Engineering Mechanics
Centroid

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12.1 Introduction

Centre of gravity is defined as a point through which the weight of the body is assumed to act. The line of action of the weight of the body passes through it. A rigid body consists of a large number of particles, and the gravitational forces acting on these particles constitute a system of parallel forces, acting towards the centre of the earth. The point at which the resultant of all these parallel forces acts is called the centre of gravity of the rigid body. This point is generally denoted by G. In many engineering applications, the location of centre of gravity of plane and solid figures are required.

12.2 Centroid and centre of gravity

The centre of gravity of a body is defined as the point through which the entire weight of the body acts; when this is referred to weightless laminas or plane areas (plane areas have no mass) is called the centroid of the area or in otherwords, centroid is the term referred to one and two dimensional figures and, centre of gravity is referred to three dimensional figures. Both are represented by 'G'.

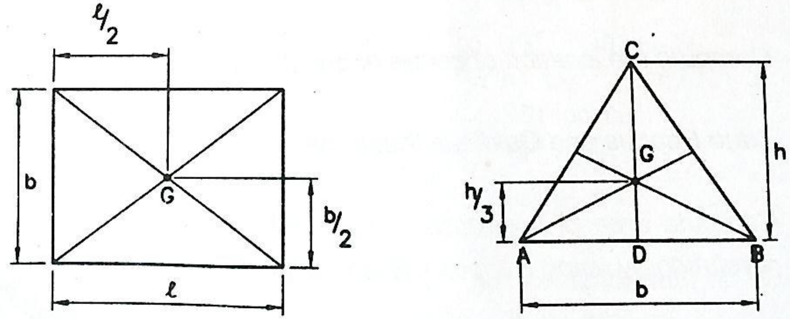
It is to be noted that, every body has one and only one, centre of gravity, which may be within the body or even outside the body.

12.3 Centroid of plane figures.

The centroid of plane figures (two dimensional figures) are determined by geometrical consideration, by the method of moments, by integration etc., For our convenience, let us divide the centroid of plane figures into centroid of simple plane figures and centroid of composite plane figures. The plane areas of well known geometric shapes like rectangle, square, circle, triangle etc, are called simple plane figures, and the plane areas, combination of two or more simple plane figures are called the composite plane figures.

12.3.1 Centroid of simple plane figures.

The centroid of simple plane figures are determined by the geometrical considerations and integration. For example, if we consider a rectangle of length '*l*', and breadth 'b', shown in fig 12.1, the centroid is the point of intersection of its diagonals. From the geometry of the figure, the centroid G is the mid point of the length as well the mid point of the breadth.



Similarly, if we consider a triangle of base 'b' and height h, shown in fig 12.2, the centroid is the point of intersection of three medians (median is the line joining a vertex and the mid-point of the opposite side). From the geometry of the figure, the centroid G, divides each median internally, in the ratio of 2:1 (2 parts from the apex and one part from the opposite side). Hence for the triangle shown in fig 12.2, the centroid G is at a height of $\frac{h}{3}$ from the base.

12.3.2 Reference axes and centroidal axes.

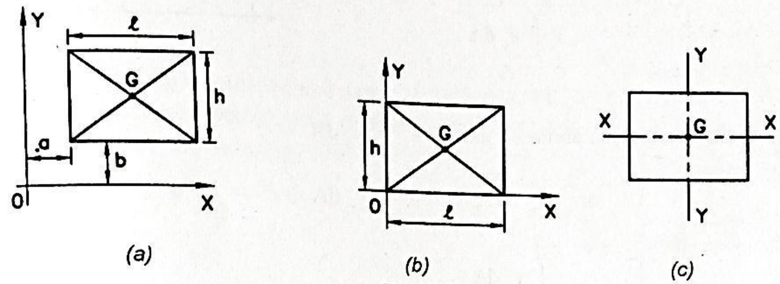


Fig 12.3

Consider a rectangle of length 'l', and height 'h'. We know its centroid (G) is the point of intersection of the diagonals. But the location of centroid (or the distance of centroid) is always measured with reference to some reference axes. Generally two axes are used for the centroid of a plane figure. When the reference axes OX and OY, with the point of origin O, are taken as shown in fig 12.3 (a) then the horizontal distance of the centroid from the vertical reference axis, $\bar{x} = a + \frac{l}{2}$; similarly the vertical distance of the centroid from the horizontal reference axis, $\bar{y} = b + \frac{h}{2}$.

