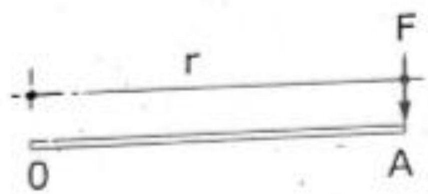
UNIT 2

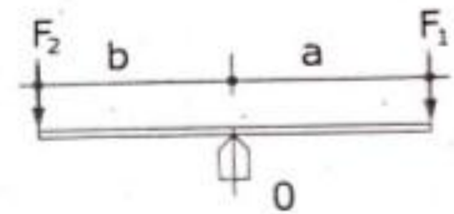
Moment and Couple

Moment of a force

Moment of a force.

Moment of a force about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from that point.





F is a downward force applied at 'A' and 'r' is the perpendicular distance of the line of action of the force F, from the point 'O'.

The Moment (M) of the force F about 'O' is given by,

$$M_o = F \times r \quad \text{(i.e. } \textit{Force} \times \textit{perpendicular distance}\text{)}$$

Downward force applied at A, will have a tendency to rotate OA about the point 'O'. Hence, moment may also be defined as the turning effect produced by a force. Moment is classified into two types:

(i) Clock wise moment (↻) and (ii) Anticlock wise (↺) moment

downward force F_1 acting on the right hand side of the fulcrum 'O', at a distance of 'a', produces clock wise moment about 'O'.

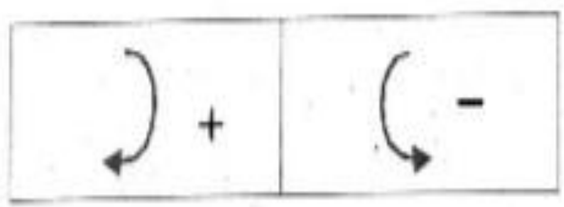
$\therefore$ Moment of F_1 about 'O' $= F_1 \times a$ (Clock wise ↻)

But, downward force F_2, applied on the left hand side of the fulcrum 'O', at a distan of b, produces anticlockwise moment about 'O'.

$\therefore$ Moment of F_2 about 'O' $= F_2 \times b$ (Anticlock wise ↺)

Sign convention.

In the proceeding articles, we use positive sign for clockwise moment and negative si for anticlockwise moment.

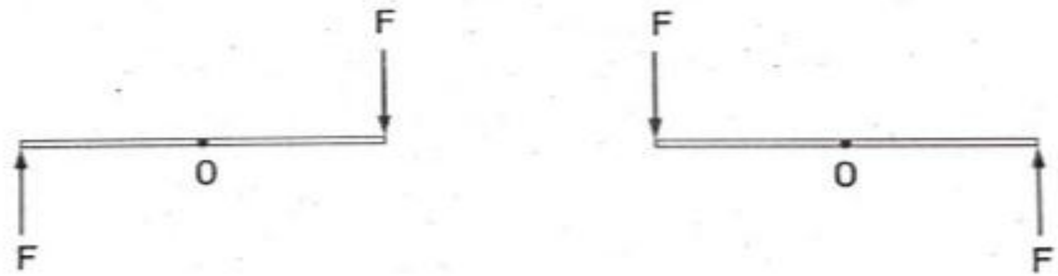


Unit of Moment.

In S.I. system, unit of moment is Newton-metre (Nm). i.e., Force is measured in Newt and the distance is measured in metre.

Moment of vertical and horizontal forces

Moment of Vertical forces.



About a point (at 'O'), right hand side downward force and left hand side upward force produces clockwise moment, shown in Similarly, about a point (at 'O'), right hand side upward force and left hand side downward force produces anticlockwise moment,

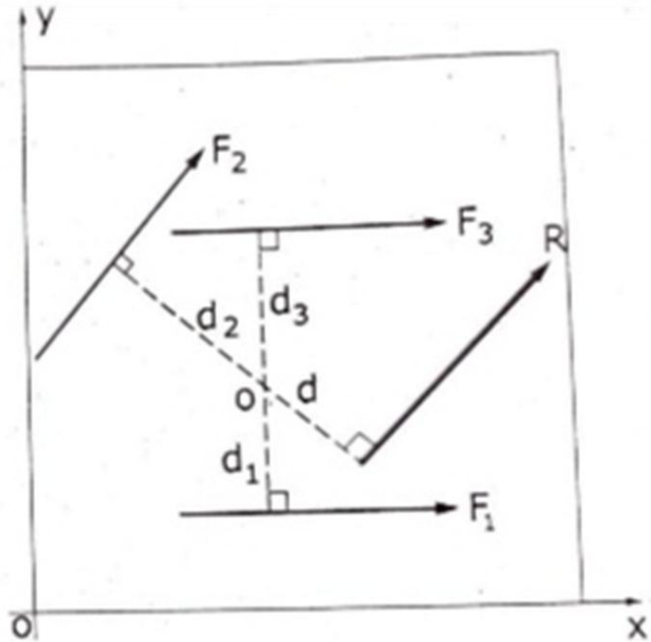
Moment of Horizontal forces

Varignons theorm

Varignon's theorem

The algebraic sum of the moments of any number of forces about any point in their plane is equal to the moment of their resultant about the same point. Varignon's theorem is also known as theorem of moments.

Consider a rigid body subjected to three coplanar forces F_1, F_2 and F_3 at perpendicular distances d_1, d_2 and d_3 from a point O. Let the resultant force R is at a distance 'd' from 'O'.



