



UNIT 2 – ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

Nature of the quadratic form

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Find rank, index, signature and nature

$$2x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 = 0$$

The matrix form is

$$A = [2 \ 2 \ 0 \ 2 \ 2 \ 0 \ 0 \ 0 \ 1]$$

Characteristic equation, Eigen values, Eigen vectors

C_1 = Sum of leading diagonal elements

$$= 2 + 2 + 1 = 5$$

C_2 = Sum of minors of leading diagonal elements

$$= 4$$

$C_3 = |A|$

$$= |2 \ 2 \ 0 \ 2 \ 2 \ 0 \ 0 \ 0 \ 1|$$

$$= 0$$

The characteristic equation is

$$\lambda^3 - 5\lambda^2 + 4\lambda = 0$$

The eigen values are 0, 1, 4

The index $p=2$

Rank $r=2$

Signature $s=2p-r=2$

The nature is semi positive