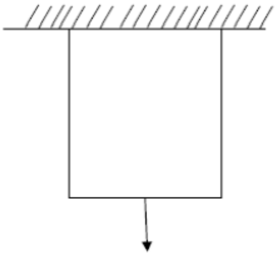
1	**Why are polynomial types of interpolation functions preferred over trigonometric functions?**	***(May/June 2013)***
	Polynomial functions are preferred over trigonometric functions due to the following reasons: 1. It is easy to formulate and computerize the finite element equations 2. It is easy to perform differentiation or integration 3. The accuracy of the results can be improved by increasing the order of the polynomial.	
2.	**Distinguish Natural & Essential boundary condition**	**(May/June 2009)**
	There are two types of boundary conditions. They are: 1. **Primary boundary condition (or) Essential boundary condition** The boundary condition, which in terms of field variable, is known as primary boundary condition. 2. Secondary boundary condition or natural boundary conditions The boundary conditions, which are in the differential form of field variables, are known as secondary boundary condition. Example: A bar is subjected to axial load shown in fig.  In this problem, displacement u at node 1 = 0, that is primary boundary condition. $AE\frac{du}{dx}=P$, that is secondary boundary condition.	
3.	**What do you mean by Boundary value problem?**	
	The solution of differential equation is obtained for physical problems, which satisfies some specified conditions known as boundary conditions. The differential equation together with these boundary conditions, subjected to a boundary value problem. Examples: Boundary value problem. $\frac{d^2y}{dx^2}-a(x)\frac{dy}{dx}-b(x)y=0$ with boundary conditions, y(m) = S and y(n) = T.	
4.	**What do you mean by weak formulation? State its advantages.** A weak form is a weighted integral statement of a differential equation in which the differentiation is distributed among the dependent variable and the weight function and also includes the natural boundary conditions of the problem. • A much wider choice of trial functions can be used.	***(April/May 2015), (May/June 2013)***

	• The weak form can be developed for any higher order differential equation. • Natural boundary conditions are directly applied in the differential equation. • The trial solution satisfies the essential boundary conditions	
5.	**What is Rayleigh-Ritz method?**	**(NOV/DEC 2015)**
	Rayleigh-Ritz method is a integral approach method which is useful for solving complex structural problems, encountered in finite element analysis. This method is possible only if a suitable functional is available.	
6.	**What is meant by degrees of freedom?** When the force or reactions act at nodal point, node is subjected to deformation. The deformation includes displacement, rotations, and/or strains. These are collectively known as degrees of freedom.	
7.	**What is "Aspect ratio"?**	
	Aspect ratio is defined as the ratio of the largest dimension of the element to the smallest dimension. In many cases, as the aspect ratio increases, the inaccuracy of the solution increases. The conclusion of many researches is that the aspect ratio should be close to unity as possible.	
8.	**What are 'h' and 'p' versions of finite element method?**	
	h' versions and 'p' versions are used to improve the accuracy of the finite element method. In 'h' versions, the order of polynomial approximation for all elements is kept constant and the numbers of elements are increased. In 'p' version, the numbers of elements are maintained constant and the order of polynomial approximation of element is increased.	
9.	**What is Discretization?**	**(NOV/DEC 2015)**
	The art of subdividing a structure into a convenient number of smaller components is known as Discretization.	
10.	**During Discretization, mention the places where it is necessary to place a node?**	
	The following places are necessary to place a node during Discretization process. (i) Concentrated load acting point. (ii) Cross-section changing point. (iii) Different material inter junction point. (iv) Sudden change in load point.	
11.	**What is natural co-ordinate?**	***(Nov/Dec 2014), (April/May 2011)***
	A natural co-ordinate system is used to define any point inside the element by a set of dimensionless numbers, whose magnitude never exceeds unity, this system is useful in assembling of stiffness matrices.	
12.	**Explain force method and stiffness method?** In force method, internal forces are considered as the unknowns of the problem. In displacement or stiffness method, displacements of the	

	nodes are considered as the unknowns of the problem. Among them two approaches, displacement method is desirable.	
13.	**Define shape function. State its characteristics**	(May/June 2014), (Nov/Dec 2014), (Nov/Dec 2012)
	In finite element method, field variables within an element are generally expressed by the following approximate relation: $u(x,y) = N_1(x,y)u_1 + N_2(x,y)u_2 + N_3(x,y)u_3$ Where u_1, u_2, u_3 are the values of the field variable at the nodes and N_1 ,N_2 ,N_3 are interpolation function. N_1 ,N_2 ,N_3 is called shape functions because they are used to express the geometry or shape of the element. The characteristics of the shape functions are follows: 1. The shape function has unit value at one nodal point and zero value at the other nodes. 2.The sum of the shape function is equal to one.	
14.	**How do you calculate the size of the global stiffness matrix?**	
	Global the matrix size=Number of nodes $\times$ $\{Degress\ of\ \text{freedom}\}$	
15.	**Give the general expression for element stiffness matrix.**	**(Nov/Dec 2015)**
	$stiffness\ matrix,\ [\text{K}] = \int_v [B]^T [D][B] dv$ $[B] \Rightarrow strain\ displacement\ matrix$ [Row matrix] $[D] \Rightarrow stress, strain\ relationship$ matrix $[Row\ matrix]$	
16.	**Write down the expression of stiffness matrix for one dimensional bar element.**	
	For 1D linear bar element $[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ A-Area of the element mm^2 E-Young's Modulus of the element N/mm^2 L-length of the element	
17.	**State the properties of a stiffness matrix.**	**[AU, Jan 2006]**
	The properties of a stiffness matrix [K] are: 1. It is symmetric matrix. 2. The sum of elements in any column must be equal to zero. 3. It is an unstable element. So, the determinant is equal to zero	
18.	**Write down the general finite element equation**	
	General finite element equation is, $\{F\} = [K]\{u\}$ where, $\{F\}$ $\rightarrow$ Force vector [Column matrix]. $[K]$ $\rightarrow$ Stiffness matrix [Row matrix]. $\{u\}$ $\rightarrow$ Degrees of freedom [Column matrix].	
19.	**Write down the finite element equation for one dimensional two noded bar element.**	
	The finite element equation for one dimensional two noded bar element is, $\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$	

20.	**Define total potential energy.** The total potential energy 1t of an elastic body is defined as the sum of total strain energy U andthe potential energy ofthe external forces, (W). Total potential energy, π = Strain energy (U) + $\left\{ \begin{array}{c} \text{Potential energy of} \\ \text{the external forces (W)} \end{array} \right\}$	

The differential equation of physical phenomenon is given by $\frac{d^2y}{dx^2} + 500x^2 = 0,\ 0 \le x \le 1,$ Trial function, $y = a_1(x - x^4)$, Boundary condition are, y(0)=0, y(1)=0 calculate the value of the parameter a_1 by the following methods. (i) Point collocation method (ii) Sub-domain collocation method (iii) least Square Method and (iv) Galerkin's method.