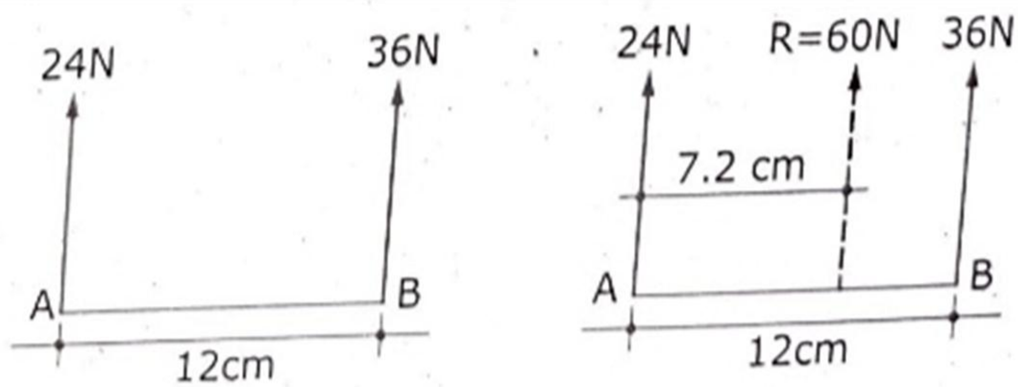
From Varignon's theorem,

Sum of the moments of the forces F_1, F_2 and F_3 about 'O' is equal to the moment of resultant force R, about the same point 'O'.

i.e., $F_1 d_1 + F_2 d_2 + F_3 d_3 = R.d.$

Sum of the moment of all the forces about a point = Moment of their resultant force about the same point. Varignon's theorem is used in locating the resultant force.

Find the resultant force for the parallel force system.



Solution.

Force system shown above is like parallel force system (as they act in same direction). Hence, resultant force will be in between the given forces. (whereas in unlike parallel force system, resultant force will be either in between the given forces or outside).

Using the sign convention, described in article.

Magnitude of Resultant force, $R = 24 + 36 = 60N$

Direction of Resultant force:- Upwards. (As 'R' is positive)

(In like parallel force system, direction of Resultant force will be same as the direction of the given forces).

Location of Resultant force

To find the Location of Resultant force, i.e., point of application of Resultant force, Varignon's theorem is applied.

Let us locate the Resultant force with reference to the point A. Hence, we will find the sum of the moments of two forces $24N$ and $36N$ about the point A.

Algebraic sum of the moments about A,

$$\Sigma M_A = (24 \times 0) - (36 \times 12) \qquad (-\text{ sign for anticlock wise moment})$$

$$= -432\ Ncm.$$

ΣM_A is negative measure, that is an anticlockwise moment. Hence the Resultant force should also produce the anticlockwise moment about A. To have anticlockwise moment by resultant force and also to act upwards, resultant force should be taken on the right side of A. (If it is taken upward, on the left hand side of A, then it will produce clockwise moment).

Equating the moment of Resultant force and the algebraic sum of the moments of the given forces,

Moment of Resultant force about $A = \Sigma M_A$

i.e., $R \times x = 432\ Ncm$

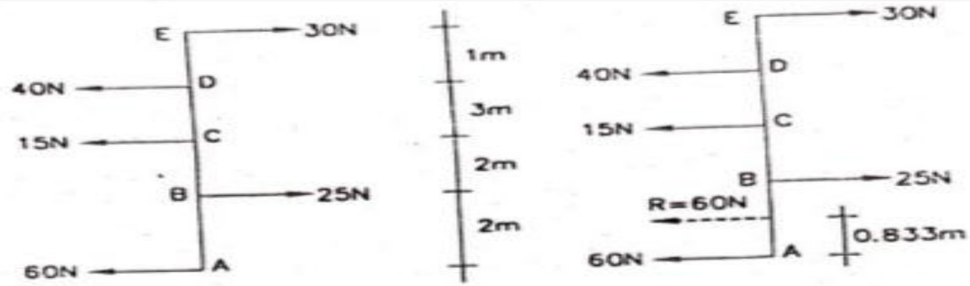
where x is the distance of Resultant force from the point A.

or $60 \times x = 432$

$$\therefore \quad x = \frac{432}{60} = 7.2cm$$

$\therefore$ The Resultant force for the given like parallel forces is $60N$, acting upwards, on t right hand side of A, at a distance of $7.2cm$.

For the parallel forces in diagram,determine the magnitude ,direction and location of the resultant force.



Solution

Force system given in the problem is unlike parallel force system.

Magnitude of Resultant force, $R = 30 - 40 - 15 + 25 - 60$

$= -60N$

(Force acting towards right is positive and the force acting towards left is negative)

(–) sign of Resultant force shows that, it is acting in the negative direction (i.e., towards left)

Let us find the location of Resultant force with reference to the point A.

$$\Sigma M_A = (30 \times 8) - (40 \times 7) - (15 \times 4) + (25 \times 2) + (60 \times 0)$$

$$= 240 - 280 - 60 + 50 = -50Nm$$

(–) sign shows that, the net moment is an anticlockwise one. Hence Resultant force should also produce anticlockwise moment about A. Satisfying 'R' is left hand side force and producing anticlockwise moment about A, it should be taken above the point A.

