

1. Static And Variable Stresses

Design

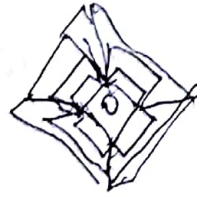
Machine element

Design of machine element

Factors influencing machine design

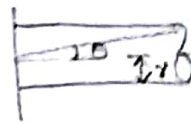
Procedures to design a machine element

Mechanical Properties of elements



Direct Tensile and Compression Stress:-

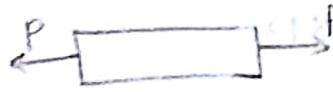
$$\frac{T}{J} = \frac{\tau}{r} = \frac{C\theta}{l}$$



Torsion

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$\sigma_t = \frac{P}{A}$$



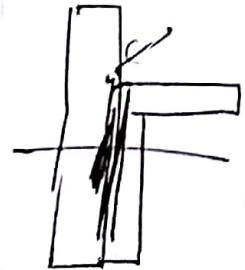
Tensile



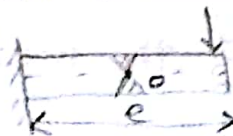
Direct Compressive load



$$\sigma_c = -\frac{P}{A}$$



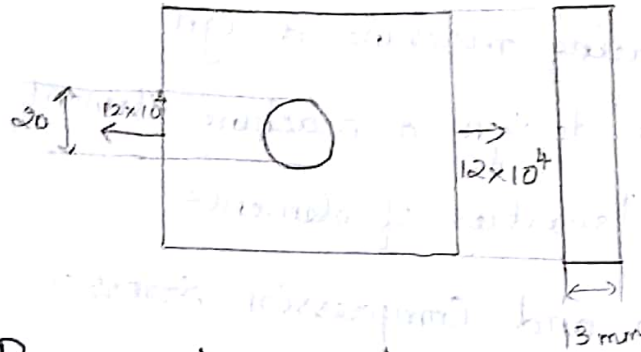
Direct Bending load



$$\sigma_{bt} = \frac{P \times e}{Z} = \frac{M}{Z}$$

$$\sigma_{bc} = -\frac{P \times e}{Z}$$

1. A tie bar carry a load of $12 \times 10^4 \text{ N}$. What must be the width of the bar 13 mm thick, if hole of 20 mm diameter on its centre. Working stress of the bar is 75 MPascal.



$$P = 12 \times 10^4 \text{ N}$$

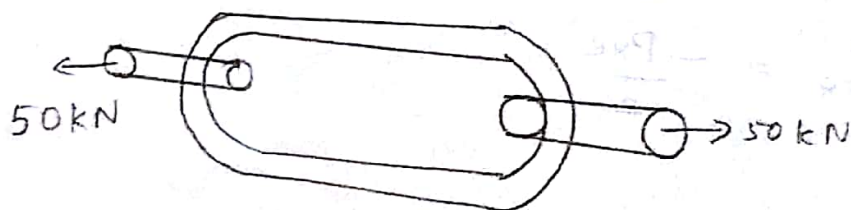
$$\begin{aligned} \sigma &= 75 \text{ MPa} \\ &= 75 \text{ N/mm}^2 \end{aligned}$$

$$\sigma = \frac{P}{A} = \frac{12 \times 10^4}{(W - 20) \times 13}$$

$$75 = \frac{12 \times 10^4}{(W - 20) \times 13}$$

$$W = 143.07 \text{ mm}$$

2. Find the diameter of line stock if the permissible tensile stress of the material is not to exceed 75 MPascal?



$$\sigma = P/A$$

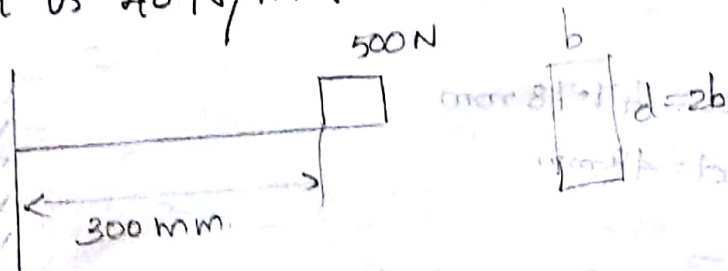
$$75 = \frac{50 \times 10^3}{\pi/4 \times d^2}$$

$$d^2 = \frac{50 \times 10^3 \times 4}{\pi \times 75}$$

$$d = 29.13 \text{ mm}$$

2/1/20 Bending Stress

1. An electric motor weighing 500 N is mounted on a sharp cantilever beam of uniform rectangular cross section. The weight of the motor acts at a distance of 300 mm from the support. The depth section is twice the width. Determine the cross section of beam - Allowable stress in the beam is 40 N/mm².



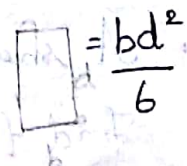
$$\sigma_b = M/z = \frac{P \times e}{z}$$

$$40 = \frac{500 \times 300}{\frac{b \times (2b)^2}{6}}$$

$$b^3 = \frac{500 \times 300 \times 6}{40 \times 4}$$

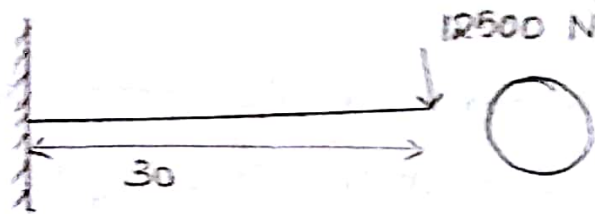
$$b = 17.78 = 18 \text{ mm}$$

PS G DB 6.1



$$\begin{aligned}
 d &= 2b \\
 &= 2 \times 18 \\
 &= 36 \text{ mm}
 \end{aligned}$$

2. A trunion of mixing machine has a effective length is 30 mm and weight which comes on the trunion is 12500 N. What should be the diameter of trunion stress not to exceed 35 N/mm^2



PGGDB 61

$$Z = \frac{\pi}{32} d^3$$

$$35 = \frac{12500 \times 30}{\frac{\pi}{32} d^3}$$

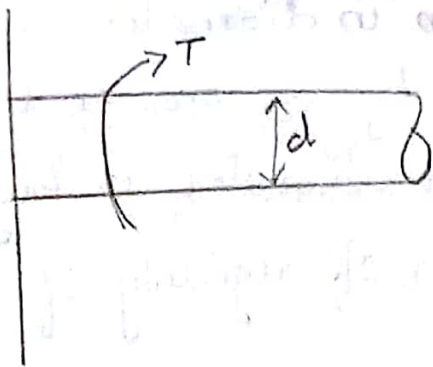
$$d^3 = \frac{12500 \times 30 \times 32}{35 \times \pi}$$

$$d = 47.78 \text{ mm}$$

$$d = 48 \text{ mm}$$

Problem on Shear Stress

1. A shaft transmitting 100 kW at 1600 rpm. Find the suitable diameter for shaft the maximum torque transmitted exceed mean by 25%. Take minimum allowable shear stress of 70 MPa.



