HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as :

Hydraulic Gradient Line. It is defined as the line which gives the sum of pressure head $\left(\frac{p}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

Total Energy Line. It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L. (Total Energy Line).

FLOW THROUGH SYPHON

Syphon is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high level ground as shown in Fig.

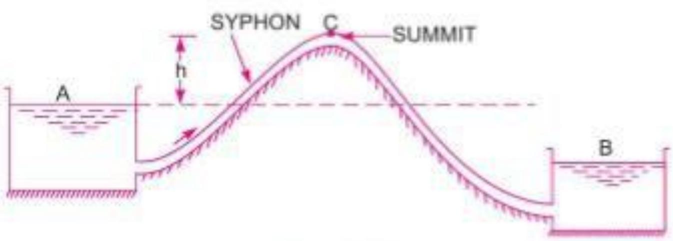


Fig.22.Flow through Syphon

The point C which is at the highest of the syphon is called the summit. As the point C is above the free surface of the water in the tank A, the pressure at C will be less than atmospheric pressure. Theoretically, the pressure at C may be reduced to – 10.3 m of water but in actual practice this pressure is only – 7.6 m of water or 10.3 – 7.6 = 2.7 m of water absolute. If the pressure at C becomes less than 2.7 m of water absolute, the dissolved air and other gases would come out from water and collect at the summit. The flow of water will be obstructed. Syphon is used in the following cases :

1. To carry water from one reservoir to another reservoir separated by a hill or ridge.
2. To take out the liquid from a tank which is not having any outlet.
3. To empty a channel not provided with any outlet sluice.

FLOW THROUGH PIPES IN SERIES OR FLOW THROUGH COMPOUND PIPES

Pipes in series or compound pipes are defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line as shown in Fig.

Let, L_1, L_2, L_3 = length of pipes 1, 2 and 3 respectively

d_1, d_2, d_3 = diameter of pipes 1, 2, 3 respectively

V_1, V_2, V_3 = velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 = co-efficient of frictions for pipes 1, 2, 3

H = difference of water level in the two tanks.

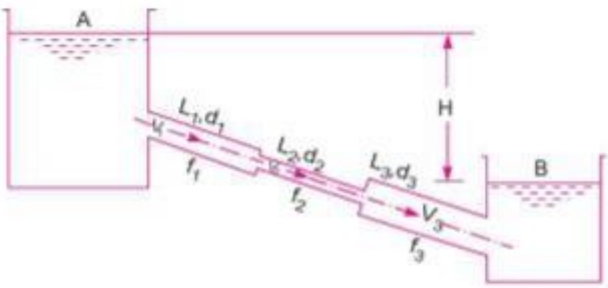


Fig.23.Pipes in series

The discharge passing through each pipe is same.

$\therefore \qquad Q = A_1V_1 = A_2V_2 = A_3V_3$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$\therefore \qquad H = \frac{0.5\,V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

If minor losses are neglected, then above equation becomes as

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

If the co-efficient of friction is same for all pipes

i.e., $f_1 = f_2 = f_3 = f$, then equation becomes as

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

Equivalent Pipe

This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe. The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.

Let L_1 = length of pipe 1 and d_1 = diameter of pipe 1

L_2 = length of pipe 2 and d_2 = diameter of pipe 2

L_3 = length of pipe 3 and d_3 = diameter of pipe 3

H = total head loss

L = length of equivalent pipe

d = diameter of the equivalent pipe

Then $L = L_1 + L_2 + L_3$

Total head loss in the compound pipe, neglecting minor losses

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

Assuming $f_1 = f_2 = f_3 = f$

Discharge, $Q = A_1V_1 = A_2V_2 = A_3V_3 = \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2 = \frac{\pi}{4}d_3^2V_3$

$\therefore$ $V_1 = \frac{4Q}{\pi d_1^2}$, $V_2 = \frac{4Q}{\pi d_2^2}$ and $V_3 = \frac{4Q}{\pi d_3^2}$

