UNIT 2 – ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

Diagonalization of a real symmetric matrix

Method to diagonalyse the symmetric matrix by orthogonal transformation

STEP 1:Find characteristic equation ,Eigen values,eigen vectors of the given matrix A.

STEP 2:Eigen vector should be pair wise orthogonal

$X_1^T X_2=0$; $X_2^T X_3=0$; $X_3^T X_1=0$

STEP 3: Normalize each Eigen vector, if $X = \left(x_1\ x_2\ x_3 \right)$ the normalized vector =

$\left(x_1/l(x_1)\ x_2/l(x_2)\ x_3/l(x_3) \right)$ where l(x)=$\sqrt{x_1^2 + x_2^2 + x_3^2}$

STEP 4: Form the normalized model matrix N, using normalized Eigen vectors

STEP 5: The normalized modal matrix N should be orthogonal matrix

$NN^T = N^T N = I$

STEP 6: Find the $N^T AN = D$,Where D is diagonal matrix.

Problem

1.Reduce the matrix to diagonalise A=$[8\ -6\ 2\ -6\ 7\ -4\ 2\ -4\ 3]$

Let A = $[8\ -6\ 2\ -6\ 7\ -4\ 2\ -4\ 3]$

To find characteristic equation

D_1 =Sum of leading diadonal elements

=8+7+3 =18

D_2= Sum of minors of leading diagonal elements

=45

$D_3 = |A|$

$= |8 \ -6 \ 2 \ -6 \ 7 \ -4 \ 2 \ -4 \ 3|$

$= 0$

The characteristic equation is

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

The eigen values are 0,3,15

The eigen vectors are $(A - \lambda I)X = 0$

$$[(8 \ -6 \ 2 \ -6 \ 7 \ -4 \ 2 \ -4 \ 3) - \lambda(1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1)](x_1 \ x_2 \ x_3) = 0$$

$$(8 \ -6 \ 2 \ -6 \ 7 \ -4 \ 2 \ -4 \ 3)(x_1 \ x_2 \ x_3) = 0$$

$$(8-\lambda \ -6 \ 2 \ -6 \ 7-\lambda \ -4 \ 2 \ -4 \ 3-\lambda)(x_1 \ x_2 \ x_3) = 0$$

CASE (i)

When $\lambda = 0$

$$(8 \ -6 \ 2 \ -6 \ 7 \ -4 \ 2 \ -4 \ 3)(x_1 \ x_2 \ x_3) = 0$$

The cofactor of first row elements are $(5 \ 10 \ 10)$ ie $(1 \ 2 \ 2)$

The Eigen vector when $\lambda = 0$ is $(1 \ 2 \ 2)$

CASE (ii)

When $\lambda = 3$

$$(5 \ -6 \ 2 \ -6 \ 4 \ -4 \ 2 \ -4 \ 0)(x_1 \ x_2 \ x_3) = 0$$

The cofactor of first row elements are $(-16 \ -8 \ 16)$ ie $(2 \ 1 \ -2)$

The Eigen vector when $\lambda = 3$ is $(2\ 1\ -2)$

CASE (iii)

When $\lambda = 15$

$$(-7\ -6\ 2\ -6\ -8\ -4\ 2\ -4\ -12)(x_1\ x_2\ x_3) = 0$$

The cofactor of first row elements are $(80\ -80\ 40)$ ie $(2\ -2\ 1)$

The Eigen vector when $\lambda = 15$ is $(2\ -2\ 1)$

STEP 2:

To check pair wise orthgonality

$X_1^T X_2 = (1\ 2\ 2)(2\ 1\ -2) = 0$

$X_2^T X_3 = (2\ 1\ -2)(2\ -2\ 1) = 0$

$X_3^T X_1 = (2\ -2\ 1)(1\ 2\ 2) = 0$

STEP 3:

To find normalized vector

Eigen vector	l(x)=$\sqrt{x_1^2 + x_2^2 + x_3^2}$	Normalized vector = $(x_1/l(x_1)\ x_2/l(x_2)\ x_3/l(x_3))$
$(1\ 2\ 2)$	$\sqrt{1+4+4}$=3	$(1/3\ 2/3\ 2/3)$

$(2\ 1\ \ -2)$	$\sqrt{1+4+4}=3$	$(2/3\ 1/3\ \ -2/3)$
$(2\ \ -2\ 1)$	$\sqrt{1+4+4}=3$	$(2/3\ \ -2/3\ 1/3)$

STEP 4:

Normalized modal matrix

$N = [1/3\ 2/3\ 2/3\ 2/3\ 1/3\ \ -2/3\ 2/3\ \ -2/3\ 1/3]$

$N^T = [1/3\ 2/3\ 2/3\ 2/3\ 1/3\ \ -2/3\ 2/3\ \ -2/3\ 1/3]$

STEP 5:

$NN^T = N^TN = I$

$N^TN =$

$(1/3\ 2/3\ 2/3\ 2/3\ 1/3\ \ -2/3\ 2/3\ \ -2/3\ 1/3)(1/3\ 2/3\ 2/3\ 2/3\ 1/3\ \ -2/3\ 2/3\ \ -2/3\ 1/3)$

$= (1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1)$

$= I$

STEP 6:

To find diagonalyze matrix

$N^TAN = D$

$N^TA = \frac{1}{3}(1\ 2\ 2\ 2\ 1\ \ -2\ 2\ \ -2\ 1)(8\ \ -6\ 2\ \ -6\ 7\ \ -4\ 2\ \ -4\ 3)$

$= \frac{1}{3}(0\ 0\ 0\ 6\ 3\ \ -6\ 30\ 30\ 15)$

$N^T AN = \frac{1}{9}(0\ 0\ 0\ 6\ 3\ -6\ 30\ 30\ 15)(1\ 2\ 2\ 2\ 1\ -2\ 2\ -2\ 1)$

$= \frac{1}{9}(0\ 0\ 0\ 0\ 27\ 0\ 0\ 0\ 15)$

$=(0\ 0\ 0\ 0\ 3\ 0\ 0\ 0\ 15)$

$= D$