$$P = 100 \times 10^3 \text{ W}$$

$$N = 1600 \text{ rpm}$$

$$\sigma = 70 \text{ MPas}$$

$$P = \frac{2\pi NT}{60}$$

$$100 \times 10^3 = \frac{2\pi \times 1600}{60} \times T$$

$$T = \frac{100 \times 10^3 \times 60}{2\pi \times 1600}$$

$$= 596.83 \text{ Nm} = 596.8 \times 10^3 \text{ N-mm}$$

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$T_{\text{max}} = T_{\text{mean}} \times 1.25$$

$$= 1.25 \times 596.8 \times 10^3$$

$$= 746 \times 10^3$$

$$746 \times 10^3 = \frac{\pi}{16} \times 70 \times d^3$$

$$d = 38 \text{ mm}$$

1, 2, 9, 14, 15, 18

26, 22, 23, 32, 35

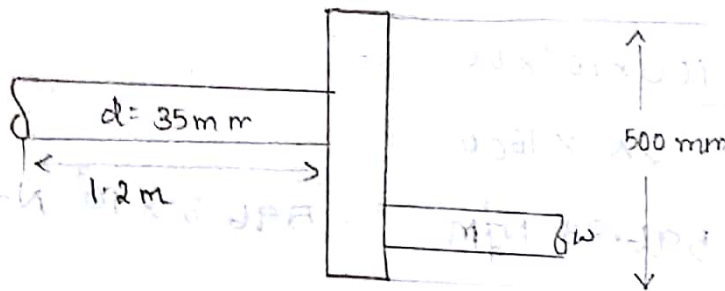
37, 51, 54, 50, 502

504, 505, 500

Q. A steel shaft $\phi 35$ mm in diameter and 1.2 m long held rigidly at one end as a handwheel 500 mm in diameter is keyed to other end modulus of rigidity of steel is 80 GPa.

Case i) What load applied tangent to the rim of the wheel produce a torsional shear of 60 MPa.

Case ii) How many degree will wheel turn when load is applied.



$$T = W \cdot R$$

$$= W \cdot 250 \cdot \text{N-mm}$$

$$J = \frac{\pi}{32} d^4$$

$$= \frac{\pi}{32} \times 35^4$$

$$= 147.323 \times 10^3 \text{ mm}^4$$

$$\frac{T}{J} = \frac{60}{80000} \times \frac{T}{147.323 \times 10^3}$$

$$\frac{250 \text{ W}}{147.323 \times 10^3} = \frac{60}{17.5}$$

$$W = 2020 \text{ N}$$

$$T = 250 \times 2020 \\ = 505.109 \times 10^3 \text{ N-mm}$$

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\frac{505.109 \times 10^3}{147.323 \times 10^3} = \frac{80 \times 10^3 \times \theta}{1200}$$

$$\theta = 0.05^\circ$$

1. A shaft transmitting 97.5 kW at 1800 rpm if allowable shear stress in the material is 60 MPa. Find the suitable dia of the shaft if the shaft is not to twist more than 1° if the length of 3m. Take $C = 80 \text{ GPa}$.

Soln: -

$$P = 97.5 \times 10^3 \text{ W}$$

$$N = 1800 \text{ rpm}$$

$$\tau = 60 \text{ MPa}$$

$$\theta = 1^\circ$$

$$l = 3000 \text{ mm}$$

$$C = 80 \text{ GPa}$$

$$P = \frac{2\pi NT}{60}$$

$$97.5 \times 10^3 = \frac{2\pi \times 1800 \times T}{60}$$

$$T = 517.25 \times 10^3 \text{ N-mm}$$

$$J = \frac{\pi d^4}{32}$$

$$= \frac{\pi d^4}{32}$$

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\frac{517.25 \times 10^3}{\frac{\pi}{32} d^4} = \frac{80 \times 10^3 \times 0.0174}{3000}$$

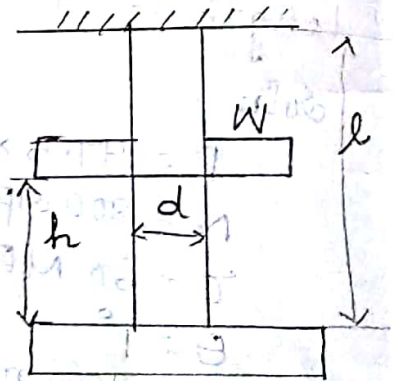
$$d^4 = \frac{517.25 \times 10^3 \times 32 \times 3000}{80 \times 10^3 \times 0.0174 \times \pi}$$

$$d = 59 \text{ mm}$$

Impact stress

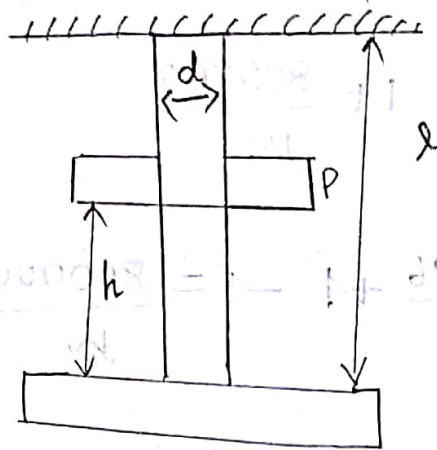
The machine element subjected to load with impact, the stress produced in the member due to falling load is known as impact stress.

$$\sigma_i = \frac{Wl}{A} \left[1 + \sqrt{1 + \frac{2hAE}{Wl}} \right]$$



1. A unknown weight falls through 10 mm on the collar rigidly attached to lower end of vertical bar 3m length and 600 mm^2 in cross section. The maximum extension is to be 2mm

What is the corresponding stress value of unknown weight. $E = 200 \text{ kN/mm}^2$.