But, for the same rectangle, if the reference axes are fixed as shown in fig 12.3 (b) then $\bar{x} = \frac{l}{2}$ and $\bar{y} = \frac{h}{2}$

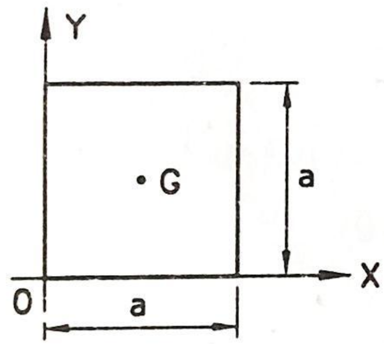
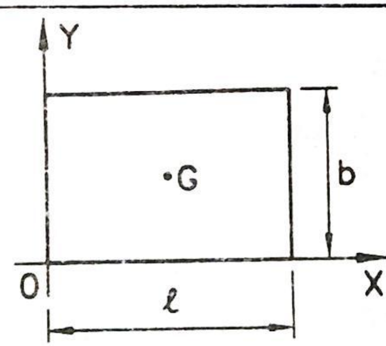
Hence, while locating the centroid of a plane figure, location of the reference axes plays a major role. **In this text, the horizontal reference axis (OX) is taken on the bottom edge of the figure and the vertical reference axis OY is taken on the left edge of the figure.** (Fig 12.3 (b)).

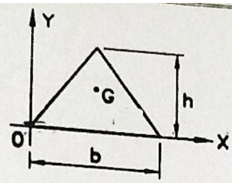
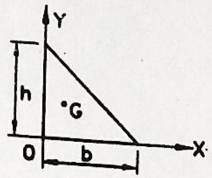
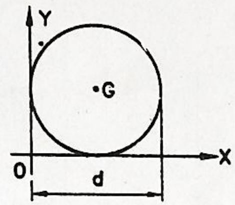
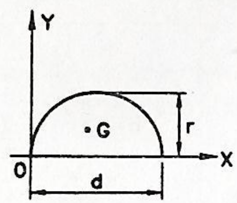
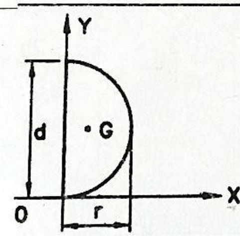
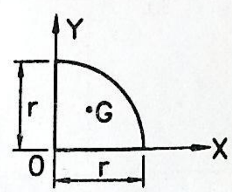
The axes which are passing through the centroid of the figure are called the centroidal axes. The horizontal and vertical centroidal axes passing through G, are XX and YY axes respectively, Shown in fig 12.3 (c)

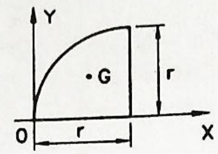
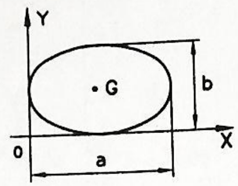
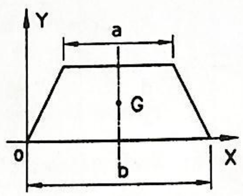
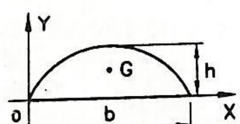
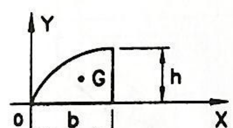
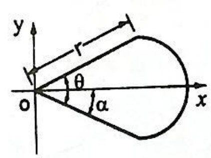
12.4 Centroid of plane figures by Integration

Consider a plane figure as shown in fig, for which the centroid is required. First of all take an elementary strip (elementary strip may have any shape, rectangle or triangle) and find its area. Let it be dA. Let x_s and y_s are the centroidal distances of the elementary strip from