Let 'x' is the perpendicular distance of Resultant force from the point A.

$$\therefore R \times x = 50$$

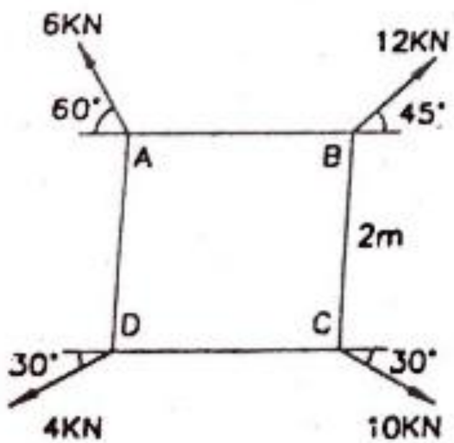
$$\text{i.e., } 60 \times x = 50$$

$$\therefore x = \frac{50}{60} = 0.833m$$

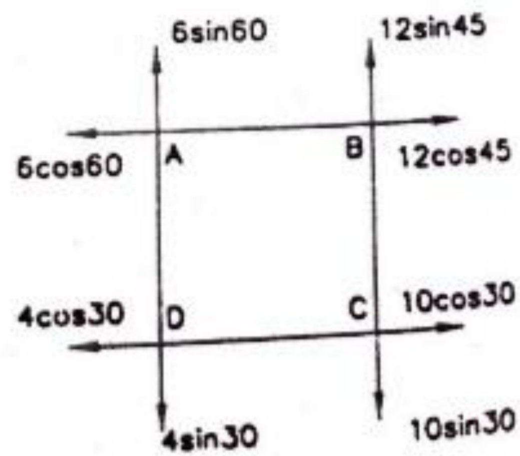
Resultant force is shown in dotted line

Four forces of magnitude and direction acting on a square ABCD of side 2m are in diagram, calculate the resultant in magnitude and direction and also locate its point of application with respect to the sides AB and AD.

Four forces of magnitude and direction acting on a square ABCD of side 2m are shown in fig Calculate the resultant in magnitude and direction and also locate its point of application with respect to the sides AB and AD.



(a)



(b)

Solution:

The given inclined forces are resolved into horizontal and vertical components

Now, Algebraic sum of Horizontal forces,

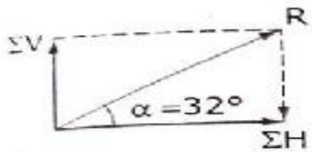
$$(\rightarrow +)\quad \Sigma H = 12\cos 45 + 10\cos 30 - 6\cos 60 - 4\cos 30$$
$$= 8.485 + 8.66 - 3 - 3.464 = 10.681 KN$$

Algebraic sum of vertical forces,

$$(\uparrow +)\quad \Sigma V = 12\sin 45 - 10\sin 30 + 6\sin 60 - 4\sin 30$$
$$= 8.485 - 5 + 5.196 - 2 = 6.681 KN$$

Magnitude of Resultant force, $R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(10.681)^2 + (6.681)^2}$

$$= \sqrt{158.71}$$
$$R = 12.598 KN$$



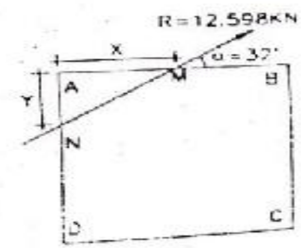
Direction of Resultant force, $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{6.681}{10.681}\right)$

$$= 32°.$$

Location of Resultant force.

In the problem, Location of resultant force with respect to the sides AB and AD is required. Hence, find the algebraic sum of moments of all the given forces about the point A.

$$\therefore \Sigma M_A = (4\cos 30° \times 2) + (10\sin 30° \times 2) - (10\cos 30° \times 2) - (12\sin 45° \times 2)$$
$$= 6.928 + 10 - 17.32 - 16.97$$
$$= -17.36\ KNm \text{ (anticlock wise).}$$



(moment of all other forces about A are zero).

Hence, to have anticlockwise moment by the resultant force, R is to be taken on the right hand side of A as shown ... which will cut the sides AB and AD.

Now, we will find the point of application of Resultant force with respect to the sides AB and AD at M and N. Let the Resultant force cuts at distances x and y from A on AB and AD respectively.

Location of Resultant force w.r.t. AB. (ie, finding x value)

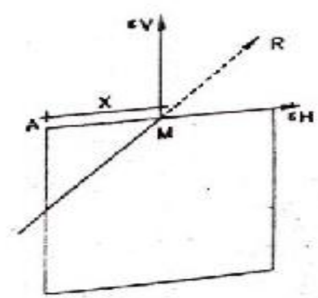
Resolve the resultant force into two components ΣH and ΣV at M as shown in .

Now, ΣM_A = sum of the moments of ΣH & ΣV

or $\Sigma M_A = \Sigma V \times x$

(as ΣH moment about A is zero)

$$\therefore 17.36 = 6.681 \times x$$



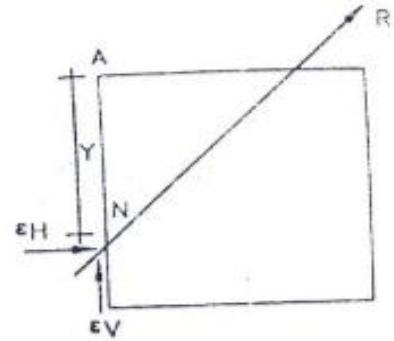
$$\therefore x = \frac{17.36}{6.681}$$

$$= 2.598m$$

But the side AB is $2m$ only. So, the Resultant force cuts the side AB extended at a distance of $2.598m$ from A.

Location of Resultant force w.r.t. *AD*. (ie, finding y value)

Resolve the resultant force into two components ΣH and ΣV at N as shown in .