Given: Differential equation $\frac{d^2y}{dx^2} + 500x^2 = 0,\ 0 \le x \le 1$
Trial function, $y = a_1(x - x^4)$
Boundary condition are, $y(0) = 0, y(1) = 0$

To find: The value of parameter a_1 by,
i. point collocation method, ii. Subdomain method, iii. Least Squares method, iv. Galerkin

Solution: First we have to verify, whether the trial function satisfies the boundary condition or not.

Trial function is, $y = a_1(x - x^4)$
When $x = 0$, $y = a_1(0 - 0) = 0$
$x = 1$, $y = a_1(1 - 1^4) = 0$

Hence it satisfies the boundary conditions,

(i) point collocation method:

$y = a_1(x - x^4)$

$dy/dx = a_1(1 - 4x^3)$

$d^2y/dx^2 = a_1(0 - 12x^2)$

$\boxed{d^2y/dx^2 = -12a_1x^2}$

Substituting $\frac{d^2y}{dx^2}$ value in given differential equation (1), $\rightarrow$ $\frac{d^2y}{dx^2} + 500x^2 = 0$

$\Rightarrow$ Residual, $R = -12a_1x^2 + 500x^2$ $\rightarrow (2)$

In point collocation method, residuals are set to zero.

$R = -12a_1x^2 + 500x^2 = 0$ $\rightarrow (3)$

In this problem, we have to find only one parameter, a_1. So, only one collocation point is needed.

The point may be chosen between 0 and 1. Let us take $1/2$,

Substituting $x = \frac{1}{2}$ in equation (3)

$$R = -12\,a_1\left[\frac{1}{2}\right]^2 + 500\left[\frac{1}{2}\right]^2 = 0$$

$$\Rightarrow -\cancel{12}^{3}\,a_1\left[\frac{1}{\cancel{4}}\right] + \cancel{500}^{125}\left[\frac{1}{\cancel{4}}\right] = 0$$

$$-3a_1 + 125 = 0$$

$$a_1 = 41.66$$

Hence the trial function is $y = 41.66(x - x^4)$ $\rightarrow (4)$

(ii) Subdomain collocation method:

This method requires $\int_0^1 R\,dx = 0$

Substitute R Value

$$\Rightarrow \int_0^1 [-12a_1x^2 + 500x^2]\,dx = 0$$

$$-12a_1\left[\frac{x^3}{3}\right]_0^1 + 500\left[\frac{x^3}{3}\right]_0^1 = 0$$

$$-\frac{12a_1}{3}[1-0] + \frac{500}{3}[1-0] = 0$$

$$-\frac{12a_1}{\cancel{3}} + \frac{500}{\cancel{3}} = 0$$

$$-12a_1 + 500 = 0$$

$$-12a_1 = -500$$

$$a_1 = \frac{500}{12} = 41.66 \quad \rightarrow 5$$

Trial function is $y = 41.66(x-x^4)$

(iii) Least squares method:

This method require, $I = \int_0^1 R^2\,dx$

It can also be written as

$$\frac{\partial I}{\partial a_1} = \int_0^1 R\,\frac{\partial R}{\partial a_1}\,dx \quad \rightarrow 6$$

We know that, $R = -12a_1x^2 + 500x^2$

$$\frac{\partial R}{\partial a_1} = -12x^2$$

Substitute R and $\frac{\partial R}{\partial a_1}$ values in eq ⑥

$$\Rightarrow \frac{\partial I}{\partial a_1} = \int_0^1 [-12a_1x^2 + 500x^2](-12x^2)\,dx$$

The requirement is, $\frac{\partial I}{\partial a_1} = 0$

$$\Rightarrow \int_0^1 [-12a_1x^2 + 500x^2)(-12x^2)\,dx = 0$$

$$\int_0^1 [144a_1x^4 - 6000x^4)\,dx = 0$$

$$144a_1\left[\frac{x^5}{5}\right]_0^1 - 6000\left[\frac{x^5}{5}\right]_0^1 = 0$$

$$\frac{144a_1}{5}[1-0] - \frac{6000}{5}[1-0] = 0$$

$$\frac{144a_1}{5} - \frac{6000}{5} = 0 \Rightarrow a_1 = \frac{6000}{144} = 41.66$$

$$a_1 = 41.66$$

(iv) Galerkin's method: In this method, the trial function itselfs is considered as the weighting function, $w_i \Rightarrow \int w_i R\,dx = 0$ ⟶ (8)

Here, the trial function is $y = w_i = a_1(x - x^4)$

Substitute w_i and R values in equation (8)

$$\int_0^1 \cancel{a_1}(x - x^4)(-12a_1x^2 + 500x^2)\,dx = 0$$

$$a_1\int_0^1 (x - x^4)(-12a_1x^2 + 500x^2)\,dx = 0$$

$$a_1\int_0^1 [-12a_1x^3 + 500x^3 + 12a_1x^6 - 500x^6]\,dx = 0$$

$$a_1\left[-12a_1\left[\frac{x^4}{4}\right]_0^1 + 500\left[\frac{x^4}{4}\right]_0^1 + 12a_1\left[\frac{x^7}{7}\right]_0^1 - 500\left[\frac{x^7}{7}\right]_0^1\right] = 0$$

$$-\frac{12a_1}{4}[1-0] + \frac{500}{4}[1-0] + \frac{12a_1}{7}(1-0) - \frac{500}{7}(1-0) = 0$$

$$-3a_1 + 125 + 1.714a_1 - 71.428 = 0$$

$$-1.286\,a_1 = -53.572$$

$$a_1 = 41.66 \quad \cdots (9)$$

Trial function is $y = 41.66(x - x^4)$

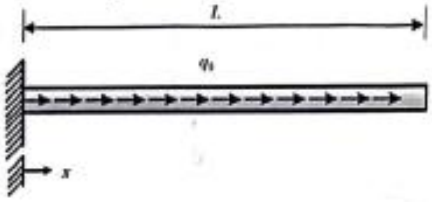
From equation 4, 5, 7, and 9, we know that the value of parameter a_1 is same for all the four methods.