Table 12.1 Centroid of simple plane figures

Sl No.	Name	Shape	$\bar{x}$	$\bar{y}$	Area
1.	Square		$\frac{a}{2}$	$\frac{a}{2}$	a^2
2.	Rectangle		$\frac{l}{2}$	$\frac{b}{2}$	$l\,b$

3.	Triangle (Isosceles)		$\frac{b}{2}$	$\frac{h}{3}$	$\frac{1}{2}bh$
4.	Triangle (Right-angled)		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{1}{2}bh$
5.	Circle		$\frac{d}{2}$	$\frac{d}{2}$	$\frac{\pi d^2}{4}$
6.	Semi-circle		$\frac{d}{2}$	$\frac{4r}{3\pi}$	$\frac{1}{2}\times\frac{\pi d^2}{4}$
7.	Semi-circle		$\frac{4r}{3\pi}$	$\frac{d}{2}$	$\frac{1}{2}\times\frac{\pi d^2}{4}$
8.	Quadrant		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{1}{4}\times\pi r^2$

9.	Quadrant		$\left(r - \frac{4r}{3\pi}\right)$	$\frac{4r}{3\pi}$	$\frac{1}{4} \times \pi r^2$
10.	Ellipse		$\frac{a}{2}$	$\frac{b}{2}$	$\frac{\pi ab}{4}$
11.	Trape-zium		$\frac{b}{2}$	$\left(\frac{b+2a}{b+a}\right)\frac{h}{3}$	$\frac{1}{2}(a+b)h$
12.	Parabola		$\frac{b}{2}$	$\frac{2}{5}h$	$\frac{2}{3}bh$
13.	Semi-par abola		$\frac{5}{8}b$	$\frac{2h}{5}$	$\frac{2}{3}bh$
14.	Sector of circle	 $\theta = 2\alpha$	$\frac{2r \sin\alpha}{3\alpha}$	**0**	$\frac{\theta}{360} \times \pi r^2$

12.5. Centroid of Composite Plane Figures.

If a plane figure is a combination of two or more simple plane figures, the algebraic sum of moments of the individual areas about any axis of reference will be equal to the moment of the whole area about the same axis. Hence, the centroid of the composite plane figures are determined by the method of moments.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + ... + a_n x_n}{a_1 + a_2 + ... + a_n} \qquad |||ly \qquad \bar{y} = \frac{a_1 y_1 + a_2 y_2 + ... + a_n y_n}{a_1 + a_2 + ... + a_n}$$

where,

$a_1, a_2 ..... a_n$ are the area of the simple plane figures 1, 2 ..... of the composite plane figure

$x_1, x_2 ..... x_n$ are the horizontal distance of the centroid of simple plane figures 1, 2, ... from the vertical reference axis oy.

$y_1, y_2, ... y_n$ are the vertical distance of the centroid of simple plane figures 1, 2.... from the horizontal reference axis ox.

$\bar{x}$ and $\bar{y}$ are the horizontal and vertical distance of the centroid of the composite plane figure from the vertical and horizontal reference axes.

12.5.1. Axis of symmetry

If a composite plane figure has an axis of symmetry (i.e., an axis about which similar configuration is seen on eitherside), centroid lies on it. This concept reduce the work of locating the centroid. Axis of symmetry concept is illustrated by some examples given below :

i) Symmetrical about both the axes

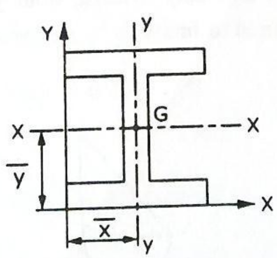


Fig. 12.11(a)

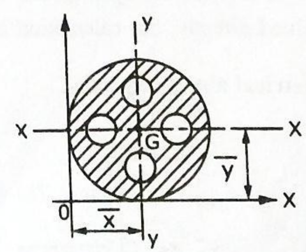


Fig. 12.11(b)