Substituting these values in equation

$$H = \frac{4fL_1 \times \left(\frac{4Q}{\pi d_1^2}\right)^2}{d_1 \times 2g} + \frac{4fL_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{d_2 \times 2g} + \frac{4fL_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{d_3 \times 2g}$$

$$= \frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

Head loss in the equivalent pipe, $H = \frac{4f.L.V^2}{d \times 2g}$ [Taking same value of f as in compound pipe]

where $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

$\therefore$
$$H = \frac{4f.L.\left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g} = \frac{4 \times 16Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

Head loss in compound pipe and in equivalent pipe is same hence equating equations

$$\frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] = \frac{4 \times 16Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

or
$$\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} = \frac{L}{d^5} \quad \text{or} \quad \frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

Equation is known as Dupuit's equation. In this equation $L = L_1 + L_2 + L_3$ and d_1, d_2 and d_3 are known. Hence the equivalent size of the pipe, *i.e.*, value of d can be obtained.

Flow through Parallel Pipes:

Consider a main pipe which divides into two or more branches as shown in Fig. and again join together downstream to form a single pipe, then the branch pipes are said to be connected in parallel. The discharge through the main is increased by connecting pipes in parallel.

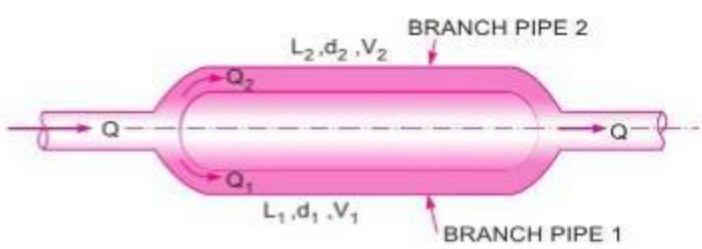


Fig.24.Parallel Pipes

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes. Hence from Fig.

$$Q = Q_1 + Q_2$$

In this, arrangement, the loss of head for each branch pipe is same.

∴ Loss of head for branch pipe 1 = Loss of head for branch pipe 2

$$\frac{4f_1L_1V_1^2}{d_1 \times 2g} = \frac{4f_2L_2V_2^2}{d_2 \times 2g}$$

$$f_1 = f_2, \text{ then } \frac{L_1V_1^2}{d_1 \times 2g} = \frac{L_2V_2^2}{d_2 \times 2g}$$

Flow through Branched Pipes:

When three or more reservoirs are connected by means of pipes, having one or more junctions, the system is called a branching pipe system. Fig. shows three reservoirs at different levels connected to a single junction, by means of pipes which are called branched pipes. The lengths, diameters and co-efficient of friction of each pipes is given. It is required to find the discharge and direction of flow in each pipe. The basic equations used for solving such problems are :

1. **Continuity equation** which means the inflow of fluid at the junction should be equal to the outflow of fluid.
2. **Bernoulli's equation**, and
3. **Darcy-Weisbach equation**

Also it is assumed that reservoirs are very large and the water surface levels in the reservoirs are constant so that steady conditions exist in the pipes. Also minor losses are assumed very small. The flow from reservoir *A* takes place to junction *D*. The flow from junction *D* is towards reservoirs *C*. Now the flow from junction *D* towards reservoir *B* will take place only when piezometric head at *D* $\left(\text{which is equal to } \frac{p_D}{\rho g} + Z_D\right)$ is more than the piezometric head at *B* (*i.e.*, Z_B). Let us consider that flow is from *D* to reservoir *B*.