$$\begin{aligned}
 h &= 10 \text{ mm} \\
 l &= 3000 \text{ mm} \\
 A &= 600 \text{ mm}^2 \\
 \delta l &= 2 \text{ mm} \\
 E &= 200 \times 10^3 \text{ N/mm}^2
 \end{aligned}$$

$$\sigma_i = \frac{W}{A} \left[1 + \sqrt{1 + \frac{2hAE}{Wl}} \right]$$

$$e = \frac{\delta l}{l} = \frac{2}{3000} = 6.6 \times 10^{-4}$$

$$\begin{aligned}
 \sigma_i &= E \times e \\
 &= 200 \times 10^3 \times 6.6 \times 10^{-4}
 \end{aligned}$$

$$\sigma_i = 133.33$$

$$133.33 = \frac{W}{600} \left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}} \right]$$

$$79998 = W \left[1 + \sqrt{1 + \frac{800000}{W}} \right]$$

$$\frac{79998}{W} = 1 + \sqrt{1 + \frac{800000}{W}}$$

$$\frac{79998}{W} - 1 = \sqrt{\frac{800000}{W} + 1}$$

$$\left(\frac{79998}{W} - 1\right)^2 = 1 + \frac{800000}{W}$$

$$\frac{6399 \times 10^6}{W^2} - \frac{159996}{W} + 1 = 1 + \frac{800000}{W}$$

$$\frac{6399 \times 10^6}{W^2} = \frac{800000}{W} + \frac{159996}{W}$$

$$\frac{6399 \times 10^6}{W^2} = \frac{959996}{W}$$

$$W = 6665.6 \text{ N}$$

1. Unknown weight falls from 15 mm on to the collar rigidly attached to lower end of the vertical bar 2.5 m long and 500 mm² is section. The maximum instantaneous extension is to be 2 mm. Find the corresponding stress and the value of weight falling. Assume $E = 2 \times 10^5 \text{ N/mm}^2$.

$$h = 15 \text{ mm}$$

$$l = 2.5 \text{ m} = 2500 \text{ mm}$$

$$A = 500 \text{ mm}^2$$

$$\delta l = 2 \text{ mm}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\sigma_i = \frac{W}{A} \left[1 + \sqrt{1 + \frac{2hAE}{Wl}} \right]$$

$$e = \frac{\delta l}{l} = \frac{2}{2500} = 8 \times 10^{-4}$$

$$\begin{aligned} \sigma_i &= E \times e \\ &= 2 \times 10^5 \times 8 \times 10^{-4} \\ &= 160 \text{ N/mm} \end{aligned}$$

$$160 = \frac{W}{500} \left[1 + \sqrt{1 + \frac{2 \times 15 \times 500 \times 2 \times 10^5}{W \times 2500}} \right]$$

$$80000 = W \left[1 + \sqrt{1 + \frac{1200000}{W}} \right]$$

$$\frac{80000}{W} = 1 + \sqrt{1 + \frac{1200000}{W}}$$

$$\left[\frac{80000}{W} - 1 \right]^2 = 1 + \frac{1200000}{W}$$

$$\frac{64 \times 10^8}{W^2} - \frac{160000}{W} + 1 - 1 = \frac{1200000}{W}$$

$$\frac{64 \times 10^8}{W^2} = \frac{1360000}{W}$$

$$W = 4705.8 \text{ N}$$

$$\left[\frac{80000}{W} + 1 \right]^2 = 1 + \frac{1200000}{W}$$

$$\left[\frac{80000}{W} + 1 \right]^2 = 1 + \frac{1200000}{W}$$

$$\left[\frac{80000}{W} + 1 \right]^2 = 1 + \frac{1200000}{W}$$

Principle Stress & Principle Plane

We already discussed the direct tensile and compressive stress as well as shear and also have referred the stress in the plane which is at right angle to the line of action of force. But it has been observed that any point in the strained materials there are 3 planes mutually perpendicular to each other which carry direct stress and has no shear stress. It may be noted that out of three stresses one will be maximum, other is minimum. The perpendicular plane which has no shear stress are known as principle plane and direct stress along this plane is known as principle stress.

$$\sigma = \sigma_t + \sigma_b.$$

$$\sigma_t = P/A$$

$$\sigma_b = My/z$$

$$\tau = 16T/\pi d^3.$$

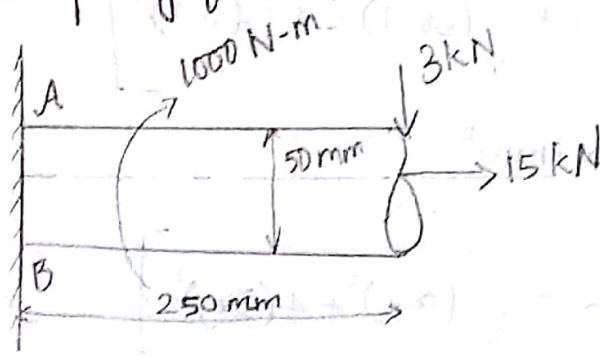
σ_1 - maximum principle stress

$$\sigma_1 = \frac{1}{2} \left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau_{xy})^2} \right]$$

$$\sigma_2 = \frac{1}{2} \left[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau_{xy})^2} \right]$$

$$\tau = \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau_{xy})^2}$$

1. A shaft is shown in figure is subjected to bending load of 3kN and pure torque of 1000Nm Axial pulley force of 15kN



$$\begin{aligned}\sigma_{tA} &= P/A \\ &= \frac{15 \times 10^3}{\frac{\pi}{4} \times (50)^2} \\ &= 7.63 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\sigma_{bA} &= M/Z \\ &= \frac{3 \times 10^3 \times 250}{\frac{\pi}{32} (50)^3} \\ &= 61.1 \text{ N/mm}^2\end{aligned}$$

$$T = \frac{\pi}{16} \tau \times d^3$$

$$\tau_{xy} = \frac{16T}{\pi d^3}$$

$$= \frac{16 \times 1000 \times 10^3}{\pi \times (50)^3}$$

$$= 40.7 \text{ N/mm}^2$$

$$\begin{aligned}\sigma_{xA} &= \sigma_t + \sigma_b \\ &= 68.73 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\sigma_{\max A} &= \frac{1}{2} \left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \right] \\ &= \frac{1}{2} \left[68.7 + \sqrt{(68.7)^2 + 4(40.7)^2} \right] \\ &= 87.60 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\sigma_{\min A} &= \frac{1}{2} \left[\sigma_x - \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \right] \\ &= \frac{1}{2} \left[68.7 - \sqrt{(68.7)^2 + 4(40.7)^2} \right] \\ &= -18.9 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\tau_{\max(A)} &= \frac{1}{2} \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \\ &= \frac{1}{2} \sqrt{(68.7)^2 + 4(40.7)^2} \\ &= 53.29 \text{ N/mm}^2\end{aligned}$$