Fig. 12.11(a) and (b) are symmetrical about both the axes. Hence, no calculation is required to locate the centroid. From geometry, $\bar{x}$ and $\bar{y}$ values can be determined directly. centroid is the point of intersection of symmetrical axes and symmetrical axes are the centroidal axes.

ii) Symmetrical about x axis

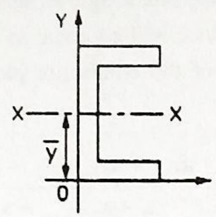


Fig. 12.12(a)

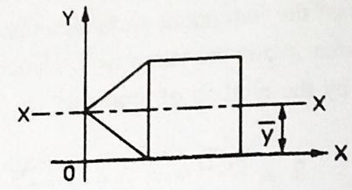


Fig. 12.12(b)

Fig. 12.12(a) and (b) are symmetrical about x axis only. The centroid lies on this axis of symmetry. Hence, no calculation is required for $\bar{y}$, but $\bar{x}$ value is to be found out.

iii) Symmetrical about y axis

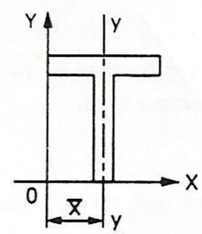


Fig. 12.13(a)

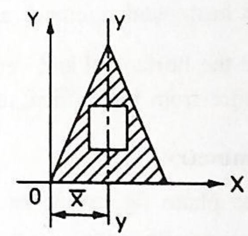


Fig. 12.13(b)

Fig. 12.13(a) and (b) are symmetrical about y axis only. Hence, from geometry $\bar{x}$ value can be determined directly, but calculation is required to find $\bar{y}$.

iv) Not symmetrical about any axis

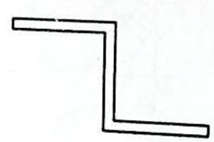


Fig. 12.14(a)

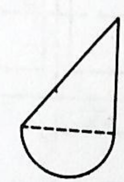


Fig. 12.14(b)

Fig. 12.14(a) and (b) are not symmetrical about any axis. Hence, calculation is required for both $\bar{x}$ and $\bar{y}$

Locate the centroid of the L-section shown in fig 12.15

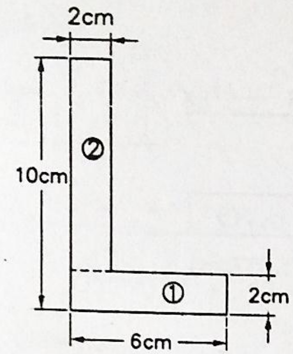


Fig 12.15

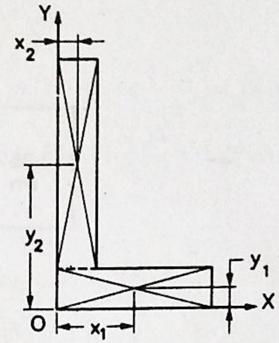


Fig 12.16

Solution

The given L-section is divided into two rectangles (ie, divided into two simple plane figures); and the centroid of each component is located. The reference axes are drawn on its left and bottom edges. The horizontal and vertical distances of the centroid of the components are measured from the reference axes (Fig. 12.16). L section is not symmetrical about any axis, hence find both $\bar{x}$ and $\bar{y}$

Portion (1) (size 6 cm × 2 cm)

area, $a_1 = 6 \times 2 = 12 \text{ cm}^2$

horizontal distance of the centroid G_1 from OY axis, $x_1 = \frac{6}{2} = 3 \text{ cm}$

vertical distance of the centroid G_1 from OX axis, $y_1 = \frac{2}{2} = 1 \text{ cm}$.

Portion (2) (size 2 cm × 8 cm)

area, $a_2 = 2 \times 8 = 16 \text{cm}^2$

horizontal distance of the centroid G_2 from OY axis, $x_2 = \frac{2}{2} = 1\text{cm}$

vertical distance of the centroid G_2, from OX axis, $y_2 = 2 + \frac{8}{2} = 6\text{cm}$

using the relation, $\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(12 \times 3) + (16 \times 1)}{12 + 16} = 1.857 \text{ cm}$

and $\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(12 \times 1) + (16 \times 6)}{12 + 16} = 3.857 \text{ cm}$

The centroid and the centroidal axes, passing through the centroid are shown in fig 12.17.