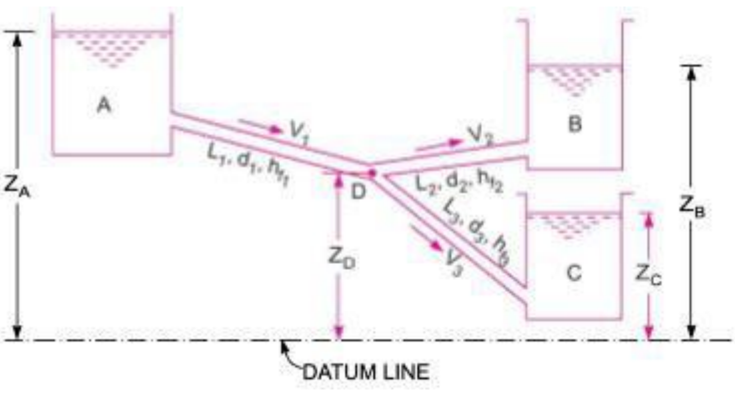


Fig.25.Branched Pipes

For flow from A to D from Bernoulli's equation

$$Z_A = Z_D + \frac{p_D}{\rho g} + h_{f_1} \quad ...(i)$$

For flow from D to B from Bernoulli's equation

$$Z_D + \frac{p_D}{\rho g} = Z_B + h_{f_2} \quad ...(ii)$$

For flow from D to C from Bernoulli's equation

$$Z_D + \frac{p_D}{\rho g} = Z_C + h_{f_3} \quad ...(iii)$$

From continuity equation,
Discharge through AD = Discharge through DB + Discharge through DC

$$\therefore \quad \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 \times V_2 + \frac{\pi}{4} d_3^2 V_3$$

or

$$d_1^2 V_1 = d_2^2 V_2 + d_3^2 V_3 \quad ...(iv)$$

There are four unknowns *i.e.*, V_1, V_2, V_3 and $\frac{p_D}{\rho g}$ and there are four equations (*i*), (*ii*), (*iii*) and (*iv*).

Hence unknown can be calculated.

Power Transmission through Pipes

Power is transmitted through pipes by flowing water or other liquids flowing through them. The power transmitted depends upon (*i*) the weight of liquid flowing through the pipe and (*ii*) the total head available at the end of the pipe. Consider a pipe AB connected to a tank as shown in Fig. The power available at the end B of the pipe and the condition for maximum transmission of power will be obtained as mentioned below :

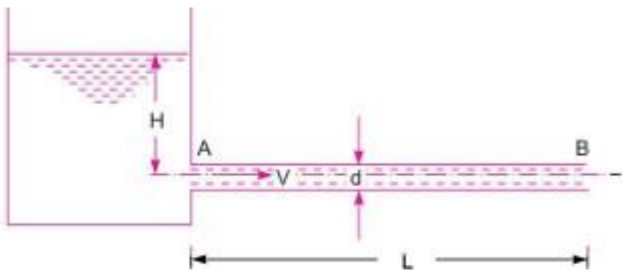


Fig.26.Power transmission through pipes

Let L = length of the pipe,
d = diameter of the pipe,
H = total head available at the inlet of pipe,
V = velocity of flow in pipe,
h_f = loss of head due to friction, and f = co-efficient of friction.

The head available at the outlet of the pipe, if minor losses are neglected

= Total head at inlet – loss of head due to friction

$$= H - h_f = H - \frac{4f \times L \times V^2}{d \times 2g} \qquad \left\{\because\ h_f = \frac{4f \times L \times V^2}{d \times 2g}\right\}$$

Weight of water flowing through pipe per sec,

$$W = \rho g \times \text{volume of water per sec} = \rho g \times \text{Area} \times \text{Velocity}$$

$$= \rho g \times \frac{\pi}{4} d^2 \times V$$

$\therefore$ The power transmitted at the outlet of the pipe

= weight of water per sec × head at outlet

$$= \left(\rho g \times \frac{\pi}{4} d^2 \times V\right) \times \left(H - \frac{4f \times L \times V^2}{d \times 2g}\right) \text{ Watts}$$

$\therefore$ Power transmitted at outlet of the pipe,

$$P = \frac{\rho g}{1000} \times \frac{\pi}{4} d^2 \times V \left(H - \frac{4fLV^2}{d \times 2g}\right) \text{ kW}$$

Efficiency of power transmission,

$$\eta = \frac{\text{Power available at outlet of the pipe}}{\text{Power supplied at the inlet of the pipe}}$$

$$= \frac{\text{Weight of water per sec} \times \text{Head available at outlet}}{\text{Weight of water per sec} \times \text{Head at inlet}}$$

$$= \frac{W \times (H - h_f)}{W \times H} = \frac{H - h_f}{H}.$$

Problem *Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s.*

Solution. Given :

Dia. of smaller pipe, $D_1 = 200$ mm $= 0.20$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400 \text{ mm} = 0.4 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$