At point B.

$$\sigma_{tB} = 7.63 \text{ N/mm}^2$$

$$\sigma_{cB} = -61.1 \text{ N/mm}^2$$

$$\tau_{xy}(B) = 40.7 \text{ N/mm}^2$$

$$\begin{aligned}\sigma_x B &= 7.63 - 61.1 \\ &= -53.47 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\sigma_{\max B} &= \frac{1}{2} \left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \right] \\ &= \frac{1}{2} \left[-53.47 + \sqrt{(-53.47)^2 + 4(40.7)^2} \right]\end{aligned}$$

$$\sigma_{\max B} = 21.96 \text{ N/mm}^2$$

$$\sigma_{\min B} = \frac{1}{2} \left[-53.47 - \sqrt{(-53.47)^2 + 4(40.7)^2} \right]$$

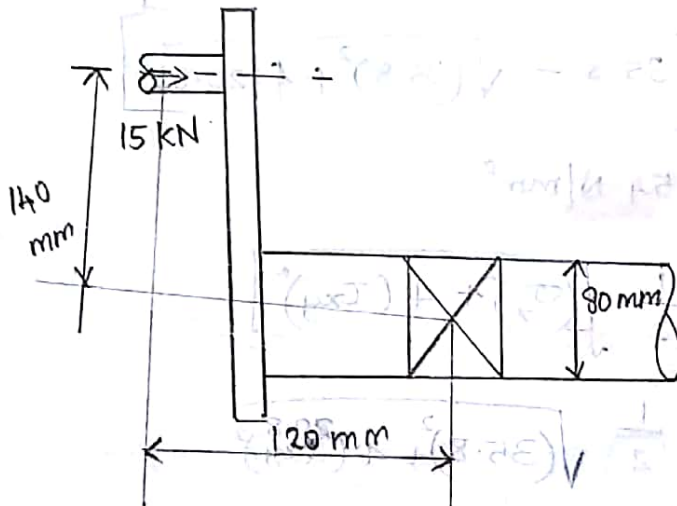
$$= -75.4 \text{ N/mm}^2$$

$$\tau_{\max (B)} = \frac{1}{2} \sqrt{(-53.47)^2 + 4(40.7)^2}$$

$$= 48.7 \text{ N/mm}^2$$

11/11/16
2.

An overhang crank with pin and shaft shown in figure. Tangential load of 15 kN act on a crank pin. Determine the maximum principal stress and minimum principal stress, max. shear stress at the centre of the crank shaft bearing.



Soln:-

$$\sigma_b = M/z = \frac{P \times e}{z}$$

$$= \frac{15 \times 10^3 \times 120}{\frac{\pi}{32} (80)^3} = 3.58 \times 10$$

$$= 35.8 \text{ N/mm}^2$$

$$T = 15 \times 10^3 \times 140$$

$$= 21 \times 10^5 \text{ N-mm}$$

$$\tau_{xy} = \frac{16T}{\pi d^3}$$

$$= \frac{16 \times 21 \times 10^5}{\pi \times (80)^3}$$

$$= 20.8 \text{ N/mm}^2$$

$$\sigma_x = \sigma_b$$

$$\sigma_{max} = \frac{1}{2} \left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \right]$$

$$= \frac{1}{2} \left[(35.8) + \sqrt{(35.8)^2 + 4(20.8)^2} \right]$$

$$= 45.34 \text{ N/mm}^2$$

$$\sigma_{min} = \frac{1}{2} \left[\sigma_x - \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \right]$$

$$= \frac{1}{2} \left[35.8 - \sqrt{(35.8)^2 + 4(20.8)^2} \right]$$

$$= -9.54 \text{ N/mm}^2$$

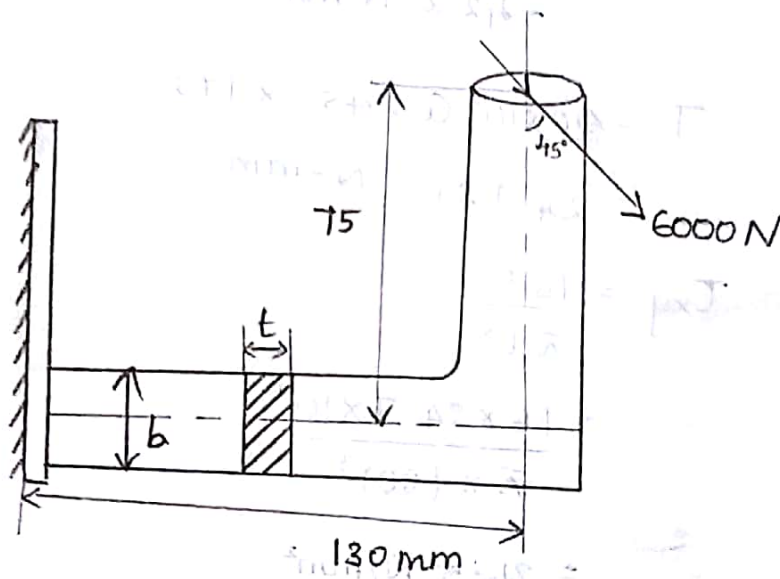
$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2}$$

$$= \frac{1}{2} \sqrt{(35.8)^2 + 4(20.8)^2}$$

$$= 27.4 \text{ N/mm}^2$$

$$\begin{aligned} \tau_{max} &= \frac{1}{2} \sqrt{(412.2)^2 + 4(24.5)^2} \\ &= \frac{1}{2} \sqrt{(412.2)^2 + 4(24.5)^2} \\ &= 22.3 \text{ N/mm}^2 \end{aligned}$$

1. A mild steel bracket is shown in figure. is subjected to axial pull of 6000N acting at 45° to horizontal axis. Bracket has rectangular section whose depth is twice the thickness. Find the cross section dimension of the bracket if the permissible stress in the material of the bracket is 60 MPa.



$$\begin{aligned} \sigma &= \sigma_{bv} + \sigma_{bh} + \sigma_d \\ \sigma_{bh} &= \frac{M_H}{Z} \\ &= \frac{6000 \cos 45^\circ \times 75}{\frac{4t^3}{6}} \\ &= \frac{477297.07}{t^3} \text{ N/mm}^2 \end{aligned}$$

