Conservation of Energy:

The law of conservation of energy says “energy cannot be created or be destroyed; One form of energy can be changed into another form only”.

Consider the Control Volume shown in Figure as a thermodynamic system. Let amount of heat ***δq*** be added to the system from the surrounding. Also let ***δw*** be the work done on the system by the surroundings. Both heat and work are the forms of energy. Addition of any form of the energy to the system, changes the amount of internal energy of the system. Lets denote this change of internal energy by ***de***. As per the principle of energy conservation,

$\delta q + \delta w = de$

Therefore in terms of rate of change, the above equation changes to

$$\frac{\delta q}{\delta t} + \frac{\delta w}{\delta t} = \frac{de}{dt} \quad \text{-----------------} \ (1)$$

For an open system there will be a change in all the forms of energies possessed by the system, like internal energy and kinetic energy. The right hand side of the equation (1) is representing change in the content of energy of the system.

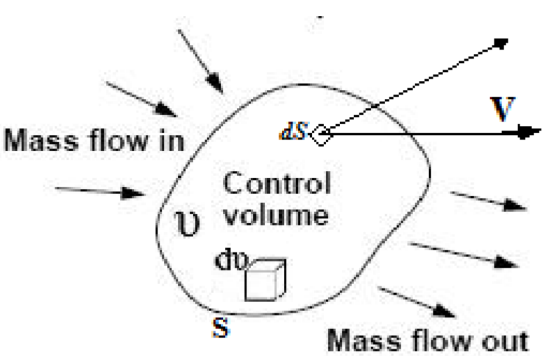
If q is the amount of heat added per unit mass, then the rate of heat addition for any elemental volume will be $q(\rho dv)$. The total external volumetric heat addition on the entire control volume and heat got added by viscous effects like conduction can be,

$$\frac{\delta q}{\delta t} = \iiint_{v} q \rho d\upsilon + \dot{Q}_{viscous} \quad \text{------------------}(2)$$

The main source of work transfer is due to the surface forces like pressure, body force etc. Consider an elemental area **ds** of the control surface. The pressure force on this elemental area is -P**ds** and the rate of work done on the fluid passing through **ds** with velocity **V** is *(-P**ds**).**V***. Integrating over the complete control surface, rate of work done due to pressure force is,

$$-\oint\!\!\oint_{s} (Pds).V \quad \text{----------------}(3)$$

In addition, consider an elemental volume **dυ** inside the control volume, as shown in Figure.



volume due to body force is $(\rho F_b du).V$. Here F_b is the body force per unit mass. Summing over the complete control volume, we obtain, rate of work done on fluid inside **υ** due to body forces is

$$\iiint_{v} (\rho F_b d\upsilon).V \quad \text{-----------------} \ (4)$$

The rate of work done on the elemental

If the flow is viscous, the shear stress on the control surface will also do work on the fluid as it passes across the surface. Let *Wviscous* denote the work done due to the shear stress. Therefore, the total work done on the fluid inside the control volume is the sum of terms given by (3) and (4) and *Wviscous,* that is

$$\frac{\delta w}{\delta t} = -\oiint_{s} PV.ds + \iiint_{v} \rho(F_b.V)d\upsilon + \dot{W}_{viscous} \quad \text{------------(5)}$$

For the open system considered, the changes in internal energy as well as kinetic energy need to be accounted. Therefore, right hand side of equation (1) should deal with total energy (sum of internal and kinetic energies) of the system. Let, ***e*** be the internal energy per unit mass of the system and kinetic energy per unit mass due to local velocity **V** be $V^2/2$.

Total energy in the control volume might also change due to influx and outflux of the fluid. The elemental mass flow across **ds** is (ρ**V**.**ds**). Therefore the elemental flow of total energy across the ds is (ρ**V**.**ds**)(e+V^2/2).

Hence the net energy change of the control volume is,

$$\frac{de}{dt} = \frac{\partial}{\partial t}\iiint_{v} \rho\left(e + \frac{V^2}{2}\right)d\upsilon + \oiint_{s}(\rho V.ds)\left(e + \frac{V^2}{2}\right)) \quad \text{--------------------(6)}$$

Thus, substituting Equations (2), (5) and (6) in equation (1), we have

This is the energy equation in the integral form. It is essentially the first law thermodynamics applied to fluid flow or open system.

