UNIT – 2 Equations of Motion

Fluid flow is described by two methods: Lagrangian method & Eulerian method In the Lagrangian method a single particle is followed over the flow field with the co ordinate system following the particle. The flow description is particle based and not space based. A moving coordinate system has to be used. In the Eulerian method, the description of flow is based on fixed coordinate system and the description of the velocity is with reference to location and time. Hence, Eulerian approach is easily adoptable to describe fluid motion mathematically.

Control Volume:

A fixed volume in space whose size and shape is entirely arbitrary, through which a fluid is continuously flowing is known as *control volume.* The boundary of a control volume is termed as the *control surface*. The size and shape is arbitrary and normally chosen such that it encloses part of the flow of particular interest.

Types of Fluid Flow:

1)Steady flow: The flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time is defined as steady flow.

Mathematically, for steady flow,

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

Unsteady Flow: The flow, in which the velocity, pressure and density at a point changes with respect to time is defined as unsteady flow.

Mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

2. Uniform flows: The flow in which the velocity at any given time does not change with respect to distance is defined as Uniform flow.

Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

Non-uniform flow: The flow in which the velocity at any given time changes with respect to distance is defined as non uniform flow.

Mathematically, for non-uniform flow,

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

3. Laminar flow: The flow in which the fluid particles move along well-defined paths which are straight and parallel is defined as laminar flow. Thus the particles move in layers and do not cross each other.

Turbulent flow: The flow in which the fluid particles do not move in a zig-zag way and the adjacent layers cross each other is defined as turbulent flow.

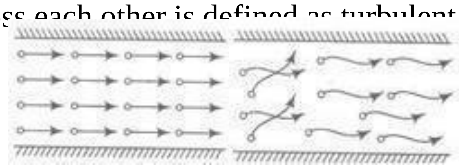

Fig.1. Laminar & Turbulent Flow

4. Compressible flow: The flow in which the density of fluid changes from point to point ie., ρ is not constant for the fluid, is defined as compressible flow.

Mathematically, for compressible flow, $\rho \neq$ constant.

Incompressible flow: The flow in which the density of fluid is constant is defined as incompressible flow.
Liquids are generally incompressible while gases are compressible.

Mathematically, for incompressible flow, $\rho = \text{Constant}$

5. Rotational flow: The flow in which the fluid particles while flowing along stream-lines, also rotate about their own axes, that type of flow is known as rotational flow.
Irrotational flow: The flow in which the fluid particles while flowing along stream-lines, do not rotate about their own axes, that type of flow is called irrotational flow.

6. One Dimensional Flow:

One dimensional flow is that type of flow in which the fluid velocity is a function of one- space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible.
Mathematically, for one-dimensional flow

$$u = f(x),\ v = 0 \quad \text{and } w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two-dimensional flow:
It is that type of flow in which the velocity is a function of two space co-ordinates only. Thus, mathematically for two dimensional flow

$$u = f_1(x, y), \quad v = f_2(x, y) \text{ and } w = 0$$

Three-dimensional flow:
It is the type of flow in which the velocity is a function of three space co-ordinates (x, y and z). Mathematically for three dimensional flow,

$$u = f_1(x, y, z), \quad v = f_2(x, y, z), \quad w = f_3(x, y, z).$$

Path Line:
A path line is the trajectory of an individual element of fluid.

Streamline:
A streamline is an imaginary continuous line within a moving fluid such that the tangent at each point is the direction of the flow velocity vector at that point.

Stream Tube:
An imaginary tube (need not be circular) formed by collection of neighboring streamlines through which the fluid flows is known as stream tube.

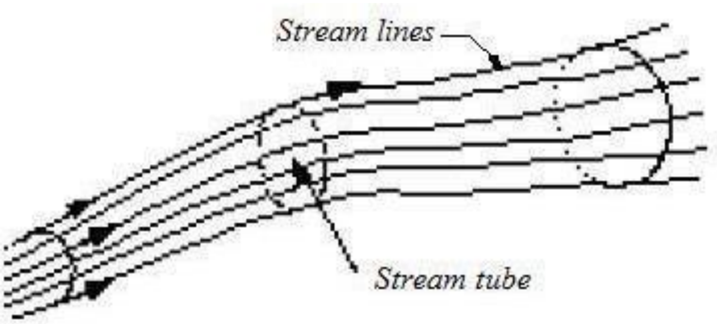

Fig.2.Conservation of mass: Integral Form

Let us consider a control volume V bounded by the control surface S. The efflux of mass across the control surface S is given by

$$\iint_S \rho \vec{V}.d\vec{A}$$

where $\vec{V}$ is the velocity vector at an elemental area(which is treated as a vector by considering its positive direction along the normal drawn outward from the surface).

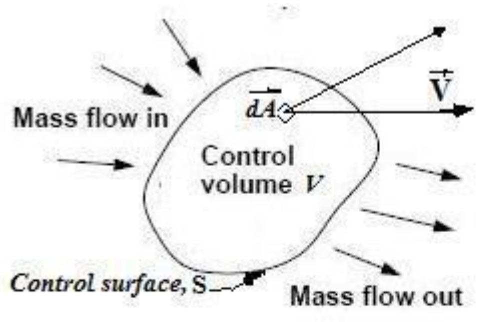

Fig.3.A Control Volume for integral form of derivation

The rate of mass accumulation within the control volume becomes

$$\frac{\partial}{\partial t}\iiint_V \rho.dV$$

where *dV* is an elemental volume, ρ is the density and *V* is the total volume bounded by the control surface S. Hence, the continuity equation becomes

$$\frac{\partial}{\partial t}\iiint_V \rho.dV + \iint_S \rho \vec{V}.d\vec{A} = 0 \quad \text{------------------------------------(1)}$$

The second term of the Equation can be converted into a volume integral by the use of the Gauss divergence theorem as

$$\iint_S \rho \vec{V}.d\vec{A} = \iiint_V \nabla.(\rho\vec{V})dV$$

Since the volume *V* does not change with time, the sequence of differentiation and integration in the first term of Equation (1) can be interchanged and it can be written as

$$\iiint_V \left[\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{V})\right]dV = 0 \quad \text{---------------------(2)}$$

Equation (2) is valid for any arbitrary control volume irrespective of its shape and size. So we can write

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{V}) = 0$$