Now, ΣM_A = sum of the moments of ΣH and ΣV.

or $\Sigma M_A = \Sigma H \times y$

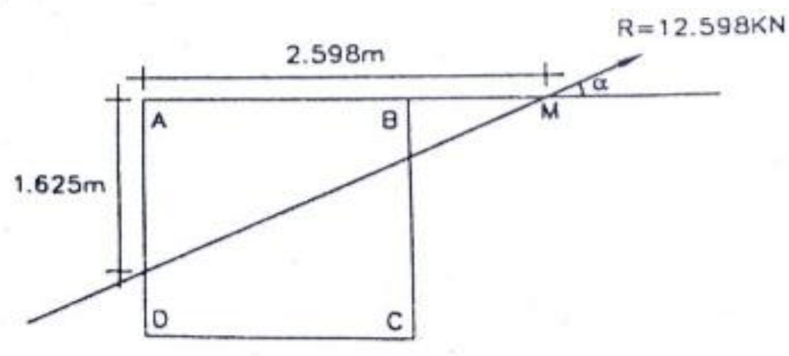
(as moment of ΣV about A is zero)

or $17.36 = 10.681 \times y$

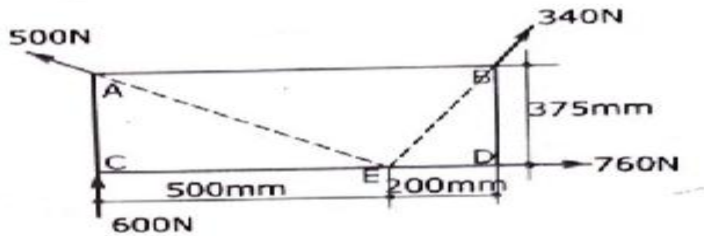
$$\therefore y = \frac{17.36}{10.681} = 1.625m$$

Resultant force cuts the side AD at N, at a distance of $1.625m$ from A.

The Resultant force is shown below



Four forces act on a 700mm plate as I diagram (a) Find the resultant of these forces (b) Locate the two points where the line of action of the resultant intersects the edge of the plate`



Solution :

Let $F_1 = 340N$; $F_2 = 760N$; $F_3 = 600N$; $F_4 = 500N$

F_2 is a horizontal force; hence $\theta = 0$.

F_3 is a vertical force; hence $\theta = 90°$

but, angle of inclination of the forces F_1 and F_4 are not directly given. Let the angle of inclinations of F_1 (340N) and F_4 (500N) forces with horizontal be θ_1 and θ_4 respectively.

To find θ_1.

In a right angled triangle *EDB*,

$$\angle AEC = \theta_1 = \tan^{-1}\left(\frac{375}{200}\right) = 61.92°$$

To find θ_4 : In a right angled triangle *ECA*,

$$\angle BED = \theta_4 = \tan^{-1}\left(\frac{375}{500}\right) = 36.86°$$

Resolved components of forces are shown in fig. 5.22.

Resultant Force :

Algebraic sum of horizontal forces,

$$(\rightarrow +) \quad \Sigma H = 340\cos 61.92 + 760 - 500\cos 36.86$$
$$= 160 + 760 - 400 = 520N$$

Algebraic sum of vertical forces,

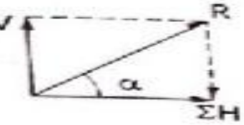
$$(\uparrow +) \quad \Sigma V = 340\sin 61.92 + 600 + 500\sin 36.86$$
$$= 300 + 600 + 300 = 1200N$$

$\therefore$ Magnitude of Resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$= \sqrt{(520)^2 + (1200)^2} = 1307.82N$$

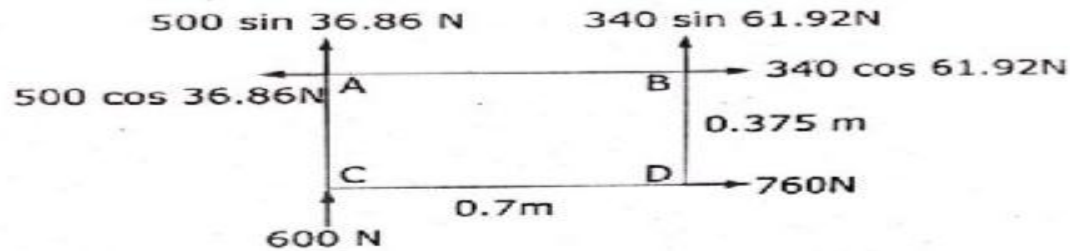
Direction of Resultant force,

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{1200}{520}\right) = 66.57^\circ$$



Location of Resultant force

We will locate the resultant force with reference to the point C.



Algebraic sum of moments of all forces about C,

$$\Sigma M_C = (340 \cos 61.92 \times 0.375) - (340 \sin 61.92 \times 0.7) - (500 \cos 36.86 \times 0.375)$$

$$= 60 - 210 - 150 = -300Nm$$

Negative sign shows that, sum of moments of all forces at C is anticlockwise. Hence resultant force should also produce an anticlockwise moment at C. Knowing the direction o resultant force to have anticlockwise moment, It should be taken below the point C

