variation of Fermi level and Carrier Concentration with temperature in an intrinsic Semiconductor

For intrinsic semiconductor number of electrons (i.e.,) electron density will be the same as that of number of holes (i.e.,) hole density.

i.e., $n_e = n_h$

From (4) and (12);

$$2\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2} e^{(E_F-E_c)/k_BT} = 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} e^{(E_v-E_F)/k_BT}$$

$$\left(\frac{m_h^*}{m_e^*}\right)^{3/2} = \frac{e^{(E_F-E_c)/k_BT}}{e^{(E_v-E_F)/k_BT}}$$

$$\left(\frac{m_h^*}{m_e^*}\right)^{3/2} = e^{(E_F-E_c-E_v+E_F)/k_BT}$$

Taking log on both sides,

$$\frac{3}{2}\log\left(\frac{m_h^*}{m_e^*}\right) = \frac{(2E_F-(E_v+E_c))}{k_BT}$$

$$2E_F = E_v+E_c + \frac{3}{2}k_BT\log\left(\frac{m_h^*}{m_e^*}\right)$$

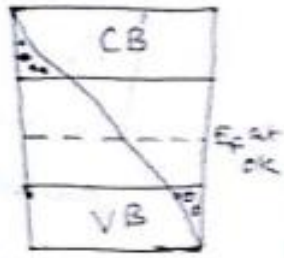
$$\boxed{E_F = \frac{E_c+E_v}{2} + \frac{3}{4} k_B T \log\left(\frac{m_h^*}{m_e^*}\right)} \quad -(13)$$

If $m_h^* = m_e^*$, then $\log \frac{m_h^*}{m_e^*} = 0$ $\quad [\because \log 1 = 0]$

$$E_F = \frac{E_c+E_v}{2} \quad -(14)$$

i.e., Fermilevel lies in the midway between E_c and E_v (at $T = 0K$). But Fermilevel slightly increases with increase in temperature.

CB - conduction band
VB - valence band



Density of electrons and holes interms of E_g:-

Interms of Energy gap $(E_g = E_c - E_v)$, we can get the expression of n_e and n_h by substituting the value of E_F interms of E_c and E_v.

We know,

$$n_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} e^{(E_F - E_c)/k_B T}$$

$$n_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \cdot \exp\left[\frac{\frac{E_c+E_v}{2} + \frac{3}{4}k_B T \log\left(\frac{m_h^*}{m_e^*}\right) - E_c}{k_B T}\right]$$

$$= 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left[\frac{2E_c + 2E_v + 3 k_B T \log\left(\frac{m_h^*}{m_e^*}\right) - 4E_c}{4 k_B T}\right]$$