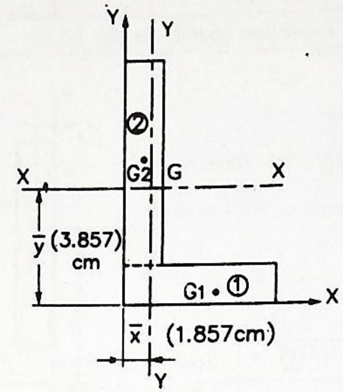


Fig. 12.17

Example 2:

Locate the centroid of the figure shown in fig 12.18

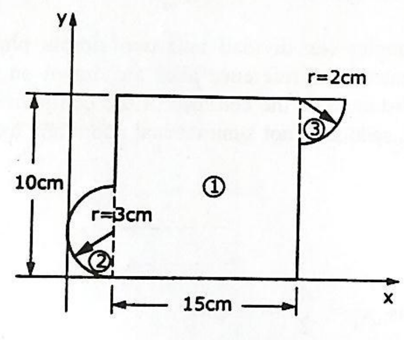


Fig 12.18

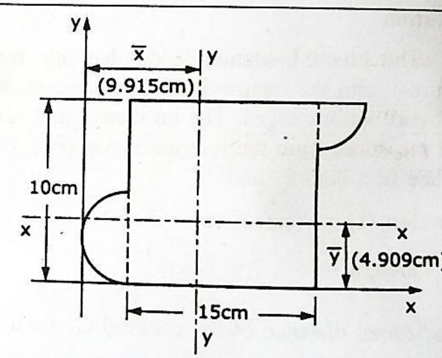


Fig. 12.19.

Solution

The reference axes OX and OY are drawn on the bottom and left edges of the given composite figure and the composite figure is divided into 3 portions.

Portion (1) (Rectangle)

area, $a_1 = 15 \times 10 = 150 \text{cm}^2$

$$x_1 = 3 + \frac{15}{2} = 10.5 \text{ cm} \quad ; \quad y_1 = \frac{10}{2} = 5 \text{ cm}$$

Portion (2) (semi - circle)

$$\text{area, } a_2 = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 3^2 = 14.137 \text{cm}^2$$

$$x_2 = r - \left(\frac{4r}{3\pi}\right) = 3 - \left(\frac{4 \times 3}{3\pi}\right) = 1.726 \text{ cm.}$$

$$y_2 = r = 3 \text{ cm}$$

Portion (3) (Quadrant)

$$\text{area, } a_3 = \frac{1}{4}\pi \times 2^2 = 3.14\text{cm}^2 \quad ; \quad x_3 = 3 + 15 + \left(\frac{4 \times 2}{3\pi}\right) = 18.84 \text{ cm}$$

$$y_3 = 10 - \left(\frac{4 \times 2}{3\pi}\right) = 9.15 \text{ cm}$$

using the relation, $\bar{x} = \dfrac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$

$$= \frac{(150 \times 10.5) + (14.137 \times 1.726) + (3.14 \times 18.84)}{150 + 14.137 + 3.14} = 9.915 \text{ cm}$$

and $\bar{y} = \dfrac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$

$$= \frac{(150 \times 5) + (14.137 \times 3) + (3.14 \times 9.15)}{150 + 14.137 + 3.14} = 4.909 \text{ cm}$$

The location of centroid is shown in fig. 12.19.

Example 3.

Locate the centroid of the lamina, shown in fig 12.20.