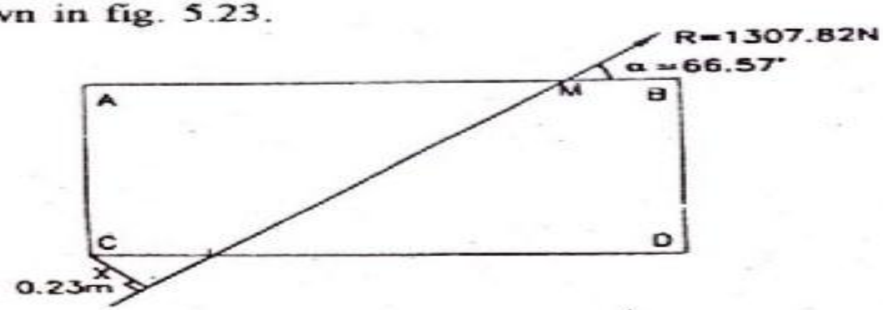
Let the perpendicular distance of resultant force from point C be 'x'm. We know,

sum of moments at C = moment of Resultant at C

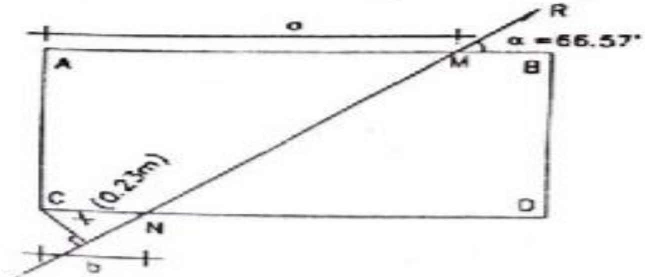
i.e., $\Sigma M_C = R \times x$

$$\therefore x = \frac{\Sigma M_C}{R} = \frac{300}{1307.82} = 0.23m$$

Resultant force is shown in fig. 5.23.



(b) Interception of Resultant force with the edges



Let the line of action of Resultant force intercepts the edges AB and CD at M an respectively as shown in

Let $AM = a$ and $CN = b$

To find 'b' (i.e., CN Distance)

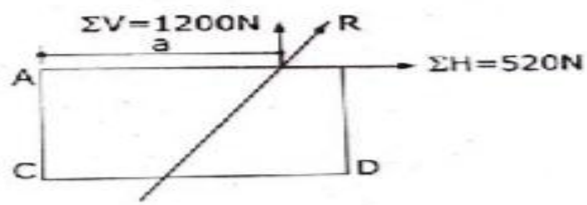
In right angled triangle CON,

$\angle CON = 90°$; $\angle CNO = 66.57°$; $CO = x = 0.23m$

$$\therefore \sin 66.57 = \frac{CO}{CN} = \frac{0.23}{b}$$

$$\therefore b = \frac{0.23}{\sin 66.57} = 0.25m \text{ or } 250mm.$$

To find 'a' (i.e., AM Distance).



Resolve the Resultant force into two components ΣH and ΣV as shown i know, sum of the moments at C, $\Sigma M_C = -300Nm$. (anticlockwise). Therefore, moment resultant force about C should also be $300Nm$. i.e., sum of the moments of its compone (ΣH and ΣV) about C is also equal to $300Nm$.

i.e., $-300 = (\Sigma H \times AC) - (\Sigma V \times a)$ (– sign due to anticlockwise)

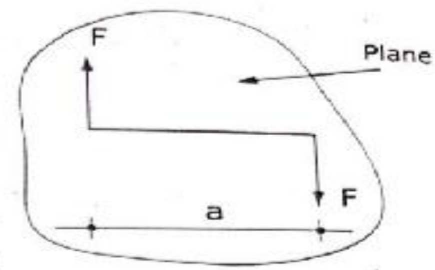
or $-300 = (520 \times 0.375) - (1200 \times a)$

Solving, we get,

$$a = 0.412m \text{ or } 412mm$$

couple

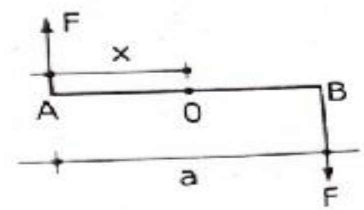
"Two forces F and $-F$ having the same magnitude, parallel lines of action and opposite sense are said to form a couple"



When two equal and opposite parallel forces act on a body, at some distance apart, the two forces form a couple. Couple has a tendency to rotate the body. The perpendicular distance between the parallel forces is called arm of the couple.

shows two parallel forces of magnitude F, but acts in opposite direction at a distance 'a'. These two forces will form a couple. This couple will have a tendency to rotate the body in clockwise direction.

In a couple sum of the moments of two forces about any po·· (either in between the forces or outside) is same.



To prove this, let us locate a point O in a couple, at a distance of x from A. There are two unlike parallel forces acting at A and B, with the arm of couple 'a'.

Hence, $OA = x$ and $OB = (a - x)$.

Sum of the moments at A:

Let us find the sum of the moments of two parallel forces about the point A.

$\Sigma M_A = (F \times a)$ (ie., Moment due to the force at B)

$= Fa$ (clockwise)

Sum of the moments at B:

Similarly, the sum of the moments of two parallel forces about the point B.

$\Sigma M_B = (F \times a)$ (ie., Moment due to the force at A)

$= Fa$ (clockwise)

Sum of the moments at 'O':

Now, let us find the sum of the moments of two parallel forces about O,

$$\Sigma M_0 = (F \times x) + F(a - x)$$
$$= (F \times x) + (F \times a) - (F \times x)$$
$$= Fa \text{ (clockwise)}$$

Therefore, **the sum of the moments of couple forces about any point is same in magnitude and nature.**

Hence, we may state that, the moment of a couple is the product of any one of the parallel forces and perpendicular distance between the forces.

Moment of a couple = Force × arm of the couple

M = F × a

Difference between Moment and Couple

The couple is a pure turning effect which may be moved anywhere in its own plane or into a parallel plane without change of its effect on the body, but the moment of a force must include a description of the reference axis about which the moment is taken.