Discharge, $Q = 250 \text{ litres/s} = 0.25 \text{ m}^3\text{/s}$

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96 \text{ m/s}$

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12564} = 1.99 \text{ m/s}$

Loss of head due to enlargement is given by equation

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \textbf{1.816 m of water. Ans.}$$

Problem *The rate of flow of water through a horizontal pipe is 0.25 m^3/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine :*

(*i*) *loss of head due to sudden enlargement,* (*ii*) *pressure intensity in the large pipe,*

(*iii*) *power lost due to enlargement.*

Solution. Given :

Discharge, $Q = 0.25 \text{ m}^3/\text{s}$

Dia. of smaller pipe, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400 \text{ mm} = 0.40 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} (0.4)^2 = 0.12566 \text{ m}^2$

Pressure in smaller pipe, $p_1 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$

Now velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96 \text{ m/s}$

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12566} = 1.99 \text{ m/s}$

(*i*) Loss of head due to sudden enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = \mathbf{1.816 \text{ m. Ans.}}$$

(*ii*) Let the pressure intensity in large pipe = p_2.

Then applying Bernoulli's equation before and after the sudden enlargement,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

But $z_1 = z_2$ (Given horizontal pipe)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e \text{ or } \frac{p_2}{\rho g} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.96^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.8160$$

$$= 15.229 - 2.0178 = 13.21 \text{ m of water}$$

$$\therefore \quad p_2 = 13.21 \times \rho g = 13.21 \times 1000 \times 9.81 \text{ N/m}^2$$

$$= 13.21 \times 1000 \times 9.81 \times 10^{-4} \text{ N/cm}^2 = \mathbf{12.96 \text{ N/cm}^2\text{. Ans.}}$$

(*iii*) Power lost due to sudden enlargement,

$$P = \frac{\rho g \cdot Q \cdot h_e}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000} = \mathbf{4.453 \text{ kW. Ans.}}$$

Problem *The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if co-efficient of friction are .005, .0052 and .0048 respectively, considering : (i) minor losses also (ii) neglecting minor losses.*

Solution. Given :

Difference of water level, $H = 12$ m

Length of pipe 1, $L_1 = 300$ m and dia., $d_1 = 300$ mm= 0.3 m

Length of pipe 2, $L_2 = 170$ m and dia., $d_2 = 200$ mm = 0.2 m

Length of pipe 3, $L_3 = 210$ m and dia., $d_3 = 400$ mm $= 0.4$ m

Also, $f_1 = .005, f_2 = .0052$ and $f_3 = .0048$

(*i*) **Considering Minor Losses.** Let V_1, V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}V_1 = \frac{d_1^2}{d_2^2}V_1 = \left(\frac{0.3}{.2}\right)^2 \times V_1 = 2.25\ V_1$$

and
$$V_3 = \frac{A_1V_1}{A_3} = \frac{d_1^2}{d_3^2}V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1 = 0.5625\ V_1$$

Now using equation (11.12), we have

$$H = \frac{0.5\,V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

Substituting V_2 and V_3, $12.0 = \frac{0.5\,V_1^2}{2g} + \frac{4 \times .005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times \left(2.25V_1^2\right)^2}{2g}$

$$+ 4 \times 0.0052 \times 170 \times \frac{(2.25\,V_1)^2}{0.2 \times 2g} + \frac{(2.25\,V_1 - .562\,V_1)^2}{2g} + \frac{4 \times .0048 \times 210 \times (.5625\,V_1)^2}{0.4 \times 2g} + \frac{(.5625\,V_1)^2}{2g}$$

$$12.0 = \frac{V_1^2}{2g}\ [0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316]$$

$$= \frac{V_1^2}{2g}\ [118.887]$$

$$\therefore \quad V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}} = 1.407 \text{ m/s}$$

$\therefore$ Rate of flow, Q = Area $\times$ Velocity $= A_1 \times V_1$

$$= \frac{\pi}{4}\ (d_1)^2 \times V_1 = \frac{\pi}{4}\ (.3)^2 \times 1.407 = 0.09945 \text{ m}^3\text{/s}$$

= 99.45 litres/s. Ans.