One dimensional form of Conservation of Energy

Consider the control volume shown in Figure for steady inviscid flow without body force, Then the equation (9) reduces to,

$$\iiint_{v} \dot{q}\,\rho d\upsilon - \oiint_{s} PV.ds = \oiint_{s} \rho\left(e + \frac{V^2}{2}\right)V.ds$$

Let us denote the first term on left hand side of above equation by $\dot{Q}$ to represent the total external heat addition in the system. Thus, above equation becomes

Evaluating the surface integrals over the control volume in Figure, we obtain

$$\dot{Q} - (-P_1u_1A + P_2u_2A) = -\rho_1\left(e_1 + \frac{u_1^2}{2}\right)u_1A + \rho_2\left(e_2 + \frac{u_2^2}{2}\right)u_2A$$

or

$$\frac{\dot{Q}}{A} + P_1 u_1 + \rho_1\left(e_1 + \frac{u_1^2}{2}\right)u_1 = P_2 u_2 + \rho_2\left(e_2 + \frac{u_2^2}{2}\right)u_2$$

or

$$\frac{\dot{Q}}{\rho_1 u_1 A} + \frac{P_1}{\rho_1} + e_1 + \frac{u_1^2}{2} = \frac{P_2}{\rho_2} + e_2 + \frac{u_2^2}{2}$$

or

$$\frac{\dot{Q}}{\rho_1 u_1 A} + P_1 \upsilon_1 + e_1 + \frac{u_1^2}{2} = P_2 \upsilon_2 + e_2 + \frac{u_2^2}{2}$$

Here, $\dot{Q}/\rho_1 u_1 A$ is the external heat added per unit mass, *q*. Also, we know e + Pu = h. Hence, above equation can be re-written as,

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

This is the energy equation for steady one-dimensional flow for inviscid flow

Conservation of Momentum (Integral Form)

Momentum is defined as the product of mass and velocity, and represents the energy of motion stored in the system. It is a vector quantity and can only be defined by specifying its direction as well as magnitude.

The conservation of momentum is defined by Newton's second law of motion.

Newton's Second Law of Motion

"The rate of change of momentum is proportional to the net force acting, and takes place in the direction of that force".

This can be expressed as

$$\frac{d}{dt}(mV) = F \qquad \text{---}(1)$$

Consider the same Control Volume shown in Figure for deriving the momentum conservation equation. Right hand side of equation (1) is the summation of all forces like surface forces and body forces. Let $\mathbf{F_b}$ and **P** be the net body force per unit mass and pressure exerted on control surface respectively. The body force on the elemental volume $\mathbf{d\upsilon}$ is therefore $\boldsymbol{\rho}$ $\boldsymbol{F_b}$ $\boldsymbol{dv}$ and the total body force exerted on the fluid in the control volume is

$$\oiiint_v \rho F_b d\upsilon \qquad \text{---}(2)$$

The surface force due to pressure acting on the element of area **ds** is –Pds, where the negative sign indicates that the force is in the opposite direction of **ds**.

The total pressure force over the entire control surface

is expressed as $-\oiint_s P ds$ ---(3)

Let $\mathbf{F_{viscous}}$ be the total viscous force exerted on the control surface. Hence, the resultant

force
experienced by the fluid is given by

$$F = -\oiint_s P ds + \oiiint_v \rho F_b d\upsilon + F_{viscous}$$ --(4)

The left hand side term of the Equation (1) gives the time rate of change of momentum following a fixed fluid element or substantial derivative of the momentum. It can be evaluated using equation (2) by evaluating the sum of net flow of momentum leaving the control volume through the control surface **S** and time rate of change of momentum due to fluctuations of flow properties inside the control volume.