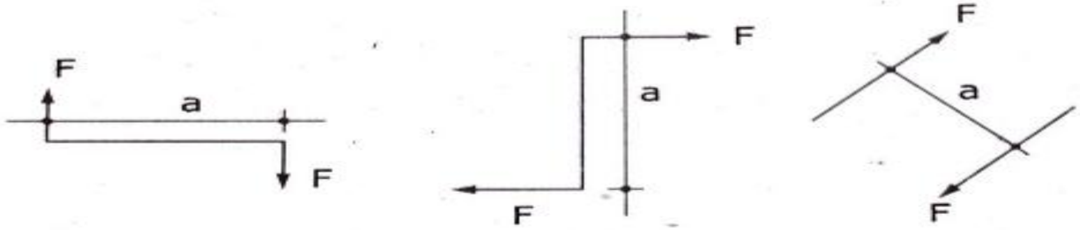
Types of couples

Types of couple.

Couple is classified into two types based on its nature.

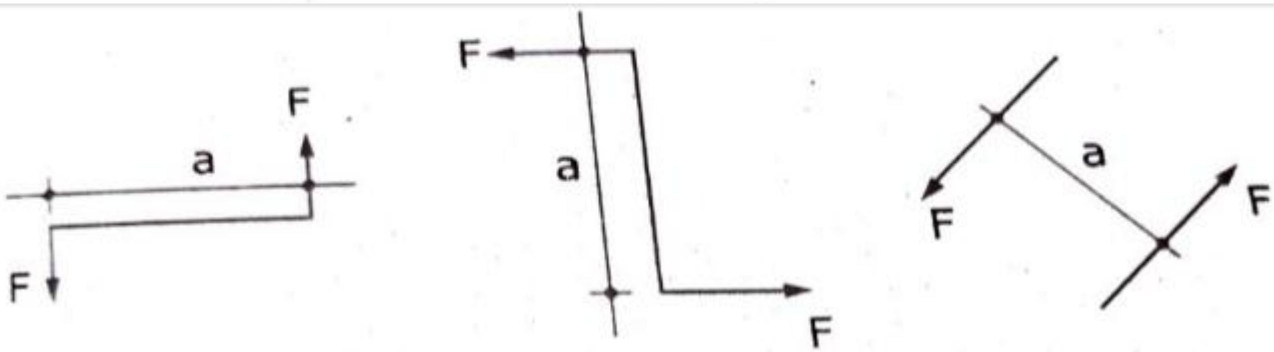
(i) Clockwise couple (ii) Anticlockwise couple

Clockwise couple acting on a body will have a tendency to rotate the body in clockwise direction, similarly anticlockwise couple will have a tendency to rotate a body in anticlockwise direction.



(i) Vertical (ii) Horizontal (iii) inclined

Clockwise Couple

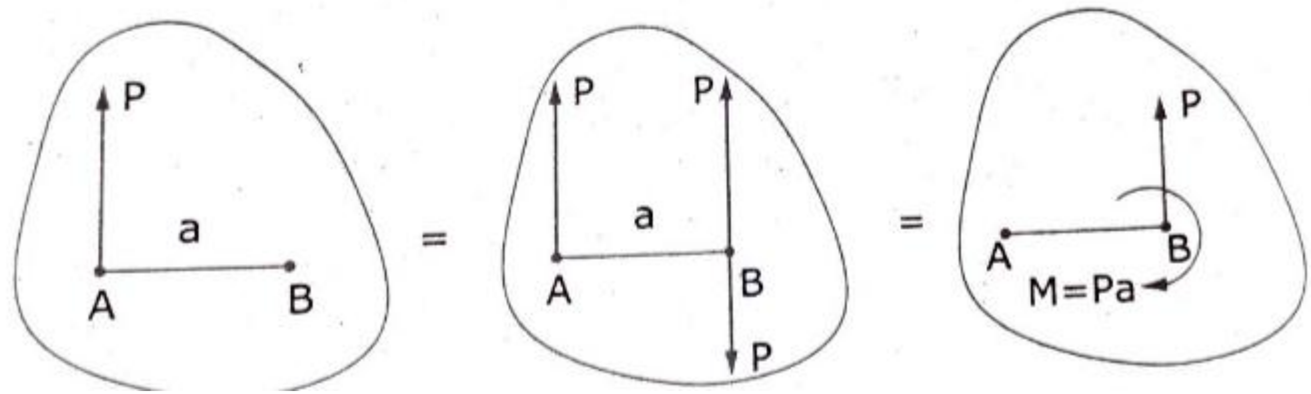


(i) Vertical (ii) Horizontal (iii) inclined

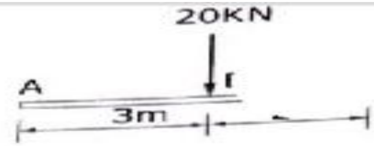
Anticlockwise Couple

Force-couple system

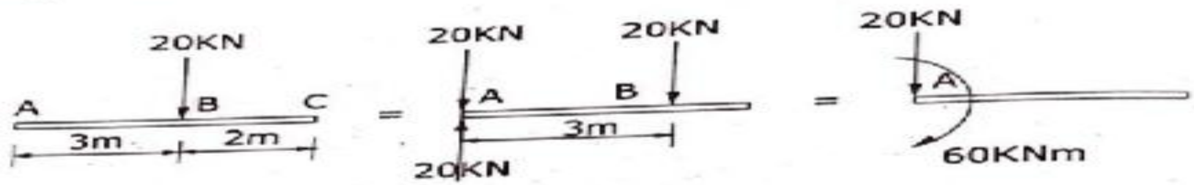
Resolution of a Force into a Force and a couple at a point (Force-Couple System).