PSA DB 6.2

$\frac{b}{d}$	$\frac{bd^2}{b}$
$\frac{t}{b}$	$\frac{tb^2}{b}$

$b = 2t$

$Z = \frac{t(2t)^2}{6}$

$= \frac{4t^3}{6}$

$$\sigma_d = \frac{R}{n}$$

$$= \frac{6000 \sin 45^\circ}{t \times 2t}$$

$$= \frac{2121.3}{t^2}$$

$$\sigma_{bv} = \frac{6000 \sin 45^\circ \times 130}{\frac{4t^3}{6}}$$

$$= \frac{827314.9}{t^3} \text{ N/mm}^2$$

$$60 = \frac{477297}{t^3} + \frac{827314}{t^3} + \frac{2121.3}{t^2}$$

$$\frac{1304611}{t^3} + \frac{2121.3}{t^2} = 60$$

$$\frac{21743.5}{t^3} + \frac{35.355}{t^2} = 1$$

$$t = 25$$

$$1.44 = 1$$

$$t = 27$$

$$1.15 = 1$$

$$t = 28$$

$$1.03 \approx 1$$

So thickness $t = 28 \text{ mm}$

$$b = 2t$$

$$= 56 \text{ mm}$$

Fits And Tolerance

Fits	Fits Factor.
R5	$5\sqrt{10} = 1.58$
R10	$10\sqrt{10} = 1.26$
R20	$20\sqrt{10} = 1.12$
R40	$40\sqrt{10} = 1.06$
R80	$80\sqrt{10} = 1.03$

Fits:-

When two parts are to be assembled the relationship between shaft & hole is called fit

Theory of failures under static load:-

Maximum Principle Stress Theory (or)
Rankine Theory:-

$$\sigma_1 = \sigma_y$$

$$\sigma_1 \text{ (or) } \sigma_2 \text{ (or) } \sigma_3 = \sigma_y$$

Maximum Shear Stress Theory (Guest
or Coulomb's Theory)

$$(\sigma_1 - \sigma_2) \text{ (or) } (\sigma_2 - \sigma_3) \text{ (or) } (\sigma_3 - \sigma_1) = \sigma_y$$

Maximum Strain Theory (St. Venant's)

$$\sigma_1 - \nu (\sigma_2 + \sigma_3) \text{ (or) } \sigma_2 - \nu (\sigma_3 + \sigma_1) = \sigma_y$$

$$\sigma_3 = 0$$

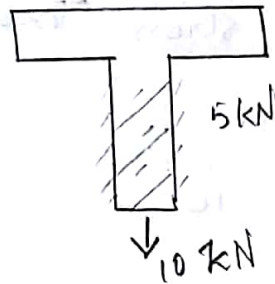
Maximum Strain Energy Theory

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = (\sigma_y)^2$$

Distortion Energy Theory (Octahedral)

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 = (\sigma_y)^2$$

1. The load on the bolt consists of axial pull of 10 kN together with shear force of 5 kN. Find the diameter of bolt according to maximum principle stress theory, maximum shear stress theory, maximum strain theory, maximum distortion energy theory. Tensile stress at elastic limit is 100 MPa $\nu = 0.3$



$$\sigma_x = \sigma_t$$

$$\sigma_t = \frac{10 \times 10^3}{\frac{\pi}{4} d^2} = \frac{12.73}{d^2} \text{ kN/mm}^2$$

$$\tau = \frac{\text{shear load}}{\text{shear Area}} \quad (\text{only for bolt})$$

$$= \frac{5}{\frac{\pi}{4} d^2} = \frac{6.36}{d^2} \text{ kN/mm}^2$$

$$\sigma_{max} = \frac{1}{2} \left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \right]$$

$$= \frac{1}{2} \left[\frac{12.73}{d^2} + \sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.36}{d^2}\right)^2} \right]$$

$$= \frac{1}{2} \left[\frac{12.73}{d^2} + \frac{17.99}{d^2} \right]$$

$$\sigma_1 = \frac{15.362}{d^2} \text{ KN/mm}^2$$

$$\sigma_2 = \frac{1}{2} \left[\sigma_x - \sqrt{(\sigma_x)^2 - 4(\tau_{xy})^2} \right]$$

$$= \frac{-2.635}{d^2} \text{ KN/mm}^2$$

Case 1: Maximum Principle Stress Theory.

$$\frac{15.365 \times 10^3}{d^2} = 100 \quad (\sigma_1 = \sigma_y)$$

$$d = 12.39 \text{ mm}$$

Case 2: Maximum Shear Stress Theory

$$\sigma_1 - \sigma_2 = \sigma_y$$

$$\left(\frac{15.365}{d^2} + \frac{2.635}{d^2} \right) 10^3 = 100.$$

$$\frac{18}{d^2} = \frac{100}{10^3}$$

$$d = 13.41 \text{ mm}$$

Case 3: Maximum Strain Theory

$$\sigma_1 - \nu(\sigma_2 + \sigma_3) = \sigma_y.$$

$$\frac{15.362}{d^2} - 0.3 \left(\frac{-2.635}{d^2} \right) = 100.$$

$$d = 12.7 \text{ mm}$$

Case 4 Maximum strain Energy

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\gamma(\sigma_1\sigma_2) = (\sigma_y)^2$$

$$\frac{235 \times 10^6}{d^4} + \frac{6.943 \times 10^6}{d^4} + 0.6 \left(\frac{40.47}{d^4} \right) 10^6 = 10^4$$

$$\frac{(235 + 6.943 + 24.28) 10^6}{10^4} = d^4$$

$$d = 12.77 \text{ mm}$$

Case 5 : Octahedral theory.

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2$$

$$\frac{235 \times 10^6}{d^4} + \frac{6.943 \times 10^6}{d^4} + \frac{40.47 \times 10^6}{d^4} = 10^4$$

$$d = 12.96 \text{ mm}$$

2. A cylindrical shaft made of steel yield strength of 700 MPa is subjected to a static load consisting of bending moment 10 kNm and torsional moment of 30 kNm.

Determine the dia of shaft using any 3 theory of failures and assuming

FOS = 2. Take Young's Modulus = 210 GPa

$\gamma = 0.3$.

