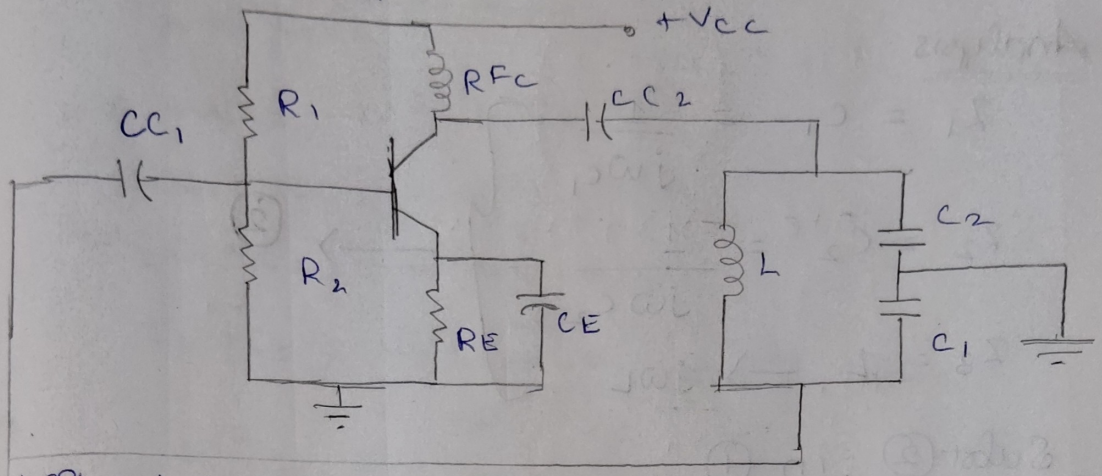
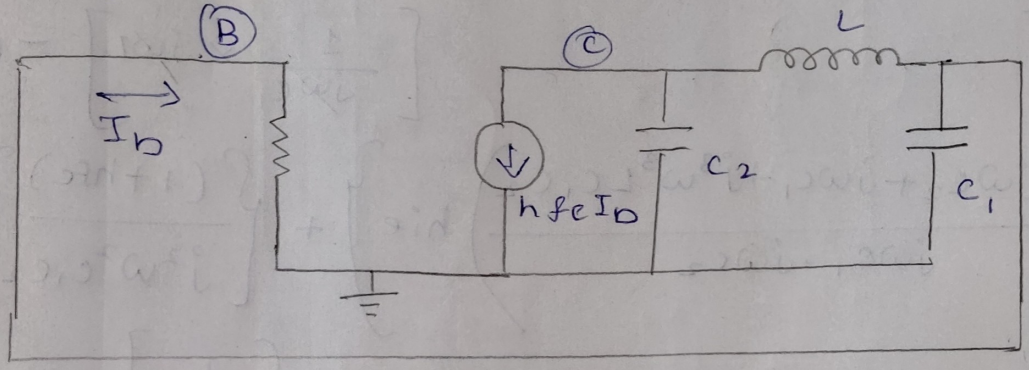
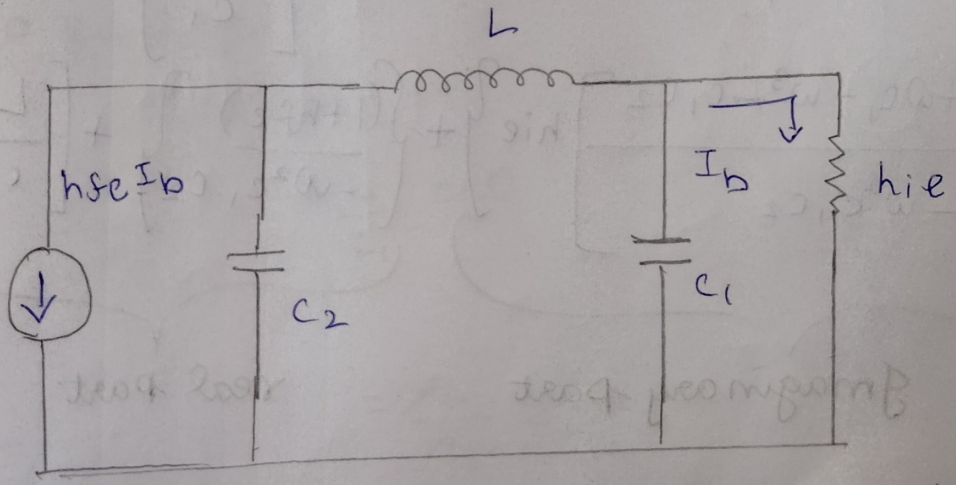
Colpitts oscillator:

The LC oscillator uses two capacitance and one inductance in the feedback network is called colpitts oscillator.