Result: parameter, a_1 [For all the four methods] $= 41.66$

5/5

Weighted residual techniques

Consider a uniform rod subjected to a uniform axial load as illustrated in figure . It can be readily shown that the deformation of the bar is governed by the differential equation $AE\frac{d^2u}{dx^2}+q_0=0$ with the boundary conditions u(0)=0, $\frac{du}{dx}_{(x=L)}=0$, Estimate the deformation of bar using the weighted residual method.



with the boundary conditions $u(0)=0$, $\left.\frac{du}{dx}\right|_{x=L}=0$.

Weighted residual Methods

(i) *Point collocation* method: $R(x)=0$

(ii) *Subdomain collocation* method: $\int R(x)dx=0$

(iii) *Least square* method: $\int R(x)\frac{dR}{da_i}dx=0,\ i=0,1,2,3......$

(iv) *Galerkin* method: $\int R(x)\,w_i\,dx=0$

Step 1: Assume a trial or guss Solution:

$$u(x) = C_0 + C_1x + C_2x^2$$

· Where the constants C_0, C_1, C_2 are yet to be determined.

Step 2: Satisfy the boundary condition. [verification of BC]

$u(x) = C_0 + C_1x + C_2x^2$ → In order to satisfy the first boundary condition

$u(0) = C_0 + C_1(0) + C_2(0)^2 \rightarrow \boxed{C_0 = 0}$

To satisfy the second boundary condition,

$\left.\frac{du}{dx}\right|_{x=L} \Rightarrow = C_1 + 2C_2x$

$= \boxed{C_1 = -2C_2L}$

Since the trial Solution contains only one free parameter C_2, it is often referred to as a parameter solution.

Step 3: Find the domain residual.

Substituting in the governing differential equation

$$AE\frac{d^2u}{dx^2}+q_0=0$$

$$u(x)=C_0+C_1x+C_2x^2$$

$$\frac{du}{dx}=C_1+C_2[2x] \Rightarrow \frac{d^2u}{dx^2}=2C_2$$

$$R_d \Rightarrow AE(2C_2)+q_0$$

Step 4: minimise the residual.

Sine there is one residual to be minimised and One parameter to be determined.

We can readily solve for the undetermined Coefficien by Setteing the residual to zero, ie $R_d \Rightarrow 0$, yielding.

$$AE[2C_2]+q_0=0$$

$$C_2=-q_0/2AE$$

The our trial solution is

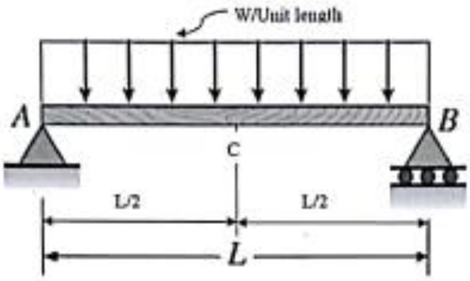
$$u(x)=C_0+C_1x+C_2x^2 \Rightarrow \quad C_0=0$$

$$= 0+[-2C_2L]x+[-q_0/2AE]x^2 \quad \text{Subst } C_1=-2C_2L, \; C_2=-q_0/2AE$$

$$=-2[-q_0/2AE]Lx - \left[\frac{q_0}{2AE}\right]x^2$$

$$=\frac{q_0}{AE}[2xL-x^2]$$

Use ***Rayleigh Ritz method*** determine the deflection at the center of the a simply supported beam of span length "L" subjected to uniformly distributed load through its length as shown in figure.



According to Rayleigh-Ritz Method,
When a displacement function make the total potential Energy minimum, that function may be considered as Approximate solution.

Consider a displacement function [i.e deflection] as

$$y = a_1 + a_2 x + a_3 x^2 \quad ----[1]$$

The boundary conditions are $y = 0$, at $x = 0$ & $x = L$

from equ ① →,

BC① → $x = 0$ $y(0) = a_1 + a_2(0) + a_3(0)^2$

$a_1 = 0$

BC② → $x = L$ $= 0 + a_2 L + a_3 L^2$

$a_2 L + a_3 L^2 = 0$ $\quad a_2 \cancel{L} = -a_3 L^{\cancel{2}}$

$$\boxed{a_2 = -a_3 L}$$

Then the deflection y can be written as,

$$y = 0 + [-a_3 L]x + a_3 x^2$$

$$= -a_3 L x + a_3 x^2 = a_3 [x^2 - Lx] \quad \rightarrow (2)$$

$$y = a_3 [x^2 - Lx]$$

Diff. two times, we get

$$\frac{dy}{dx} = a_3 [2x - L]$$

$$\frac{d^2y}{dx} = a_3 [2] = 2a_3$$

Strain Energy $U = \frac{EI}{2} \int_0^L \left[\frac{d^2y}{dx^2}\right] \cdot dx$

$$= \frac{EI}{2} \int_0^L [2a_3]^2 \cdot dx = \frac{EI}{\cancel{2}} [\cancel{4} a_3^2] \cdot [x]_0^L$$