Soln: -

$$M = 10 \text{ kNm}$$

$$T = 30 \text{ kNm}$$

$$\sigma_b = M/z$$

$$= \frac{10 \times 10^6}{\frac{\pi}{32} d^3}$$

$$= \frac{101.85}{d^3} \times 10^6 \text{ N/mm}^2$$

$$\tau = \frac{16T}{\pi d^3}$$

$$= \frac{16 \times 30 \times 10^6}{\pi d^3} = \frac{152.78 \times 10^6}{d^3} \text{ N/mm}^2.$$

$$\sigma_1 = \frac{1}{2} \left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \right]$$

$$= \frac{1}{2} \left[\frac{101.85 \times 10^6}{d^3} + \sqrt{\left(\frac{101.85 \times 10^6}{d^3} \right)^2 + 4 \left(\frac{152.78 \times 10^6}{d^3} \right)^2} \right]$$

$$= \frac{1}{2} \left[\frac{101.85 \times 10^6}{d^3} + \frac{322.08 \times 10^6}{d^3} \right]$$

$$= \frac{211.96 \times 10^6}{d^3} \text{ N/mm}^2$$

$$\sigma_2 = \frac{1}{2} \left[\frac{101.85 \times 10^6}{d^3} - \frac{322.08 \times 10^6}{d^3} \right]$$

$$= \frac{-110.11 \times 10^6}{d^3} \text{ N/mm}^2$$

1. Maximum Principle stress theory.

$$\frac{211.96 \times 10^6}{d^3} = \frac{700}{2}$$

$$d = 84.60 \text{ mm.}$$

2. Maximum shear stress Theory.

$$(\sigma_1 - \sigma_2) = \sigma_y / n$$

$$\frac{211.96 \times 10^6}{d^3} + \frac{110.11 \times 10^6}{d^3} = \frac{700}{2}$$

$$d = 97.2 \text{ mm}$$

3. Maximum strain theory.

$$\sigma_1 - \nu(\sigma_2) = \sigma_y / n$$

$$\frac{211.96 \times 10^6}{d^3} + 0.3 \left(\frac{110.11 \times 10^6}{d^3} \right) = \frac{700}{2}$$

$$d = 88.78 \text{ mm}$$

4. Strain Energy Theory

$$\sigma_1^2 + \sigma_2^2 - 2\nu(\sigma_1 \sigma_2) = \left(\frac{\sigma_y}{n} \right)^2$$

$$\frac{44927.04 \times 10^{12}}{d^6} + \frac{12124.21 \times 10^{12}}{d^6} + \frac{14003.3 \times 10^{12}}{d^6} = 122500$$

$$d = 91.3 \text{ mm}$$

3. Mild steel shaft of 50 mm dia is subjected to bending moment of 2000 N-m and torque T. The yield point of the steel in tension is 200 MPa. Maximum value of torque without causing yielding of shaft according to max. principal stress theory. Max. shear

-theory Max. distortion theory

Soln:

$$\begin{aligned}\tau_b &= M/z \\ &= \frac{2000 \times 10^3 \text{ N}\cdot\text{mm}}{\frac{\pi}{32} \cdot (50)^3 \text{ mm}^3} \\ &= 162.97 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\tau &= \frac{16T}{\pi d^3} \\ &= \frac{16 \times T}{\pi \times (50)^3} \\ &= 4.07 \times 10^{-5} \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\sigma_1 &= \frac{1}{2} \left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2} \right] \\ &= \frac{1}{2} \left[162.97 + \sqrt{26559.2 + 6.625 \times 10^{-9} \text{ T}^2} \right]\end{aligned}$$

$$\sigma_2 = \frac{1}{2} \left[162.97 - \sqrt{26559.2 + 6.625 \times 10^{-9} \text{ T}^2} \right]$$

Max. Principle stress theory

$$\sigma_1 = \sigma_y$$

$$\frac{1}{2} \left[162.97 + \sqrt{26559.2 + 6.625 \times 10^{-9} \text{ T}^2} \right] = 200.$$

$$162.97 + \sqrt{26559.2 + 6.625 \times 10^{-9} \text{ T}^2} = 400.$$

$$\sqrt{26559.2 + 6.625 \times 10^{-9} T^2} = 237.03$$

$$26559.2 + 6.625 \times 10^{-9} T^2 = 56183.22$$

$$6.625 \times 10^{-9} T^2 = 29624.02$$

Maximum Shear $T = 2.1 \times 10^6$ N.mm.

Stress Theory :-

$$\sigma_1 - \sigma_2 = \sigma_y$$

$$\frac{1}{2} \left[162.97 + \sqrt{26559.2 + 6.625 \times 10^{-9} T^2} \right] - \frac{1}{2} \left[162.97 - \sqrt{26559.2 + 6.625 \times 10^{-9} T^2} \right] = 200.$$

$$162.97 + \sqrt{26559.2 + 6.625 \times 10^{-9} T^2} - \left[162.97 - \sqrt{26559.2 + 6.625 \times 10^{-9} T^2} \right] = 400$$

$$\sqrt{26559.2 + 6.625 \times 10^{-9} T^2} + \sqrt{26559.2 + 6.625 \times 10^{-9} T^2} = 400$$

$$\sqrt{26559.2 + 6.625 \times 10^{-9} T^2} = 200.$$

$$26559.2 + 6.625 \times 10^{-9} T^2 = 40000.$$

$$T = 1.42 \times 10^6 \text{ N.mm}$$

Maximum Distortion Theory.

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = (\sigma_y)^2$$

$$\left\{ \frac{1}{2} \left[162.97 + \sqrt{26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2} \right] \right\}^2 +$$

$$\left\{ \frac{1}{2} \left[162.97 - \sqrt{26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2} \right] \right\}^2$$

$$- \left\{ \frac{1}{2} \left(162.97 + \sqrt{26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2} \right) \right.$$

$$\left. \frac{1}{2} \left(162.97 - \sqrt{26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2} \right) \right\}$$

$$= 40000$$

$$\left(162.97 + \sqrt{26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2} \right)^2 +$$

$$\left(162.97 - \sqrt{26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2} \right)^2 -$$

$$\left[26559 \cdot 2 - (26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2) \right]$$

$$= 10000$$

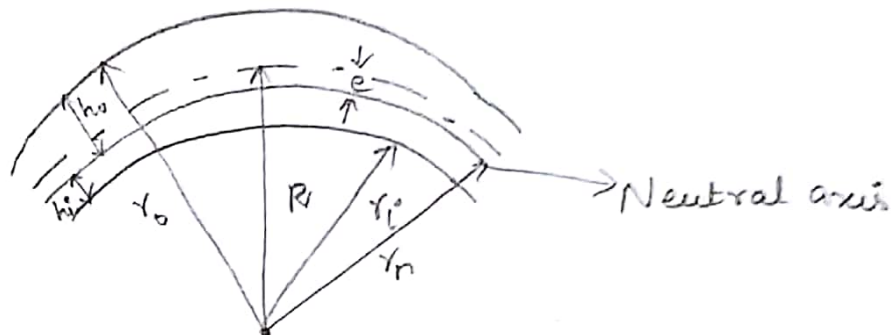
$$\left(162.97 + \sqrt{26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2} \right)^2 +$$

