flow: A local velocity which has a constant mean value but also has a statistically random fluctuating component due to turbulence in the flow. Typical plots of velocity time histories for laminar flow, turbulent flow, and the region of transition between the two are shown below.

Principal parameter used to specify the type of flow regime is the Reynolds number :

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

V- Flow velocity

D – Flow dimension

µ - Dynamic Viscosity

Ʋ – Kinematic Viscosity

Frictional Loss in Pipe flow

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero. The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity. This viscous action causes loss of energy which is usually known as frictional loss.

On the basis of his experiments, William Froude gave the following laws of fluid fraction for turbulent flow.

The frictional resistance for turbulent flow is :

(*i*) proportional to V^n, where n varies from 1.5 to 2.0,

(*ii*) proportional to the density of fluid,

(*iii*) proportional to the area of surface in contact,

(*iv*) independent of pressure,

(*v*) dependent on the nature of the surface in contact.

Expression for Loss of Head due to friction in pipes:

Consider a uniform horizontal pipe having steady flow as shown in fig 18. Let 1-1 and 2-2 are two sections of pipe.

Let P_1 = pressure intensity at section 1-1

V_1 = Velocity of flow at section 1-1

L = length of the pipe between sections 1-1 and 2-2,

d = diameter of pipe,

f' = frictional resistance per unit wetted area per unit velocity,

h_f = loss of head due to friction,

and p_2, V_2 = are values of pressure intensity and velocity at section 2-2.

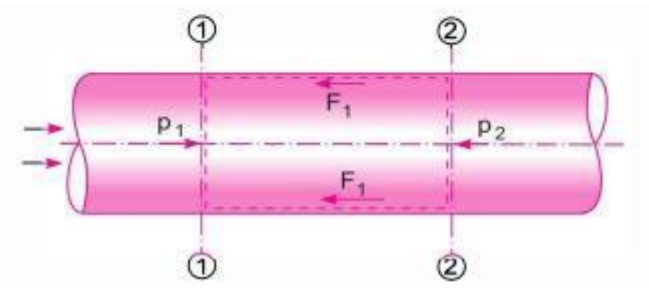

Fig.18.Uniform Horizontal Pipe

Applying Bernoulli's equations between sections 1-1 and 2-2,

Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 and 2-2

or $$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = z_2$ as pipe is horizontal

$V_1 = V_2$ as dia. of pipe is same at 1-1 and 2-2

$$\therefore \quad \frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f \text{ or } h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} \qquad ...(i)$$

But h_f is the head lost due to friction and hence intensity of pressure will be reduced in the direction of flow by frictional resistance.

Now frictional resistance = frictional resistance per unit wetted area per unit velocity × wetted area × velocity^2

or $$F_1 = f' \times \pi dL \times V^2 \quad [\because \text{ wetted area} = \pi d \times L, \text{ velocity} = V = V_1 = V_2]$$
$$= f' \times P \times L \times V^2 \qquad [\because \pi d = \text{Perimeter} = P] \;...(ii)$$

The forces acting on the fluid between sections 1-1 and 2-2 are :

1. pressure force at section 1-1 = $p_1 \times A$

where A = Area of pipe

2. pressure force at section 2-2 = $p_2 \times A$
3. frictional force F_1 as shown in Fig. 10.3.

Resolving all forces in the horizontal direction, we have

$$p_1 A - p_2 A - F_1 = 0 \qquad ...(10.1)$$

or $$(p_1 - p_2)A = F_1 = f' \times P \times L \times V^2 \qquad [\because \text{From } (ii),\ F_1 = f'PLV^2]$$

or $$p_1 - p_2 = \frac{f' \times P \times L \times V^2}{A}$$

But from equation (i), $p_1 - p_2 = \rho g h_f$

Equating the value of $(p_1 - p_2)$, we get

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

or $$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad ...(iii)$$

In equation (iii), $\frac{P}{A} = \frac{\text{Wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$

$$\therefore \quad h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4LV^2}{d} \qquad ...(iv)$$

Putting $\frac{f'}{\rho} = \frac{f}{2}$, where f is known as co-efficient of friction.

Equation (*iv*), becomes as $h_f = \frac{4 \cdot f}{2g} \cdot \frac{LV^2}{d} = \frac{4f \cdot L \cdot V^2}{d \times 2g}$

is known as Darcy-Weisbach equation. This equation is commonly used for finding loss of head due to friction in pipes.

Sometimes equation (10.2) is written as

$$h_f = \frac{f \cdot L \cdot V^2}{d \times 2g}$$

Then f is known as friction factor.

Loss of Energy in Pipes:

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses
   - This is due to friction and it is calculated by the following formulae :
     - (*a*) Darcy-Weisbach Formula
     - (*b*) Chezy's Formula
2. Minor Energy Losses
   - This is due to
     - (*a*) Sudden expansion of pipe
     - (*b*) Sudden contraction of pipe
     - (*c*) Bend in pipe
     - (*d*) Pipe fittings etc.
     - (*e*) An obstruction in pipe.

Loss of Energy due to friction:

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter and is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$$

where h_f = loss of head due to friction

f = co-efficient of friction which is a function of Reynolds number

$= \frac{16}{R_e}$ for $R_e < 2000$ (viscous flow)

$= \frac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = length of pipe,

V = mean velocity of flow,

d = diameter of pipe.