$$U = 2EI a_3^2 [[L] - 0] = \boxed{2 a_3^2 EI . L}$$

Work done, $W = \int_0^L W \cdot L \cdot dx$

$$= \int_0^L W a_3 [x^2 - Lx] \cdot dx$$

$$= W a_3 \left[\frac{x^3}{3} - L \cdot \frac{x^2}{2}\right]_0^L$$

$$= W a_3 \left[\frac{L^3}{3} - \frac{L^3}{2} \right] = -\frac{W a_3 L^3}{6}$$

Total potential Energy

Π = Strain Energy − Work done

$$\Pi = U - W$$

$$\Pi = 2a_3^2 EI.L + \frac{W a_3 L^3}{6}$$

$$\frac{\partial \Pi}{\partial a_3} = 4a_3 EI \cdot L + \frac{W L^3}{6}$$

$$\frac{\partial \Pi}{\partial a_3} = 0$$

$$4a_3 EI.L + \frac{WL^3}{6} = 0$$

$$a_3 = -\frac{W L^{\cancel{3}}}{6} \times \frac{1}{4EI.\cancel{L}}$$

$$= -\frac{WL^2}{24EI}$$

Then the deflection

$$y = a_3 [x^2 - Lx]$$

$$= -\frac{WL^2}{24EI}[x^2 - Lx]$$

$y_{max} = ?$

Maximum deflection at $x = L/2$

$$y_{max} = \frac{-wL^2}{24EI}\left[\frac{L^2}{4} - L\cdot\frac{L}{2}\right]$$

$$= \frac{-wL^2}{24EI}\left[\frac{L^2}{4} - \frac{L^2}{2}\right]$$

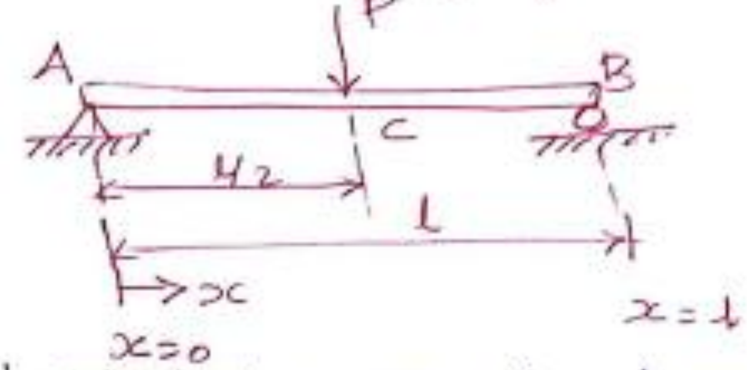
$$= \frac{-wL^2}{24EI}\left[-\frac{L^2}{4}\right]$$

$$= \frac{w\cdot L^4}{96EI}$$

$$y_{max} = \frac{wL^4}{96EI}$$

is the approximate Solution.

∴ Find the deflection at the Centre of a simply supported beam of span length l subjected to a concentrated load P at its mid point as shown in figure. Use Rayleigh-Ritz method



Total potential energy for beam is given by

$$\Pi = U - W$$

U → Strain energy

W → work done by the external force

Strain energy for a beam, $U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx$

E → Modulus of elasticity

I → area moment of Inertia.

y ⇒ deflection

$$y = a_1 + a_2x + a_3x^2 + a_4x^3 + \cdots \quad (1)$$

To simplify the problem, consider the first three terms such as

$$y = a_1 + a_2x + a_3x^2 \quad (2)$$

The boundary condition are

$y = 0$ at $x = 0$ & $x = l$

Hence equation (2) ⇒ $0 = a_1$

and $0 = a_2 l + a_3 l^2 \Rightarrow a_2 = -a_3 l$

(1)

Then y can be expressed as

$$y = -a_3 l x + a_3 x^2 = a_3 (x^2 - l x) \quad \text{--- (3)}$$

Differentiating two times we get,

$$\frac{dy}{dx} = a_3 (2x - l)$$

$$\frac{d^2y}{dx} = 2a_3$$

Then strain energy is given by

$$U = \frac{EI}{2} \int_0^l (2a_3)^2 dx$$

$$= \frac{EI}{2} 4 a_3^2 l$$

$$= 2 EI a_3^2 l$$

Work done $W = P \cdot y_{max}$

$$= P \cdot y_{at\, x = l/2}$$

$$= P a_3 (x^2 - l x) \quad \text{(from eq 3)}$$

$$\text{at } x = l/2$$

$$= P a_3 \left[\frac{l^2}{4} - \frac{l \cdot l}{2} \right]$$

$$= -P a_3 \frac{l^2}{4}$$

The Total pontential energy π is given by

$$\pi = U - W$$

$$= 2EI a_3^2 l + P a_3 \frac{l^2}{4}$$

For minimum potential energy condition

$$\partial \pi / \partial a_3 = 0$$

i.e $4EI a_3 l = \dfrac{-Pl^2}{4}$ (2) $\Rightarrow a_3 = -\dfrac{Pl^2}{4} \times \dfrac{1}{4EIl} = \dfrac{-Pl}{16EI}$

∴ Substituting the Value of a_3 in equation (9), we
$[y = a_3(x^2 - lx)]$

$$= -\frac{pl}{16EI}(x^2 - lx)$$

Maximum deflection occurs at $x = l/2$

Hence $$y_{max} = -\frac{pl}{16EI}\left[\frac{l^2}{4} - l \cdot \frac{l}{2}\right]$$

$$= -\frac{pl}{16EI}\left[-\frac{l^2}{4}\right]$$

$$= \frac{pl^3}{64EI}$$

$$\boxed{y_{max} = \frac{pl^3}{64EI}}$$ which is the approximate solution

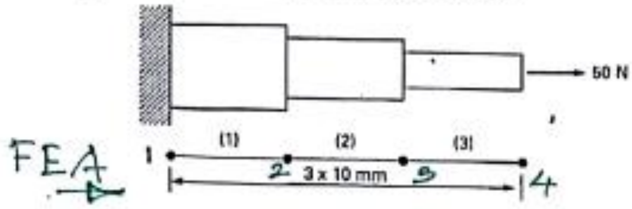
But the exact solution (i.e the exact value of maximum deflection for a Simply Supported beam to point load at the centre) is $y_{max} = \frac{pl^3}{48EI}$

To get more accurate solution by Rayleigh-Ritz method, the displacement function should contain more number of Ritz Coefficients such as

$$y = a_1 + a_2x + a_3x^2 + a_4x^3 + a_5x^4 + \ldots$$

(3)

Use the finite element method to calculate the displacements and stress of the bar in figure.1, and compare with theory.



$A^{(1)} = 50\ mm^2$
$A^{(2)} = 20\ mm^2$
$A^{(3)} = 10\ mm^2$
$E = 200 \times 10^3$ MPa

Figure .1.

Stiffness matrix for Element ①

$$K_1 = \frac{A_1 E}{L_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \frac{50 \times 200 \times 10^3}{10}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$2 \cdot 10^5 \begin{array}{cc} \begin{array}{cc}1 & 2\end{array} & \\ \begin{bmatrix}10 & -10\\ -10 & 10\end{bmatrix} & \begin{array}{c}1\\2\end{array}\end{array}$$

Stiffness matrix for Element ②

$$K_2 = \frac{A^{(2)} E}{L_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 20 \times 200 \times 10^3 \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 10^5 \begin{array}{cc} \begin{array}{cc}2 & 3\end{array} & \\ \begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix} & \begin{array}{c}2\\3\end{array}\end{array}$$

Stiffness matrix for element ③

$$K_3 = \frac{A^{(3)} E}{L_3}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 10 \times 200 \times 10^3 \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 10^5 \begin{array}{cc} \begin{array}{cc}3 & 4\end{array} & \\ \begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix} & \begin{array}{c}3\\4\end{array}\end{array}$$

Global matrix

$$K = K_1 + K_2 + K_3$$

$$= 10^5 \begin{array}{c} \begin{array}{cccc}1 & 2 & 3 & 4\end{array} \\ \begin{bmatrix}10 & -10 & 0 & 0\\ -10 & \boxed{10+4}^{\,14} & -4 & 0\\ 0 & -4 & (4+2)^{\,6} & -2\\ 0 & 0 & -2 & 2\end{bmatrix}\end{array}$$