$$\left(162.97 - \sqrt{26559 \cdot 2 + 6 \cdot 625 \times 10^{-9} T^2} \right)^2$$

$$+ 6 \cdot 625 \times 10^{-9} T^2 = 10000$$

Design of Curved beams

$$\sigma = \sigma_d + \sigma_b$$



Features	Straight beam	Curved beam
Centroidal axis and neutral axis	Coincident	Not coincident (Neutral axis is offset from geometrical axis)
Stress developed	Same throughout the section	Different at inner radii & outside radii

$$M_b = P \times R \quad R - \text{Centroidal axis (radius)}$$

$$\sigma = \pm \sigma_d \pm \sigma_b$$

$$= \frac{P}{a} + \frac{M_b h_i}{a e r_i}$$

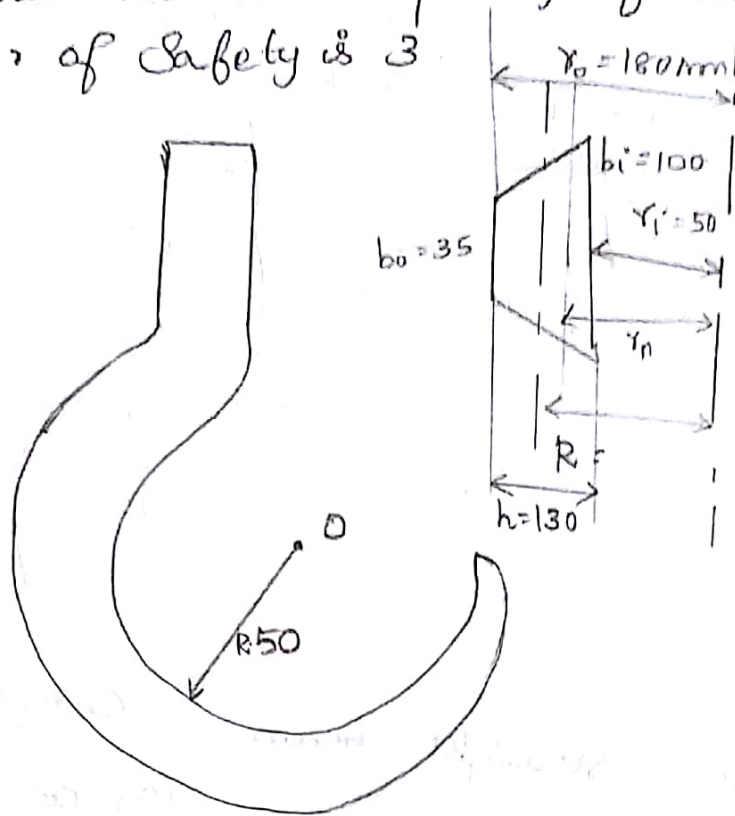
$$e = R - r_n$$

$$h_i = r_n - r_i$$

$$h_o = r_o - r_n$$

1. A crane hook has a section which for the purpose is considered trapezoidal, as shown in figure. It is made of plain carbon steel with yield strength of 380 MPa in tension

Determine the load capacity of hook.
Factor of Safety is 3



From psg dB 6.3.

$$R_i = r_i + \frac{h(b_i + 2b_o)}{3(b_i + b_o)}$$

$$= 50 + \frac{130(100 + 70)}{3(100 + 35)}$$

$$= 50 + \frac{22100}{405}$$

$$= 104.56 \text{ mm}$$

$$\sigma_y = \frac{380}{3}$$

$$= 126.6 \text{ N/mm}^2$$

$$\gamma_n = \frac{1/2 (b_i + b_o) h}{\left(\frac{b_i r_o - b_o r_i}{h} \right) \ln \left(\frac{r_o}{r_i} \right) - (b_i - b_o)}$$

$$\gamma_n = \frac{\frac{1}{2} (100 + 35) 130}{\left(\frac{100 \times 180 - 35 \times 50}{130} \right) \ln \left(\frac{180}{50} \right) - (100 - 35)}$$

$$= \frac{8775}{125 \times \ln 3.6 - 65}$$

$$= 92.36 \text{ mm}$$

$$\begin{aligned} h_i &= \gamma_n - \gamma_i \\ &= 92.36 - 50 \\ &= 42.36 \text{ mm} \end{aligned}$$

$$\begin{aligned} e &= R - \gamma_n \\ &= 104.56 - 92.36 \\ &= 12.31 \text{ mm} \end{aligned}$$

$$\begin{aligned} a &= \frac{1}{2} (b_i + b_o) h \\ &= 8775 \text{ mm}^2 \end{aligned}$$

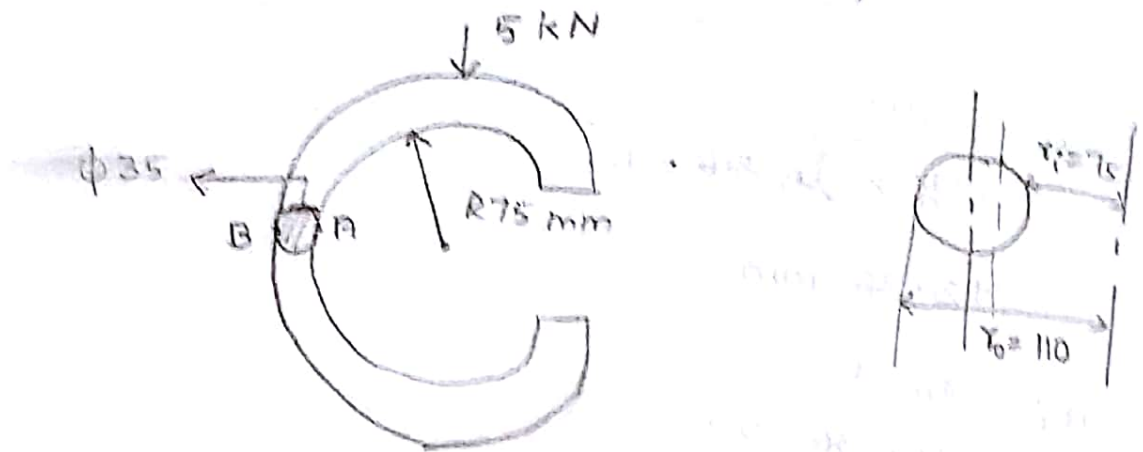
$$\begin{aligned} M_b &= P \times R \\ &= 104.56 P \end{aligned}$$

$$\begin{aligned} \sigma_i &= \sigma_{bi} + \sigma_d \\ &= \frac{M_b h_i}{a e \gamma} + \frac{P}{a} \end{aligned}$$