(*ii*) **Neglecting Minor Losses.** Using equation we have

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

or
$$12.0 = \frac{V_1^2}{2g}\left[\frac{4 \times .005 \times 300}{0.3} + \frac{4 \times .0052 \times 170 \times (2.25)^2}{0.2} + \frac{4 \times .0048 \times 210 \times (.5625)^2}{0.4}\right]$$

$$= \frac{V_1^2}{2g}\ [20.0 + 89.505 + 3.189] = \frac{V_1^2}{2g} \times 112.694$$

$$V_1 = \sqrt{\frac{2 \times 9.81 \times 12.0}{112.694}} = 1.445 \text{ m/s}$$

Discharge, $Q = V_1 \times A_1 = 1.445 \times \frac{\pi}{4}\ (.3)^2 = 0.1021 \text{ m}^3\text{/s}$ = **102.1 litres/s. Ans.**

Problem *Three pipes of lengths 800 m, 500 m and 400 m and of diameters 500 mm, 400 mm and 300 mm respectively are connected in series. These pipes are to be replaced by a single pipe of length 1700 m. Find the diameter of the single pipe.*

Solution. Given :

Length of pipe 1, $L_1 = 800$ m and dia., $d_1 = 500$ mm $= 0.5$ m

Length of pipe 2, $L_2 = 500$ m and dia., $d_2 = 400$ mm $= 0.4$ m

Length of pipe 3, $L_3 = 400$ m and dia., $d_3 = 300$ mm $= 0.3$ m

Length of single pipe, $L = 1700$ m

Let the diameter of equivalent single pipe $= d$

Applying equation $$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

or $$\frac{1700}{d^5} = \frac{800}{.5^5} + \frac{500}{.4^5} + \frac{400}{0.3^5} = 25600 + 48828.125 + 164609 = 239037$$

$\therefore$ $$d^5 = \frac{1700}{239037} = .007118$$

$\therefore$ $$d = (.007188)^{0.2} = 0.3718 = \mathbf{371.8\ mm.\ Ans.}$$

Problem 11.32 *A main pipe divides into two parallel pipes which again forms one pipe as shown in Fig. The length and diameter for the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.0 m*^{3}*/s. The co-efficient of friction for each parallel pipe is same and equal to .005.*

Solution. Given :

Length of pipe 1, $L_1 = 2000$ m

Dia. of pipe 1, $d_1 = 1.0$ m

Length of pipe 2, $L_2 = 2000$ m

Dia. of pipe 2, $d_2 = 0.8$ m

Total flow, $Q = 3.0\ m^3/s$

$f_1 = f_2 = f = .005$

Let Q_1 = discharge in pipe 1

Q_2 = discharge in pipe 2

From equation $Q = Q_1 + Q_2 = 3.0$ ...(*i*)

Using equation , we have

$$\frac{4f_1L_1V_1^2}{d_1 \times 2g} = \frac{4f_2L_2V_2^2}{d_2 \times 2g}$$

$$\frac{4 \times .005 \times 2000 \times V_1}{1.0 \times 2 \times 9.81} = \frac{4 \times .005 \times 2000 \times V_2^2}{0.8 \times 2 \times 9.81}$$

or
$$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8} \text{ or } V_1^2 = \frac{V_2^2}{0.8}$$

$\therefore$
$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{.894} \qquad ...(ii)$$

Now
$$Q_1 = \frac{\pi}{4}\, d_1^2 \times V_1 = \frac{\pi}{4}\,(1)^2 \times \frac{V_2}{.894} \qquad \left[\because V_1 = \frac{V_2}{.894}\right]$$

and
$$Q_2 = \frac{\pi}{4}\, d_2^2 \times V_2 = \frac{\pi}{4}\,(.8)^2 \times V_2 = \frac{\pi}{4} \times .64 \times V_2$$

Substituting the value of Q_1 and Q_2 in equation (*i*), we get

$$\frac{\pi}{4} \times \frac{V_2}{0.894} + \frac{\pi}{4} \times .64\ V_2 = 3.0 \text{ or } 0.8785\ V_2 + 0.5026\ V_2 = 3.0$$

$$V_2[.8785 + .5026] = 3.0 \text{ or } V = \frac{3.0}{1.3811} = 2.17 \text{ m/s.}$$