Example



Force couple system at A

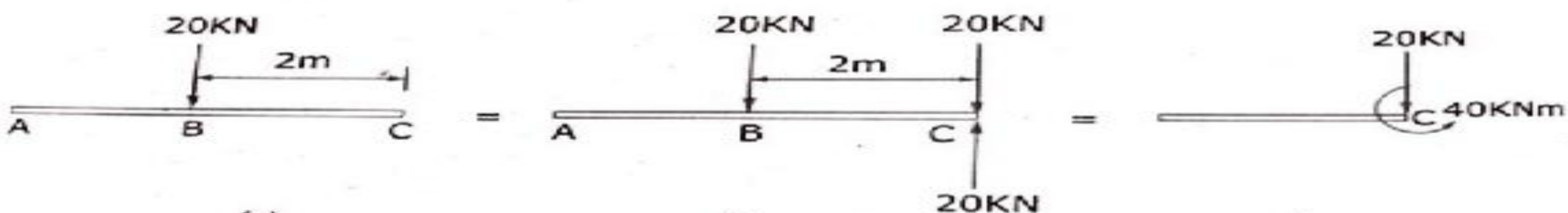


(a) (b) (c)

Force-couple system at A

The given force 20 KN magnitude acting at B, on the rod AC, at a distance of $3m$ from A is shown in fig. Now it is to be replaced by a Force-couple at A. For this, apply two equal and opposite collinear forces at A parallel to the given force. Now, 20 KN downward force at B, and 20 KN upward force at A form a couple. The moment of this couple =(20 × 3) = 60 KN-m (clockwise), hence these two forces can be replaced by a moment 60 KN-m at A. The third force, i.e, 20 KN downward force remains at A; it is unchanged. Hence, the final force 20 KN and couple 60 KN-m at A is shown in

Force-Couple system at C:

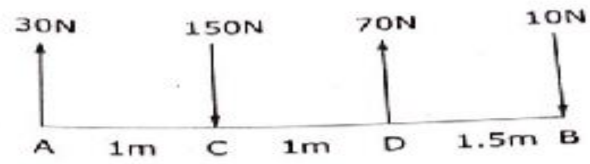


(a) (b) (c)

Force couple system at C

The given force is shown in . To replace this force by an equivalent force-couple at C, apply two equal and opposite collinear forces at C, Parallel to the given force and of same magnitude (i.e. 20 KN). Now 20 KN downward force at B, and 20 KN upward force at C form a couple. The moment of this couple about $C = 20 \times 2 = 40$ KNm (anticlockwise). The third force, i.e. 20 KN downward force remains at C. It is unchanged. (C) shows the Force couple system at C.

A system of parallel forces are acting on rigid bar in diagram . Reduce the system to (a) single force (b) a single force and a couple at A (c) a single force and a couple at B



Solution

(i) Single force system

The single force system will consist only Resultant force. The given system of force is, unlike parallel vertical forces. (Coplanar)

Magnitude of resultant force, $(\uparrow +)\ R = 30 - 150 + 70 - 10$

$= -60\ N$

Direction of resultant force: Downwards (as R is negative)

As all the forces are vertical, ΣH will be zero ; hence Resultant force is also a vertical force, parallel to the given forces.

Location of Resultant force

Let us locate the resultant force with respect to the point A. Let, the perpendicular distance of resultant from A is 'x'm. We know,

Sum of moments of all the given forces about A, $(\Sigma M_A) = (R \times x)$

Now, algebraic sum of moments of forces about A,

$$\Sigma M_A = (30 \times 0) + (150 \times 1) - (70 \times 2) + (10 \times 3.5)$$

$$= 150 - 140 + 35$$

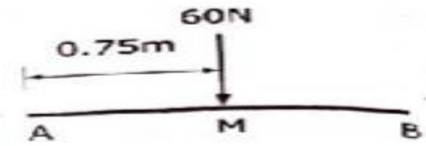
$$= 45 \text{ KNm (Clockwise)}$$

Hence, resultant force also produces clockwise moment about A, We have already seen that resultant force is acting vertically downwards. So, to produce clockwise moment an acting downwards it will be on the right side of A.

now, using $\Sigma M_A = R \times x$

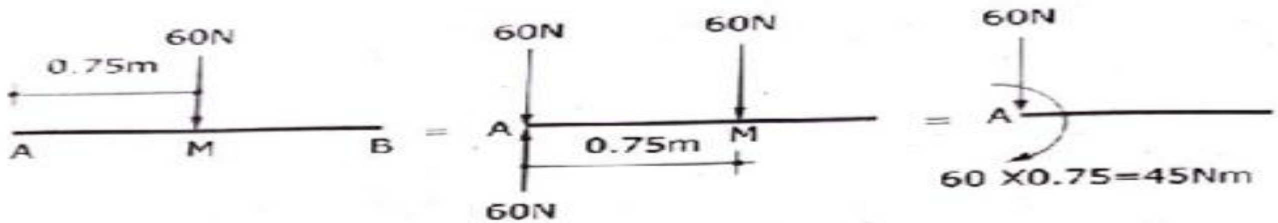
$$\text{or} \quad x = \frac{\Sigma M_A}{R} = \frac{45}{60} = 0.75\text{m}$$

Hence, the given system of parallel forces is equivalent to a single force 60N acting vertically downward at a distance of 0.75m from *A* on its right hand side as shown in ,



(ii) A single force and a couple at *A*

The resultant force of the given system of forces is again shown in Let the Resultant force cuts the rod *AB* at *M* (0.75m from *A*) Now to reduce this resultant force into a force-couple system at *A*, introduce two unlike collinear forces at *A*, parallel to the resultant force and of same magnitude (60N) as shown in fig.