The mass flow across the elemental area **ds** is *(ρV.ds)*.Therefore, the flow of momentum per second across **ds** is *(ρV.ds)V*

The net flow of momentum out of the control volume through **s** is , $$\oiint_s (\rho V.ds)V$$ ---------------------------------------(5)

The momentum of the fluid in the elemental volume **dυ** is *(ρ du)V*. The momentum contained at any instant inside the control volume is

$$\oiiint_v \rho V d\upsilon$$

and its time rate of change due to unsteady flow fluctuation is

$$\frac{\partial}{\partial t}\oiiint_v \rho V d\upsilon$$ ---(6)

Combining Equations (5) and (6) to obtain the left hand side of equation (1), we get

$$\frac{d}{dt}(mV) = \frac{\partial}{\partial t}\oiiint_v \rho V d\upsilon + \oiint_s (\rho V.ds)V$$ -------------------------(7)

Thus, substituting Equations (4) and (7) into (1), we have

$$\frac{\partial}{\partial t}\oiiint_v \rho V d\upsilon + \oiint_s (\rho V.ds)V = -\oiint_s P ds + \oiiint_v \rho F_b d\upsilon + F_{viscous}$$ --------------------------(8)

This is the momentum equation in integral form.

It is a general equation, applies to the unsteady, three-dimensional flow of any fluid, compressible or incompressible, viscous or non viscous.

One dimensional form of Momentum conservation equation

For the steady and non viscous flow with no body forces, the Equation (8) reduces to

$$\oiint_s (\rho V.ds)V = -\oiint_s P ds$$

Above equation is a vector equation. However, since we are dealing with the one-dimensional flow, we need to consider only the scalar x component of equation.

$$\oiint_s (\rho V.ds)u = -\oiint_s (P ds)_x$$

Considering the control volume shown in Figure, above equation

transforms to, $\rho_1(-u_1A)u_1 + \rho_2(-u_2A)u_2 = -(-P_1A + P_2A)$
or

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 \quad \text{---} \quad (9)$$

This is the momentum equation for steady,non viscous one-dimensional

One Dimensional flow – forces of fluid in a curved pipe

In the case where fluid flows in a curved pipe as shown in figure, let ABCD be the control volume, A_1, A_2 the areas, v_1, v_2 the velocities, and p_1,p_2 the pressures of sections AB and CD respectively. Let F be the force of fluid acting on the pipe; the force of the pipe acting on the fluid is -F. This force and the pressures acting on sections AB and CD act on the fluid, increasing the fluid momentum by such a combined force (Increase in momentum = momentum going out - momentum coming in).

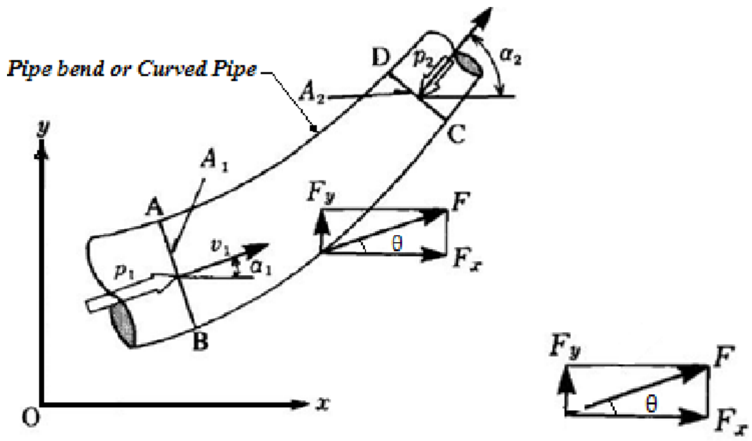


Fig.4. One dimensional flow

If F_x and F_y are the component forces in the x and y directions of F respectively, then from the equation of momentum,

$F_x + A_1p_1 \cos \alpha_1 - A_2p_2 \cos \alpha_2 = m (v_2 \cos \alpha_2 - v_1 \cos \alpha_1)$ $F_y + A_2p_2 \sin\alpha_2 - A_2p_2 \sin\alpha_2 = m(v_2 \sin\alpha_2 - v_1 \sin\alpha_1)$

From the above equations , F_x and F_y are given by

In these equations, m is the mass flow rate. If Q is the volumetric flow rate, then the following relation exists:

$m = \rho A_1 v_1 = \rho A_2 v_2 = \rho Q$

If the curved pipe is a pipe bend in a horizontal plane, then $\alpha_1 = 0$. Therefore

Rate of Flow (or) Discharge Q

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible flow of liquid, the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(i) For liquids the units of Q are m^3/s or ***litres/s***

(ii) For gases the units of Q are kgf/s or Newton/s Consider a fluid flowing through a pipe in which A= Cross-sectional area of pipe.

V= Average area of fluid across the section Then, discharge $Q = A \times V$ m^3/s