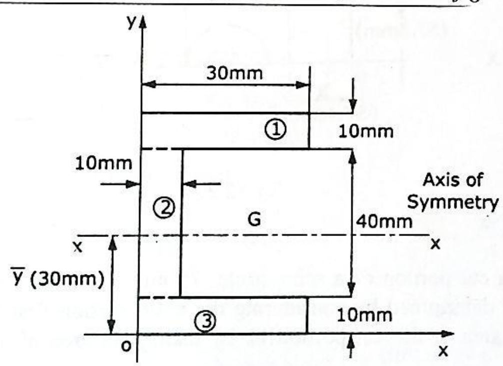


Fig 12.20

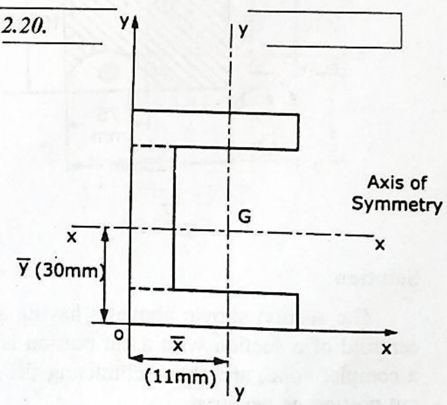


Fig. 12.21.

Solution

The given figure is symmetrical about x axis, hence $\bar{y}$ can be found directly from the geometry of the figure.

$\therefore$ due to symmetry, $\bar{y} = \dfrac{10 + 40 + 10}{2} = 30$ mm

to find $\bar{x}$, use the relation, $\bar{x} = \dfrac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$

where

$$a_1 = (30 \times 10) = 300\text{mm}^2; \qquad x_1 = \frac{30}{2} = 15 \text{ mm}$$

$$a_2 = (10 \times 40) = 400\text{mm}^2; \qquad x_2 = \frac{10}{2} = 5 \text{ mm}$$

$$a_3 = (30 \times 10) = 300\text{mm}^2; \qquad x_3 = \frac{30}{2} = 15 \text{ mm}$$

$$\therefore \quad \bar{x} = \frac{(300 \times 15) + (400 \times 5) + (300 \times 15)}{300 + 400 + 300} = 11 \text{ mm}$$

The location of centroid is shown in fig. 12.21.

Example 4

Locate the centroid of the lamina shown in fig 12.22.

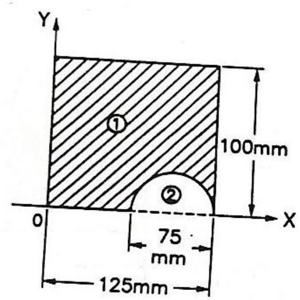


Fig 12.22.

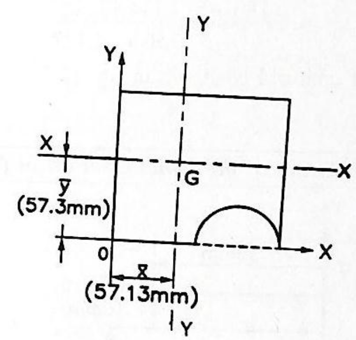


Fig 12.23.

Solution

The section shown above is having a cut portion of a semi-circle, 75 mm diameter. The centroid of a section with a cut portion is determined by considering the main section first as a complete one, and then subtracting the area of the cut portion ie, by taking the area of the cut portion as negative.

Portion (1) (rectangle)

$$\text{area,} \quad a_1 = 125 \times 100 = 12500 \text{ mm}^2$$

$$x_1 = \frac{125}{2} = 62.5 \text{ mm} \quad ; \quad y_1 = \frac{100}{2} = 50 \text{ mm}$$

Portion (2) (Semi-circle)

$$\text{area,} \quad a_2 = \frac{1}{2} \times \frac{\pi \times 75^2}{4} = 2208.93 \text{ mm}^2$$

$$x_2 = 125 - \frac{75}{2} = 87.5 \text{ mm} \quad ; \quad y_2 = \frac{4r}{3\pi} = \frac{4 \times 37.5}{3\pi} = 15.91 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{(12500 \times 62.5) - (2208.93 \times 87.5)}{(12500 - 2208.93)} = 57.13 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(12500 \times 50) - (2208.93 \times 15.91)}{(12500 - 2208.93)} = 57.31 \text{ mm}$$

Thank you..