(*b*) **Chezy's Formula for loss of head due to friction in pipes.** Refer to chapter article in which expression for loss of head due to friction in pipes is derived. Equation (*iii*) of article

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$$

where h_f = loss of head due to friction, P = wetted perimeter of pipe,
A = area of cross-section of pipe, L = length of pipe,
and V = mean velocity of flow.

Now the ratio of $\frac{A}{P}\left(= \frac{\text{Area of flow}}{\text{Perimeter (wetted)}}\right)$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation , we get

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m} \text{ or } V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L} = \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\therefore \quad V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} = \sqrt{\frac{\rho g}{f'}}\sqrt{m\frac{h_f}{L}}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant and $\frac{h_f}{L} = i$, where i is loss of head per unit length of pipe.

Substituting the values of $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$ in equation (11.3), we get

$$V = C\sqrt{mi}$$

Equation is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to $d/4$.

Minor Energy Losses

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases :

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head at the entrance of a pipe,
4. Loss of head at the exit of a pipe,
5. Loss of head due to an obstruction in a pipe,
6. Loss of head due to bend in the pipe,
7. Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

Loss of Head Due to Sudden Enlargement. Consider a liquid flowing through a pipe which has sudden enlargement as shown in Fig. Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

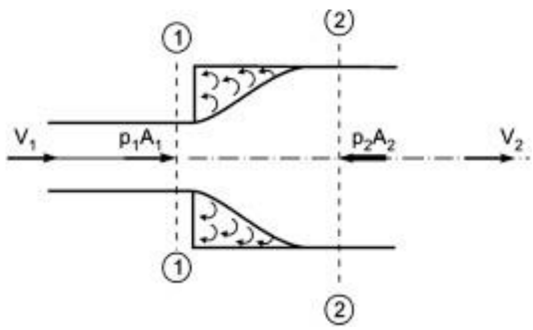

Fig.19. Sudden Enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}.$$

Loss of Head due to Sudden Contraction. Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in Fig. Consider two sections 1-1 and 2-2 before and after contraction. As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at a section *C-C* as shown in Fig. This section *C-C* is called Vena-contracta. After section *C-C*, a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from Vena-contracta to smaller pipe.

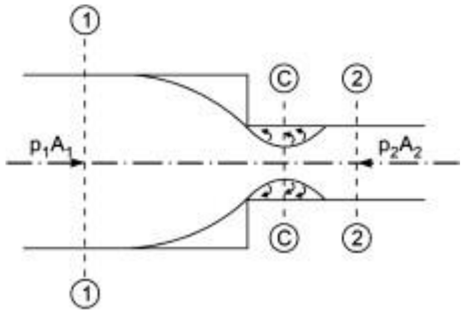

Fig.20.Sudden Contraction

$$h_c = 0.5\,\frac{V_2^2}{2g}.$$

Loss of Head at the Entrance of a Pipe. This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance. In practice the value of loss of head at the entrance (or inlet) of a pipe with sharp cornered entrance is taken = $0.5 \frac{V^2}{2g}$, where V = velocity of liquid in pipe.

This loss is denoted by h_i

$$\therefore \qquad h_i = 0.5 \frac{V^2}{2g}$$

Loss of Head at the Exit of Pipe. This is the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet (if outlet of the pipe is free) or it is lost in the tank or reservoir (if the outlet of the pipe is connected to the tank or reservoir). This loss is equal to $\frac{V^2}{2g}$, where V is the velocity of liquid at the outlet of pipe. This loss is denoted h_o.

$$\therefore \qquad h_o = \frac{V^2}{2g}$$

where V = velocity at outlet of pipe.

Loss of Head Due to an Obstruction in a Pipe. Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in Fig.

Consider a pipe of area of cross-section A having an obstruction as shown in Fig.

Let a = Maximum area of obstruction

A = Area of pipe

V = Velocity of liquid in pipe

Then $(A - a)$ = Area of flow of liquid at section 1-1.

As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1, after which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to velocity, V in the pipe. This situation is similar to the flow of liquid through sudden enlargement.

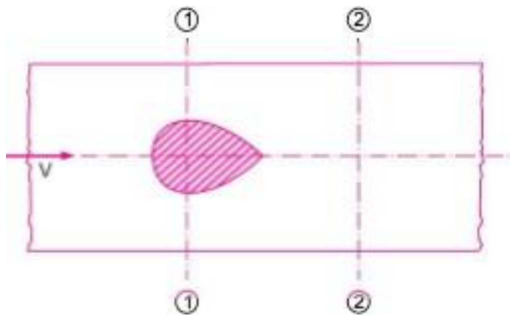

Fig.21. Obstruction in a pipe

$$\text{Head loss due to obstruction} = \frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2$$

Loss of Head due to Bend in Pipe. When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary and also formation of eddies takes place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as

$$h_b = \frac{kV^2}{2g}$$

where h_b = loss of head due to bend, V = velocity of flow, k = co-efficient of bend

The value of k depends on

(*i*) Angle of bend, (*ii*) Radius of curvature of bend, (*iii*) Diameter of pipe.

Loss of Head in Various Pipe Fittings. The loss of head in the various pipe fittings such as valves, couplings etc., is expressed as

$$= \frac{kV^2}{2g}$$

where V = velocity of flow, k = co-efficient of pipe fitting.