Displacement Vector

$$U = \begin{Bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{Bmatrix}$$

Force Vector $= \begin{Bmatrix}F_1\\ F_2\\ F_3\\ F_4\end{Bmatrix} = \begin{bmatrix}0\\ 0\\ 0\\ 50\end{bmatrix}$

Assemble the element equation $[K]\{U\}=\{F\}$

$$10^5\begin{bmatrix}10 & -10 & 0 & 0\\ -10 & 14 & -4 & 0\\ 0 & -4 & 6 & -2\\ 0 & 0 & -2 & 2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\\ F_3\\ F_4\end{bmatrix}$$

Apply Boundary condition i.e at node 1, displacement $u_1=0$

At point load of 50N is acting at node 4 as shown in fig, So add 50N in F_4 vector.

$$10^5\begin{bmatrix}10 & -10 & 0 & 0\\ -10 & 14 & -4 & 0\\ 0 & -4 & 6 & -2\\ 0 & 0 & -2 & 2\end{bmatrix}\begin{bmatrix}0\\ u_2\\ u_3\\ u_4\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\\ 50\end{bmatrix}$$

In the above equation, $u_1=0$, So, neglect first row and first column of $[K]$ matrix

Hence the equation reduces to

$$10^5\begin{bmatrix}14 & -4 & 0\\ -4 & 6 & -2\\ 0 & -2 & 2\end{bmatrix}\begin{bmatrix}U_2\\ U_3\\ U_4\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 50\end{bmatrix}$$

$(14U_2 - 4U_3 + 0)\,10^5 = 0$ — ①

$(-4U_2 + 6U_3 - 2U_4) \times 10^5 = 0$ — ②

$(0 + (-2U_3) + 2U_4) \times 10^5 = 50$ — ③

Solve

$U_2 = 50 \times 10^{-6}$ mm $\quad U_3 = 175 \times 10^{-6}$ mm

$U_4 = 425 \times 10^{-6}$ mm.

Strain

$$\varepsilon^{(1)} = \left[\frac{U_2 - U_1}{L_1}\right] = \left[\frac{50 \times 10^{-6} - 0}{10}\right] = 5 \times 10^{-6}$$

$$\varepsilon^{(2)} = \left[\frac{U_3 - U_2}{L_2}\right] = \left[\frac{175 \times 10^{-6} - 50 \times 10^{-6}}{10}\right] = 12.5 \times 10^{-6}$$

$$\varepsilon^{(3)} = \left[\frac{U_4 - U_3}{L_3}\right] = \left[\frac{(425 - 175) \times 10^{-6}}{10}\right] = 25 \times 10^{-6}$$

Stress

$\sigma^{(1)} = \varepsilon^{(1)} \times E = 200 \times 10^3 \times 5 \times 10^{-6} = 1$ N/mm^2

$\sigma^{(2)} = \varepsilon^{(2)} \times E = 200 \times 10^3 \times 12.5 \times 10^{-6} = 2.5$ N/mm^2

$\sigma^{(3)} = \varepsilon^{(3)} \times E = 200 \times 10^3 \times 25 \times 10^{-6} = 5$ N/mm^2

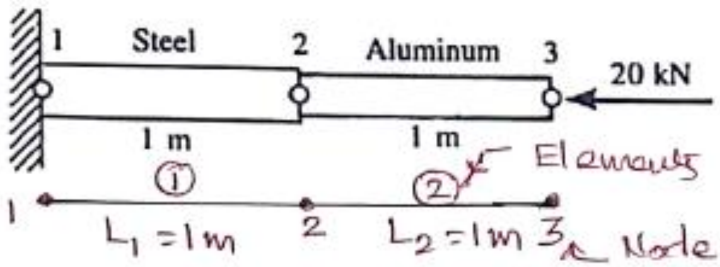
Verification.

$\sigma^{(1)} = P/A_1 = 50/50 = 1$ N/mm^2

$\sigma^{(2)} = P/A_2 = 50/20 = 2.5$ N/mm^2

$\sigma^{(3)} = P/A_3 = 50/10 = 5$ N/mm^2

For the bar assemblage shown in Figure 0.0, determine the nodal displacements, the forces in each element, and the reactions. Use the direct stiffness method for these problems.



$E_{st} = 200$ GPa
$A_{st} = 4 \times 10^{-4}$ m^2
$E_{al} = 70$ GPa
$A_{al} = 2 \times 10^{-4}$ m^2

Stiffness matrix For Element ①

$$k^{(1)} = \frac{A_1 E_1}{L_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$k^{(1)} = 4\times10^{-4}(m^2)\times 200\times10^{6}\left(\frac{kN}{m^2}\right)\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 800\times10^{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\frac{kN}{m}$$

$$= 10^2 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix}800 & -800\\ -800 & 800\end{bmatrix} \end{array}\begin{array}{c}1\\2\end{array} kN/m$$

Stiffness matrix For Element ②

$$k^{(2)} = \frac{A_2 E_2}{L_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \frac{2\times10^{-4}\times70\times10^{6}}{1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$k^{(2)} = 140\times10^{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 10^2 \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix}140 & -140\\ -140 & 140\end{bmatrix} \end{array}\begin{array}{c}2\\3\end{array}\frac{kN}{m}$$

Global matrix

$$k^{(G)} = 10^2 \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix}800 & -800 & 0\\ -800 & 800+140 & -140\\ 0 & -140 & 140\end{bmatrix} \end{array}\begin{array}{c}1\\2\\3\end{array} = 10^2\begin{bmatrix}800 & -800 & 0\\ -800 & 940 & -140\\ 0 & -140 & 140\end{bmatrix}$$

Element Equation $\Rightarrow [K][U] = \{F\}$

Global Stiff, Displacem. Vector, Force Vector

Displacement Vector.

$$U = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

Force Vector

$$F = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Formulation of Element Equation.