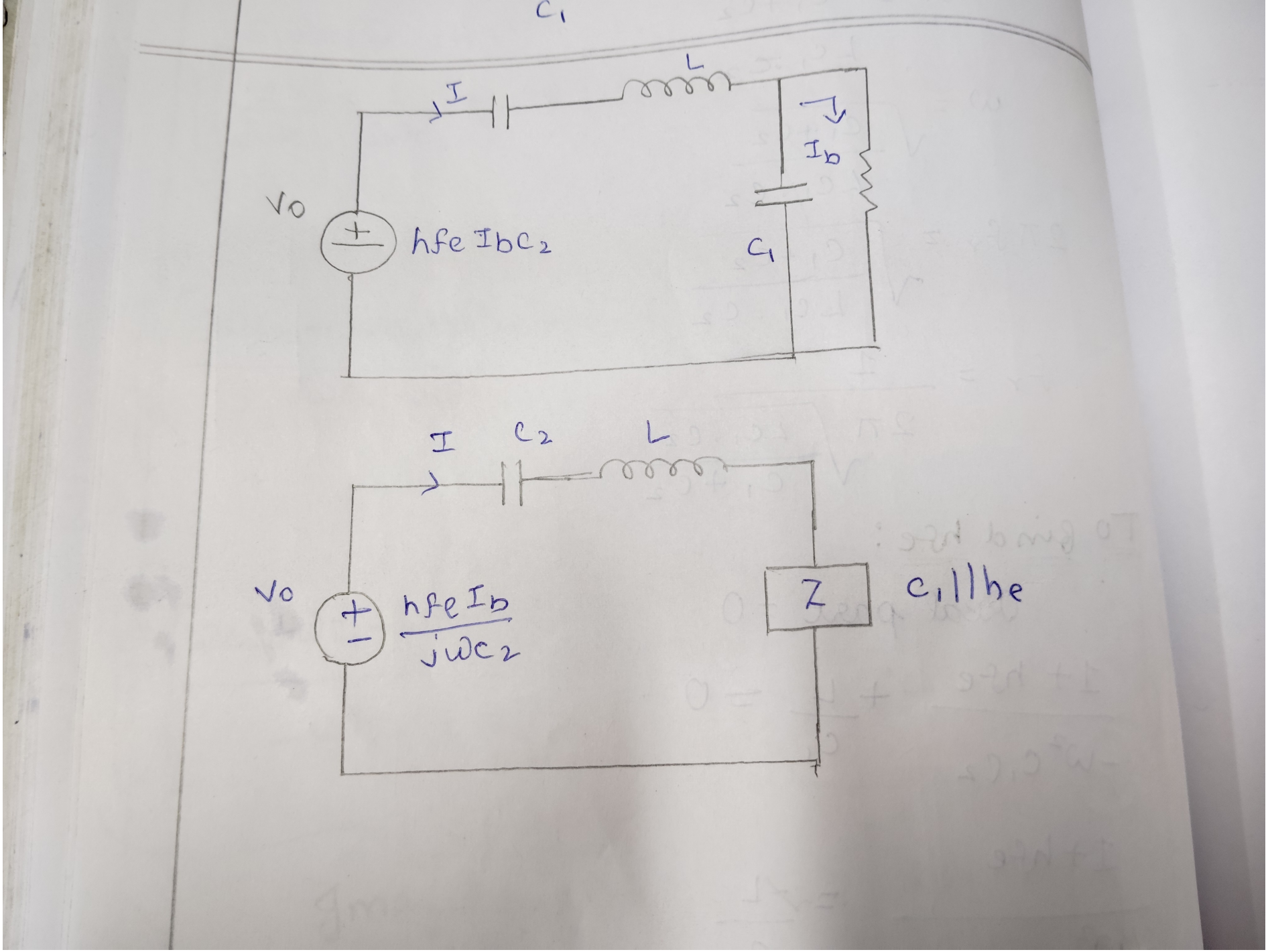


⇒ Short coupling capacitor
⇒ By pass capacitor are short circuited
⇒ neglect R_1 & R_2