Substituting this value in equation (*ii*),

$$V_1 = \frac{V_2}{.894} = \frac{2.17}{0.894} = 2.427 \text{ m/s}$$

Hence
$$Q_1 = \frac{\pi}{4}\, d_1^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427 = \mathbf{1.906\ m^3/s.\ Ans.}$$

$\therefore$
$$Q_2 = Q - Q_1 = 3.0 - 1.906 = \mathbf{1.094\ m^3/s.\ Ans.}$$

Problem *A pipe of diameter 0.4 m and of length 2000 m is connected to a reservoir at one end. The other end of the pipe is connected to a junction from which two pipes of lengths 1000 m and diameter 300 mm run in parallel. These parallel pipes are connected to another reservoir, which is having level of water 10 m below the water level of the above reservoir. Determine the total discharge if f = 0.015. Neglect minor losses.*

Solution. Given :

Dia. of pipe, $d = 0.4$ m

Length of pipe, $L = 2000$ m

Dia. of parallel pipes, $d_1 = d_2 = 300$ mm = 0.30 m

Length of parallel pipes, $L_1 = L_2 = 1000$ m

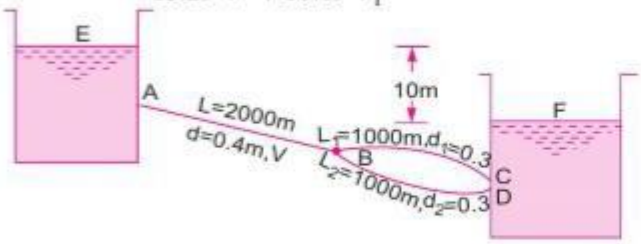
Difference of water level in two reservoir, $H = 10$ m, $f = .015$

Applying Bernoulli's equation to points E and F. Taking flow through ABC.

$$10 = \frac{4fLV^2}{d \times 2g} + \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g}$$

$$= \frac{4 \times .015 \times 2000 \times V^2}{0.4 \times 2 \times 9.81} + \frac{4 \times .015 \times 1000 \times V_1^2}{0.3 \times 2 \times 9.81}$$

$$= 15.29\ V^2 + 10.19\ V_1^2 \quad \ldots(i)$$



From continuity equation

Discharge through AB = discharge through BC + discharge through BD

or
$$\frac{\pi}{4}\, d^2 \times V = \frac{\pi}{4}\, d_1^2 \times V_1 + \frac{\pi}{4}\, d_1^2 V_2$$

But $d_1 = d_2$ and also the lengths of pipes BC and BD are equal and hence discharge through BC and BD will be same. This means $V_1 = V_2$ also

$\therefore$
$$\frac{\pi}{4}\, d^2 V = \frac{\pi}{4}\, d_1^2 \times V_1 + \frac{\pi}{4}\, d_1^2 \times V_1 \qquad [\because\ d_1 = d_2,\ V_1 = V_2]$$

$$= 2 \times \frac{\pi}{4}\, d_1^2 \times V_1 \ \text{ or } \ d^2 V = 2 d_1^2 V_1$$

or
$$(0.4)^2 \times V = 2 \times (0.3)^2 V_1 \text{ or } .16V = 0.18\ V_1$$

$\therefore$
$$V_1 = \frac{0.16}{0.18}\, V = 0.888\ V$$

Substituting this value of V_1 in equation (i), we get

$$10 = 15.29\ V^2 + (10.19)(.888)^2 V^2 = 15.29\ V^2 + 8.035\ V^2 = 23.325\ V^2$$

$\therefore$
$$V = \sqrt{\frac{10}{23.325}} = 0.654 \text{ m/s}$$

$\therefore$ Discharge $= V \times$ Area

$$= 0.654 \times \frac{\pi}{4}\, d^2 = 0.654 \times \frac{\pi}{4}\, (0.4)^2 = \mathbf{.0822\ m^3/s.\ Ans.}$$