



## DEPARTMENT OF MATHEMATICS

### UNIT - IV FUNCTIONS OF SEVERAL VARIABLES

#### METHOD OF LAGRANGIAN'S MULTIPLIERS

We can find an extreme value of the function  $f(x, y, z)$  subject to the constrained  $g(x, y, z) = 0$

Define  $F(x, y, z) = f(x, y, z) + \lambda g(x, y, z)$  where  $\lambda$  is an undetermined constant called the Lagrangian multipliers.

By solving the eqn.

$\frac{\partial F}{\partial x} = 0, \frac{\partial F}{\partial y} = 0, \frac{\partial F}{\partial z} = 0, \frac{\partial F}{\partial \lambda} = 0$ , we get the values of  $x, y, z$  and  $\lambda$ . Using  $\lambda$  value find  $x, y, z$  find the values either maximum or minimum by substituting  $x, y, z$  in  $f(x, y, z)$ .

1) Find the minimum value of  $x^2 + y^2 + z^2$ , given that  $ax + by + cz = P$

Let  $f = x^2 + y^2 + z^2$  and  $g = ax + by + cz - P$

$$\begin{aligned} F(x, y, z) &= f(x, y, z) + \lambda g(x, y, z) \\ &= x^2 + y^2 + z^2 + \lambda [ax + by + cz - P] \end{aligned}$$



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$$\frac{\partial F}{\partial x} = 2x + a\lambda$$

$$\Rightarrow \frac{\partial F}{\partial x} = 0 \Rightarrow 2x + a\lambda = 0$$
$$\Rightarrow x = -\frac{\lambda a}{2}$$

$$\frac{\partial F}{\partial y} = 2y + b\lambda$$

$$\Rightarrow \frac{\partial F}{\partial y} = 0 \Rightarrow 2y + b\lambda = 0$$
$$\Rightarrow y = -\frac{\lambda b}{2}$$

$$\frac{\partial F}{\partial z} = 2z + c\lambda$$

$$\Rightarrow \frac{\partial F}{\partial z} = 0 \Rightarrow 2z + c\lambda = 0$$
$$\Rightarrow z = -\frac{\lambda c}{2}$$

$$\frac{\partial F}{\partial \lambda} = ax + by + cz - P$$

$$\Rightarrow \frac{\partial F}{\partial \lambda} = 0 \Rightarrow ax + by + cz = P$$
$$\Rightarrow a \cdot -\frac{\lambda a}{2} + b \cdot -\frac{\lambda b}{2} + c \cdot -\frac{\lambda c}{2} = P$$
$$\Rightarrow -\frac{\lambda}{2} [a^2 + b^2 + c^2] = P$$
$$\Rightarrow \lambda = -\frac{2P}{a^2 + b^2 + c^2}$$



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$$\therefore x = -\frac{\lambda a}{2} \Rightarrow x = \frac{pa}{a^2+b^2+c^2}$$

$$y = -\frac{\lambda b}{2} \Rightarrow y = \frac{pb}{a^2+b^2+c^2}$$

$$z = -\frac{\lambda c}{2} \Rightarrow z = \frac{pc}{a^2+b^2+c^2}$$

sub  $x, y, z$  in  $f$

$$f = x^2 + y^2 + z^2$$

$$= \left(\frac{pa}{a^2+b^2+c^2}\right)^2 + \left(\frac{pb}{a^2+b^2+c^2}\right)^2 + \left(\frac{pc}{a^2+b^2+c^2}\right)^2$$

$$= \frac{p^2(a^2+b^2+c^2)^2}{(a^2+b^2+c^2)^2}$$

$$= \frac{p^2}{a^2+b^2+c^2}$$



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3) A rectangular box open at the top is to have a volume of 32 cc. Find the dimension of the box, that require the least material for its construction.

Let the dimension of the box be  $x, y, z$ .

$$\text{Volume} = xyz$$

$$\text{Volume} = lbh$$

$$\text{Surface area} = xy + 2yz + 2zx$$

$$\text{Surface Area} = lb + 2l(l + 2(lb + bh + hl))$$

$$\text{Given volume} = 32 \text{ cc} \Rightarrow xyz = 32$$

$$(1) \quad xyz - 32 = 0$$

$$\therefore f = xyz - 32$$

$$g = xy + 2yz + 2zx$$

$$\therefore F(x, y, z) = xy + 2yz + 2zx + \lambda [xyz - 32]$$

$$\frac{\partial F}{\partial x} = y + 2z + \lambda yz \Rightarrow y + 2z + \lambda yz = 0 \quad \left[ \because \frac{\partial F}{\partial x} = 0 \right]$$

$$\frac{\partial F}{\partial y} = x + 2z + \lambda xz$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow x + 2z + \lambda xz = 0 \quad \text{--- (2)}$$

$$\frac{\partial F}{\partial z} = 2y + 2x + \lambda xy$$

$$\frac{\partial F}{\partial z} = 0 \Rightarrow 2y + 2x + \lambda xy = 0 \quad \text{--- (3)}$$



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$$\textcircled{1} x^2 \Rightarrow xy + 2xz + \lambda xyz = 0$$

$$\textcircled{2} xy \Rightarrow xy + 2yz + \lambda xyz = 0$$

$$2xz - 2yz = 0$$

$$\Rightarrow x = y$$

Solve  $\textcircled{2}$  &  $\textcircled{3}$ :

$$\textcircled{2} xy : xy + 2yz + \lambda xyz = 0$$

$$\textcircled{3} xz : 2yz + 2xz + \lambda xyz = 0$$

$$xy - 2zx = 0$$

$$\Rightarrow y = 2z$$

$$\therefore x = y = 2z$$

Sub:  $x = y = 2z$  in  $\textcircled{1}$

$$x + 2\left(\frac{x}{2}\right) + \lambda(2)\left(\frac{x}{2}\right) = 0$$

$$x + x + \frac{\lambda x^2}{2} = 0$$

$$\Rightarrow \frac{\lambda x^2}{2} = -2x$$

$$\Rightarrow x = -\frac{4}{\lambda}$$



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Sub  $x = y = 2z$  in (2)

$$x + 2z + \lambda(xz) = 0$$

$$y + 2\frac{y}{2} + \lambda y \cdot \frac{y}{2} = 0$$

$$\Rightarrow y = -4/\lambda$$

Sub  $x = y = 2z$  in (3)

$$2y + 2x + \lambda xy = 0$$

$$2(2z) + 2(2z) + \lambda(2z)(2z) = 0$$

$$4z + 4z + 4z^2\lambda = 0$$

$$2 + 4z\lambda = 0$$

$$\Rightarrow z = -2/\lambda$$

$$\frac{\partial F}{\partial \lambda} = xyz - 32$$

$$\Rightarrow \frac{\partial F}{\partial \lambda} = 0 \Rightarrow xyz = 32$$

$$\Rightarrow -\frac{4}{\lambda} \times -\frac{4}{\lambda} \times -\frac{2}{\lambda} = 32$$

$$\Rightarrow -1 = \lambda \quad \text{or } \lambda = -1$$

$$\therefore x = 4, \quad y = 4, \quad z = 2$$

$$\therefore F = xy + 2yz + 2zx$$

$$= 4(4) + 2(4)(2) + 2(2)(4)$$

$$= 16 + 16 + 16 = 48$$



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3) Find the dimensions of the rectangular box without top of maxi. capacity with surface area 432 Sq. metre.

Ans:  $x = y = 2z, 12, 12, 6. \quad \therefore V = 864$  cubic metres.