$[K]\{U\} = \{F\}$

$$10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

Apply Boundary Condition. $U_1 = 0$, $F_1 = 0$, $F_2 = 0$, $F_3 = -20$

$$10^2 \begin{bmatrix} \cancel{800} & \cancel{-800} & \cancel{0} \\ \cancel{-800} & 940 & -140 \\ \cancel{0} & -140 & 140 \end{bmatrix} \begin{bmatrix} \cancel{0} \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \cancel{0} \\ 0 \\ -20\,kN \end{bmatrix}$$

Apply Elimination Approach

$$10^2 \begin{bmatrix} 940 & -140 \\ -140 & 140 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -20\,kN \end{bmatrix}$$

generation of Simultaneous equation from Eqn

$$10^2 [940 u_2 - 140 u_3] = 0 \quad \cdots (1)$$

$$u_3 = 6.741 u_2$$

$$10^2 [-140 u_2 + 140 u_3] = -20 \quad -(2)$$

Substituting $u_3 = 6.741 u_2 \rightarrow (2)$

$$10^2 [-140 u_2 + 140 [6.74 u_2]] = -20$$

$$u_2 = -0.25 \times 10^{-3} \, m$$

$$u_3 = -1.678 \times 10^{-3} \, m \quad [-1.678571429 \times 10^{-3}]$$

Strain Calculation

$$\varepsilon^{(1)} = \frac{u_2 - u_1}{L_1} = \frac{-0.25 \times 10^{-3} - 0}{1} = -0.25 \times 10^{-3}$$

$$\varepsilon^{(2)} = \frac{u_3 - u_2}{L_2} = \frac{-1.678571429 \times 10^{-3} + 0.25 \times 10^{-3}}{1}$$

$$= -1.428571429 \times 10^{-3}$$

Stress Calculation [By Hooke's law]

$$\sigma^{(1)} = E\varepsilon^{(1)} = 200 \times 10^6 \times -0.25 \times 10^{-3} = 50000$$

$$\sigma^{(2)} = E_2 \varepsilon^{(2)} = 70 \times 10^6 \times -1.428571429 \times 10^{-3} = 10,000$$

Theoretical Stresses for this problem are easily Calculated by

$$\sigma^{(1)}_{Theory} = \frac{P}{A_1} = \frac{20}{4\times10^{-4}} = 50{,}000$$

$$\sigma^{(2)}_{Theory} = P/A_2 = \frac{20}{2\times10^{-4}} = 10{,}000$$

Reaction force

$$[R] = [K][U] - [F]$$

$$\begin{bmatrix} R_1 \\ R_2 \to 0 \\ R_3 \to 0 \end{bmatrix} = 10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} u_1 \to 0 \\ u_2 \\ u_3 \end{bmatrix} - \begin{bmatrix} F_1 \to 0 \\ F_2 \to 0 \\ F_2 \to (-20) \end{bmatrix}$$

Apply BC

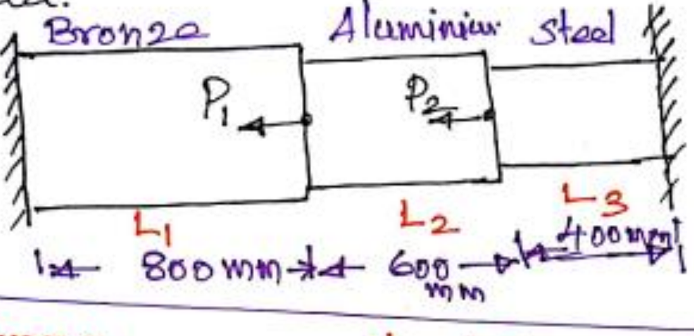
$$\begin{bmatrix} R_1 \\ 0 \\ 0 \end{bmatrix} = 10^2 \begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -20 \end{bmatrix}$$

$$-800\times10^2 \times u_2 - 0 = R_1$$

$$-800\times10^2 \times -1.67857142 9\times10^{-3} = \underline{20\,kN}$$

4/4

The structure shown in figure is subjected to an increase in temperature of 80°C. Determine the displacements, stresses and support reactions. Assume the following data.



$P_1 = 80\,KN$

$P_2 = 75\,KN$

$\Delta T = 80^\circ C$

Bronze	Aluminium	Steel
$A_1 = 2400\,mm^2$	$A_2 = 1200\,mm^2$	$A_3 = 600\,mm^2$
$E_1 = 83\,Gpa$	$E_2 = 70\,Gpa$	$E_3 = 200\,GPa$
$\alpha_1 = 18.9\times10^{-6}/^\circ c$	$\alpha_2 = 23\times10^{-6}/^\circ c$	$\alpha_3 = 11.7\times10^{-6}/^\circ c$

Solution: FEA Model

$$1 \overset{①}{—} 2 \overset{②}{—} 3 \overset{③}{—} 4$$

Finite element equation for one dimensional two noded bar element is given by

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{A_1 E_1}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \Rightarrow [K]\{U\} = \{F\}$$

Stiffness matrix

↳ Element ①

$$K^{(1)} = \frac{2400\times83\times10^3}{800}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 249\times10^3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^3 \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} 249 & -249 \\ -249 & 249 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

Element ②

$$K^{(2)} = \frac{1200\times70\times10^3}{600}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 140\times10^3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^3 \begin{array}{cc} \begin{array}{cc} 2 & 3 \end{array} & \\ \begin{bmatrix} 140 & -140 \\ -140 & 140 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array}$$

Element ③

$$K^{(3)} = 600\times200\times1$$

$$= \frac{600\times200\times10^3}{400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^3 \begin{array}{cc} \begin{array}{cc} 3 & 4 \end{array} & \\ \begin{bmatrix} 300 & -300 \\ -300 & 300 \end{bmatrix} & \begin{array}{c} 3 \\ 4 \end{array} \end{array}$$

1/5

Global matrix $[K] = K^{(1)} + K^{(2)} + K^{(3)}$

$$10^3 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 249+140 & -140 & 0 \\ 0 & -140 & 140+300 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

Displacement Vector

$$U = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

Load Vector $\{F\} = EA\alpha\Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

Element$^{(1)}$ $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = 83\times10^3 \times 2400 \times 18.9\times10^{-6} \times 80 \times \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 10^3 \begin{Bmatrix} -301.1904 \\ 301.1904 \end{Bmatrix} \begin{array}{c} 1 \\ 2 \end{array}$$

Element$^{(2)}$ $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = 70\times10^3 \times 1200 \times 23\times10^{-6} \times 80 \times \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

$$= 10^3 \begin{Bmatrix} -154.56 \\ 154.56 \end{Bmatrix} \begin{array}{c} 2 \\ 3 \end{array}$$

Element$^{(3)}$ $\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = 200\times10^3 \times 600 \times 11.7\times10^{-6} \times 80 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$.

$$= 10^3 \begin{Bmatrix} -112.32 \\ 112.32 \end{Bmatrix}$$

$\boxed{2/5}$

Global force Vector

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = 10^3 \times \begin{Bmatrix} -301.1904 \\ 301.1904 - 154.56 \\ 154.56 - 112.32 \\ 112.32 \end{Bmatrix}$$

$$= 10^3 \times \begin{Bmatrix} -301.1904 \\ 146.6304 \\ 42.24 \\ 112.32 \end{Bmatrix} \quad * \; 10^3 \times \begin{bmatrix} -301.1904 \\ 146.6304 - 60 \\ 42.24 - 75 \\ 112.32 \end{bmatrix}$$