$$\begin{aligned} 126.6 &= \frac{104.56 P \times 42.36}{8775 \times 12.31 \times 50} + \frac{P}{8775} \\ &= 8.200 \times 10^{-4} P + 1.13 \times 10^{-4} P \\ &= 9.33 \times 10^{-4} P \end{aligned}$$

$$P = 135.69 \text{ kN}$$

2. Calculate stress at point A & B of a circular arch shown in figure.



$$\sigma_A = -\sigma_2 + \sigma_{bi}$$

$$a = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} \times 35^2$$

$$= 962.11 \text{ mm}^2$$

$$R = r_i + d/2$$

$$= 75 + 17.5$$

$$= 92.5 \text{ mm}$$

$$r_n = \frac{[\sqrt{r_o} + \sqrt{r_i}]^2}{4} = 91.6 \text{ mm}$$

$$e = R - r_n$$

$$= 0.835$$

$$M_b = P \times R$$

$$= 5 \times 10^3 \times 92.5$$

$$= 462500 \text{ Nmm}$$

$$h_i = r_n - r_i$$

$$= 91.6 - 75$$

$$= 16.6 \text{ mm}$$

$$\sigma_A = -\frac{P}{a} - \frac{M_b h_i}{a e r_i}$$

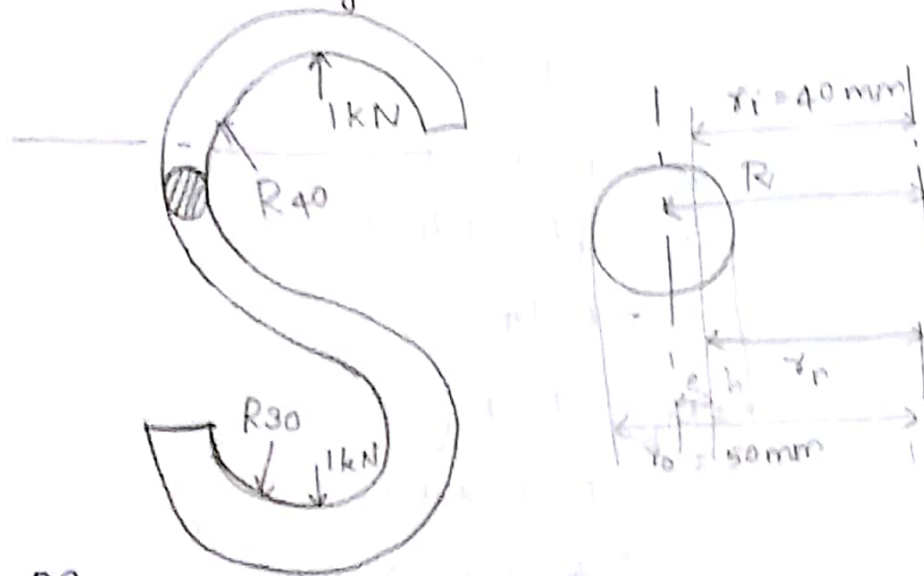
$$= -\frac{5 \times 10^3}{962.11} - \frac{462500 \times 16.6}{962.11 \times 0.835 \times 75}$$

$$= -5.196 - 127.42$$

$$= -132.61 \text{ N/mm}^2$$

$$\begin{aligned}\sigma_B &= -\sigma_d + \sigma_{bi} \\ &= -P/a + \frac{M_b h_i}{a e r_i} \\ &= 122.24 \text{ N/mm}^2\end{aligned}$$

1. A open ϕ -link shown in figure is made of steel rod of diameter 10 mm. Determine the maximum tensile stress for the given cross section.



Section AA

$$\begin{aligned}\sigma_c &= \sigma_{di} + \sigma_{bi} \\ &= P/a + \frac{M_b h_i}{a e r_i}\end{aligned}$$

$$e = R - r_i$$

$$\begin{aligned}a &= \pi/4 d^2 \\ &= \pi/4 \times 10^2 \\ &= 78.53 \text{ mm}^2\end{aligned}$$

$$\begin{aligned}R &= r_i + d/2 \\ &= 40 + 5 = 45 \text{ mm}\end{aligned}$$

$$r_n = \frac{[\sqrt{r_o} + \sqrt{r_i}]^2}{4}$$

$$= \frac{[\sqrt{50} + \sqrt{40}]^2}{4}$$

$$= 44.86 \text{ mm.}$$

$$h_i = r_n - r_i$$

$$= 44.86 - 40$$

$$= 4.86$$

$$e = R - r_n$$

$$= 45 - 44.86$$

$$= 0.14$$

$$M_b = P \times R$$

$$= 1 \times 10^3 \times 45$$

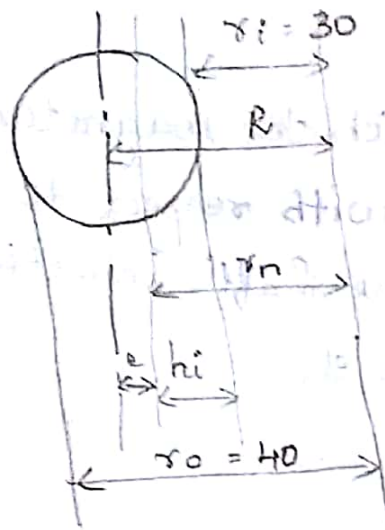
$$= 45 \times 10^3 \text{ N-mm.}$$

$$\sigma_c = \sigma_{di} + \sigma_{bi}$$

$$= \left(\frac{1 \times 10^3}{78.53} \right) + \left(\frac{45 \times 10^3 \times 4.86}{78.53 \times 0.14 \times 40} \right)$$

$$= 12.73 + 497.3$$

$$= 510 \text{ N/mm}^2$$



$$R = r_i + d/2$$

$$= 30 + 5$$

$$= 35 \text{ mm}$$

$$r_n = \frac{[\sqrt{40} + \sqrt{30}]^2}{4}$$

$$= 34.82 \text{ mm}$$

$$h_i = r_n - r_i$$

$$= 34.82 - 30$$

$$= 4.82$$

$$e = R - r_n$$

$$= 35 - 34.82$$

$$= 0.18$$

$$M_b = 1 \times 10^3 \times 35$$

$$= 35 \times 10^3$$

$$\sigma_i = \frac{1 \times 10^3}{78.53} + \left(\frac{35 \times 10^3 \times 4.82}{78.53 \times 0.18 \times 30} \right)$$

$$= 410 \text{ N/mm}^2$$