(convert current source into Volt source)



General form of LC oscillator is

$$(Z_1+Z_2+Z_3)h_{ie} + Z_1Z_2(1+h_{fe}) + Z_1Z_3 = 0 \longrightarrow (1)$$

Analysis

$$\left.\begin{aligned} Z_1 = C_1 &= \frac{1}{j\omega C_1} \\ Z_2 = C_2 &= \frac{1}{j\omega C_2} \\ Z_3 = L &\longrightarrow j\omega L \end{aligned}\right\} \longrightarrow (2)$$

Subst (2) in (1)

$$\left\{\left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + j\omega L\right) h_{ie}\right\} + \left\{\frac{1}{j\omega C_1} \times \frac{1}{j\omega C_2}(1+h_{fe})\right\} + \left[\frac{1}{j\omega C_1} \times j\omega L\right] = 0$$

$$\left\{\left(\frac{j\omega C_2 + j\omega C_1 + j^3\omega^3 L C_1 C_2}{j\omega C_1 \cdot j\omega C_2}\right) h_{ie}\right\} + \left\{\frac{(1+h_{fe})}{j^2\omega^2 C_1 C_2}\right\} + \left[\frac{L}{C_1}\right] = 0$$

$$\left\{j\left[\frac{\omega C_2 + \omega C_1 - \omega^3 L C_1 C_2}{-\omega^2 C_1 C_2}\right] h_{ie}\right\} + \left\{\frac{(1+h_{fe})}{-\omega^2 C_1 \cdot C_2}\right\} + \left[\frac{L}{C_1}\right] = 0$$

Imaginary part

real part

To find frequency:

Imaginary part = 0

$$\left(\frac{\omega C_1 + \omega C_2 - \omega^3 L C_1 C_2}{-\omega^2 C_1 C_2}\right) h_{ie} = 0$$

$$\omega C_1 + \omega C_2 - \omega^3 L C_1 C_2 = 0$$

$$\omega C_1 + \omega C_2 = \omega^3 L C_1 \cdot C_2$$

$$\omega (C_1 + C_2) = \omega^3 L C_1 \cdot C_2$$

$$C_1 + C_2 = \frac{\omega^3 L C_1 \cdot C_2}{\omega} = \omega^2 L C_1 \cdot C_2$$

$$\omega^2 = \frac{C_1 + C_2}{L C_1 \cdot C_2}$$

$$\omega = \sqrt{\frac{C_1 + C_2}{L C_1 \cdot C_2}}$$

$$2\pi f_r = \sqrt{\frac{C_1 + C_2}{L C_1 \cdot C_2}}$$

$$f_r = \frac{1}{2\pi \sqrt{\frac{L C_1 \cdot C_2}{C_1 + C_2}}}$$

To find h_{fe}:

Real part = 0

$$\frac{1 + h_{fe}}{-\omega^2 C_1 C_2} + \frac{L}{C_1} = 0$$

$$\frac{1 + h_{fe}}{\omega^2 C_1 C_2} = \frac{L}{C_1}$$

$$(1 + h_{fe}) = \frac{L \cdot \omega^2 C_1 \cdot C_2}{C_1}$$

$$(1 + h_{fe}) = L \cdot \omega^2 C_2$$

$$h_{fe} = L\omega^2 C_2 - 1$$

$$h_{fe} = \frac{1}{1} \cdot \frac{C_1 + C_2}{C_1 \cdot C_2} \; C_2 - 1$$

$$\because \omega^2 = \frac{C_1 + C_2}{LC_1 \cdot C_2}$$

$$h_{fe} = \left(\frac{C_1 + C_2}{C_1}\right) - \frac{1}{}$$

$$h_{fe} \Rightarrow \frac{C_1 + C_2 - C_1}{C_1} = \frac{C_2}{C_1}$$

$$h_{fe} = \frac{C_2}{C_1}$$