* load is acting towards left $P_1 = -60 \times 10^3$
$P_2 = -75 \times 10^3$

Element Equation.

$$\cancel{10^3} \begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 389 & -140 & 0 \\ 0 & -140 & 440 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \cancel{10^3} \times \begin{bmatrix} -301.1904 \\ 86.6304 \\ -32.76 \\ 112.32 \end{bmatrix}$$

Apply the boundary Condition
$u_1 = 0, \; u_4 = 0$

$$\begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 389 & -140 & 0 \\ 0 & -140 & 440 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{bmatrix} = \begin{bmatrix} -301.1904 \\ 86.6304 \\ -32.76 \\ 112.32 \end{bmatrix}$$

In above equation, $u_1 = 0$, so, neglect first row and first column of [K] matrix, $U_4 = 0$ So, neglect fourth row and fourth column of [K] matrix. Hence the equation reduces to

[3/5]

$$\begin{bmatrix} 389 & -140 \\ -140 & 440 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 86.6304 \\ -32.76 \end{bmatrix}$$

$$389u_2 - 140u_3 = 86.6304$$

$$-140u_2 + 440u_3 = -32.76$$

Solving

$u_2 = 0.2212$ mm $\quad u_3 = -0.00345$ mm

Thermal stress $\sigma = E\frac{du}{dx} - E\alpha\Delta T$

For element(1) $\sigma_{(1)} = E_1\frac{(u_2 - u_1)}{L_1} - E_1\alpha_1\Delta T$

$$= \frac{83\times10^3[0.2212 - 0]}{800} - 83\times10^3 \times 18.9\times10^{-6}\times 80$$

$= -102.5455$ N/mm^2 [Compressive Stress]

For element(2) $\sigma_{(2)} = \frac{70\times10^3[-0.00345 - 0.2212]}{600} - 70\times10^3\times23\times10^{-6}\times80$

$\sigma_2 = -155.009$ N/mm^2 [Compressive Stress]

For Element(3)

$$\sigma_{(3)} = \frac{200\times10^3[0 + 0.00345]}{400} - 200\times10^3\times11.7\times10^{-6}\times80$$

$\sigma_{(3)} = -185.475$ N/mm^2 [Compressive stress]

[4/5]

Reaction force

$\{R\} = [K]\{U^*\} - \{F\}$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = 10^3 \begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 389 & -140 & 0 \\ 0 & -140 & 440 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \begin{bmatrix} 0 \\ 0.2212 \\ -0.00345 \\ 0 \end{bmatrix} - 10^3 \begin{bmatrix} -301.1904 \\ 86.6304 \\ -32.76 \\ 112.32 \end{bmatrix}$$

$$= 10^3 \begin{bmatrix} -55.0788 \\ 86.5018 \\ -32.486 \\ 1.035 \end{bmatrix} - 10^3 \begin{bmatrix} -301.1904 \\ 86.6304 \\ -32.76 \\ 112.32 \end{bmatrix}$$

$$= 10^3 \times \begin{bmatrix} 246.1116 \\ 0 \\ 0 \\ -113.35 \end{bmatrix}$$

Result.

Displacement

$u_1 = 0$

$u_2 = 0.2212\,mm$

$u_3 = -0.00345\,mm$

$u_4 = 0$

Stress

$\sigma_1 = -102.5465$

$\sigma_2 = -155.009\ N/mm^2$

$\sigma_3 = -185.475\ N/mm^2$

Reaction force

$R_1 = 246.1166 \times 10^3\ N$

$R_2 = -113.35 \times 10^3\ N$

[5/5]

For a tapered plate of uniform thickness t=10mm as shown in Figure.1, find the displacements at the nodes by meshing into two element model. The bar has mass density $\rho = 7800 kg/m^3$, $E = 2\times10^5 MN/m^2$. In addition to self-weight, the plate is subjected to a point load P=10kN at its center. Also determine the reaction force at the support.

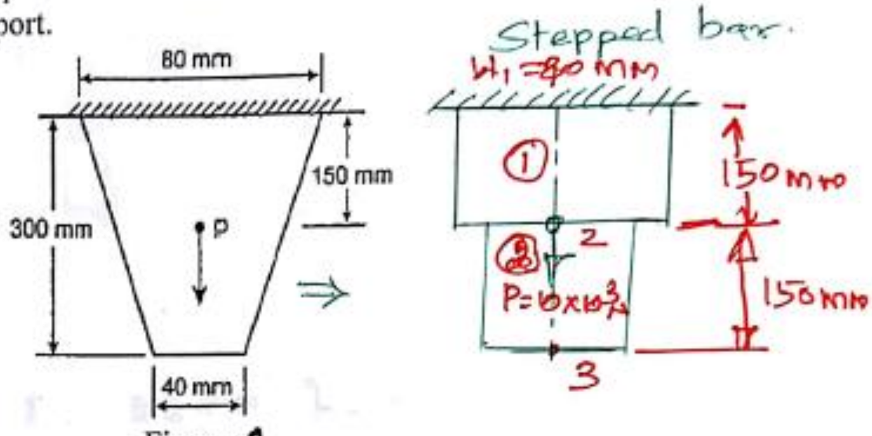


Figure.1

Solution

Area at node 1, A_1

$= \text{Width} \times \text{thickness} = W_1 \times t_1$

$= 80\times10$

$A_1 = 800\, mm^2$

Area at node 2, A_2

$A_2 = \left[\frac{W_1 + W_3}{2}\right] \times t_2$

$= \left[\frac{80+40}{2}\right] \times 10$

$A_2 = 600\, mm^2$

$[t_1, t_2, t_3 = 10\, mm]$

Area at node 3, A_3

$A_3 = W_3 \times t_3 = 40\times10$

$= 400\, mm^2$

Average area of element (1) $A_1^{(1)} =$

$= \frac{\text{Area at node 1} + \text{Area at node 2}}{2}$

$= \frac{800+600}{2}$

$A_1^{(1)} = 700\, mm^2$

Average Area of element (2) $A_2^{(2)}$

$= \frac{A_2 + A_3}{2}$

$= \frac{600+400}{2}$

$A_2^{(2)} = 500\, mm^2$

Mass density $\rho = 7800\ kg/m^3 = 7800 \times 9.81\ N/m^3$
$= 76518\ N/m^3 = 76518 \times 10^{-9}\ N/mm^3$
$= 7.6518 \times 10^{-5}\ N/mm^3$