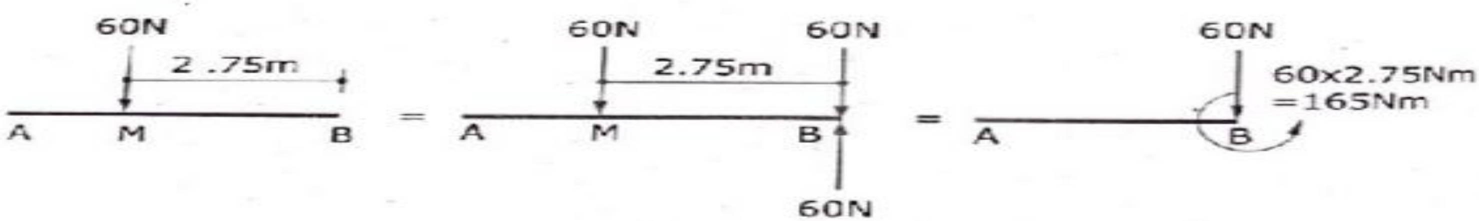


(a) (b) (c)

Force-couple at A

Now, the downward 60N force at *M* and upward 60N force at *A* form a clockwise couple. The moment of this couple at *A* is (60 × 0.75) = 45Nm and the third force at *A* (60N downward) is unchanged. :) shows the force-couple system at *A*.

(iii) A single force and a couple at *B*



(a) (b) (c)

Force-couple at B

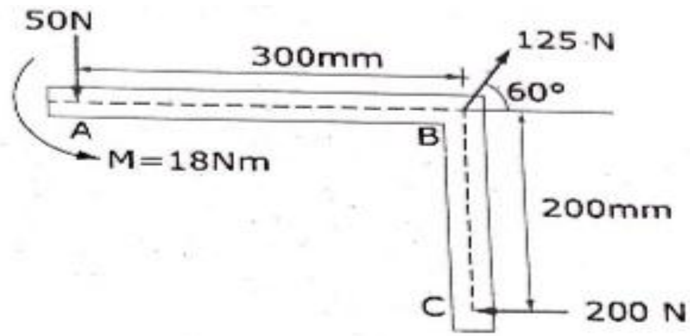
The resultant force of the given system of forces is again shown i but the distance of resultant force is measured from *B*. The distance $MB = AB - AM$

$= 3.5 - 0.75 = 2.75M$

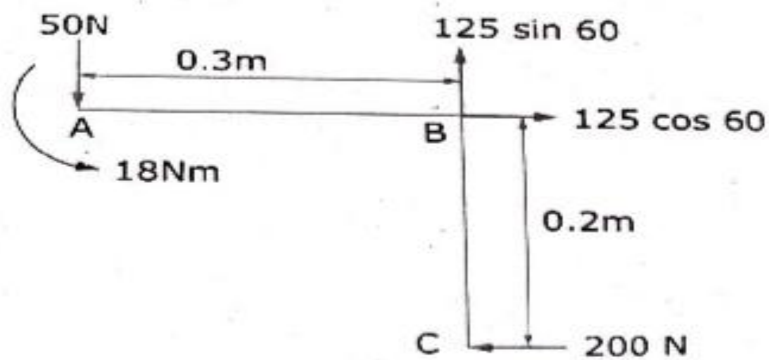
Now, to reduce this resultance force into a force-couple system at *B*, introduce two unlike collinear forces at *B*, parallel to the resultant force and of same magnitude (60N). as shown

Now, the downward 60N force at M and upward 60N force at B form an a...clockwise couple. The moment of this couple is $(60 \times 2.75) = 165$ Nm, and the third force at B (60N downward) is unchanged. ...hows the force-couple system at B.

The three forces and a couple of magnitude ,M=18Nm are applied to an angled bracket as shown (a) find the resultant of system of forces (b) locate the points where the line of action of the resultant intersects line AB and line AC



Solution :



The given force system is again drawn in fig. with the components of the inclined force, 125N.

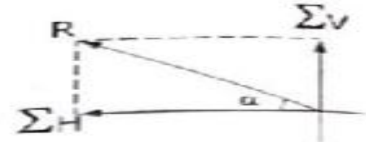
(i) **Resultant force:**

Algebraic sum of Horizontal forces,

$$\Sigma H = 125 \cos 60 - 200 = -137.5\ N.$$

Algebraic sum of vertical forces,

$$\Sigma V = 125 \sin 60 - 50 = 58.25 \text{ N}$$



Magnitude of resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(137.5)^2 + (58.25)^2}$$

$$= 149.32\ N$$

Direction of resultant force, $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{58.25}{137.5}\right)$

$$= 22.95°$$

Whatever be the magnitude and nature of couple *M*, it will not affect the magnitude and direction of resultant force.

Location of resultant force :

Let us locate the resultant force with respect to the point *A*. Hence, we will find th algebraic sum of moments about *A*.

$$\Sigma M_A = (200 \times 0.2) - (125 \sin 60 \times 0.3) - 18$$

$$= 40 - 32.475 - 18$$

$= -10.475\ Nm$ Negative sign shows anticlockwise moment

1) **In ΣM_A equation −18 refers to the anticlockwise couple at *A*; and also the moments of 50N and 125 cos 60 *N* forces about *A* are zero.**

2) **ΣM_A is Negative, hence resultant force should also produce an anticlockwise moment about *A*. To have the anticlockwise moment of resultant force and to act in direction as shown above, resultant force should be taken on the right hand side of *A*.**

Let the perpendicular distance of resultant force from *A* be '*x*' *m*.

Applying Varignon's theorem,

i.e., $\Sigma M_A = R \times x$

$$\therefore\ x = \frac{\Sigma M_A}{R} = \frac{10.475}{149.32} = 0.07 \text{ m} = 70 \text{ mm}$$