Young's Modulus $E = 2 \times 10^5\ MN/m^2$
$= 2 \times 10^5 \times 10^6\ N/m^2$
$= 2 \times 10^5 \times 10^6 \times 10^{-6}\ N/mm^2$
$= 2 \times 10^5\ N/mm^2$

Stiffness matrix for element 1

$$K_1 = \frac{A_1^{(1)} E}{L_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{700 \times 2 \times 10^5}{150}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 4.666 \times 2 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^5 \begin{array}{cc} 1 & 2 \\ \begin{bmatrix} 9.332 & -9.332 \\ -9.332 & 9.332 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

Stiffness matrix for element 2

$$K_2 = \frac{A_2^{(2)} E}{L_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{500 \times 2 \times 10^5}{150}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 6.666 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 10^5 \begin{array}{cc} 2 & 3 \\ \begin{bmatrix} 6.666 & -6.666 \\ -6.666 & 6.666 \end{bmatrix} \end{array}$$

Global matrix $[K] = K_1 + K_2$

$$= 10^5 \times \begin{bmatrix} 9.332 & -9.332 & 0 \\ -9.332 & \boxed{9.332 + 6.666} \xrightarrow{} 15.998 & -6.666 \\ 0 & -6.666 & 6.666 \end{bmatrix}$$

Displacement vector $U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

Body force Vector $\{F\} = \frac{\rho A L}{2}\begin{bmatrix}1\\1\end{bmatrix}$

Force Vector For element ①

$$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{\rho_1 A_1^{(1)} L_1}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= 7.6518\times10^{-5}\times700\times150\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \begin{array}{c}\begin{bmatrix}4.017\\4.017\end{bmatrix}\end{array}\begin{array}{c}1\\2\end{array}$$

Force Vector for element ②

$$\begin{bmatrix}F_2\\F_3\end{bmatrix} = \frac{\rho_2 A_2^{(2)} L_2}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= 7.6518\times10^{-5}\times500\times150\begin{Bmatrix}1\\1\end{Bmatrix}$$

$$= \begin{array}{c}\begin{bmatrix}2.869\\2.869\end{bmatrix}\end{array}\begin{array}{c}2\\3\end{array}$$

Global force vector

$$\begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}4.017\\4.017+2.869\\2.869\end{Bmatrix} = \begin{Bmatrix}4.017\\6.886\\2.869\end{Bmatrix}$$

Assemble the finite element equation $[K]\{u\}=\{F\}$

$$10^5\begin{bmatrix}9.332 & -9.332 & 0\\-9.332 & 15.998 & -6.666\\0 & -6.666 & 6.666\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}4.017\\6.886\\2.869\end{bmatrix}$$

Apply the boundary condition ie at node 1 displace $u_1 = 0$.

A point load of 10×10^3 N is acting at node 2, so add 10,000 N in F_2 vector

$$10^5 \begin{bmatrix} 9.332 & -9.332 & 0 \\ -9.332 & 15.998 & -6.666 \\ 0 & -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 4.017 \\ 10006.886 \\ 2.869 \end{bmatrix}$$

In the above equation $u_1 = 0$, so neglect first row and first column of [K] matrix. The reduced equation is.

$$10^5 \begin{bmatrix} 15.998 & -6.666 \\ -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 10006.886 \\ 2.869 \end{bmatrix}$$

$$[15.998 U_2 - 6.666 U_3] 10^5 = 100006.886 \quad \text{—} (1)$$

$$[-6.666 U_2 + 6.666 U_3] 10^5 = 2.869 \quad \text{—} (2)$$

Solve above equation

$u_2 = 0.01073$ mm

$u_3 = 0.01073$ mm

4/5

Reaction force $\{R\} = [K]\{U\} - \{F\}$

$$= 10^5 \begin{bmatrix} 9.332 & -9.332 & 0 \\ -9.332 & 15.998 & -6.666 \\ 0 & -6.666 & 6.666 \end{bmatrix} \begin{bmatrix} 0 \\ 0.01073 \\ 0.01073 \end{bmatrix} = \begin{bmatrix} 4.017 \\ 10,006.886 \\ 2.869 \end{bmatrix}$$

$$= \begin{bmatrix} -10,000 \\ 10,000 \\ 0 \end{bmatrix} - \begin{bmatrix} 4.017 \\ 10,006.886 \\ 2.869 \end{bmatrix}$$

$$= \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -10004.017 \\ -6.886 \\ -2.869 \end{Bmatrix} N$$

Reaction force is equivalent and opposite to applied force.

Verification $R_1 + R_2 + R_3$

$= -10004.017 - 6.886 - 2.869$

$= -10013.772$ N

Applied force $= 4.017 + 10,006.886 + 2.869$

$= 10013.772$ N

Result: Displacement $U_1 = 0$

$U_2, U_3 = 0.01073$ mm.

Reaction force at the suppor

$R_1 = -10004.017$ N

5/5

01-Solve the following system of equation using gauss elimination method.

$$x_1 - x_2 + x_3 = 1,\ \ -3x_1 + 2x_2 - 3x_3 = -6,\ \ 2x_1 - 5x_2 + 4x_3 = 5$$

In form matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -3 \\ 2 & -5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & -3 & 2 & 3 \end{bmatrix}$$

$\Rightarrow R_2 = [R_2 + 3R_1]$

$\Rightarrow R_3 = [R_3 - 2R_1]$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 2 & 12 \end{bmatrix}$$

$\Rightarrow R_3 = [R_3 - 3R_2]$

$$2x_3 = 12$$
$$-x_2 = -3$$
$$x_1 - x_2 + x_3 = 1$$

$\mathrm{Re}\,sult:$

$x_3 = 6, x_2 = 3, x_1 = -2$

It is a method which is easily adapted to the computer and is based on triangularization of the coefficient matrix and evaluation of the unknowns by back- substitution starting from the last equation

02-Describe the step by step procedure of solving FEA.

GeneralStepstobefollowedwhilesolvingastructuralproblembyusingFEM:
1.Discretize and select the element type
2.Choose a displacement function
3.Define the strain/displacement and stress/ strain relationships
4.Derive the element stiffness matrix and equations by using direct or variational or Galerkin's approach
5.Assemble the element equations to obtain the global equations and introduce boundary conditions
6.Solve for the unknown degrees of freedom or generalized displacements
7.Solve for the element strains and stresses
8.Interpret the results

Step 1. Discretize and Select element type

- Dividing the body in to an equivalent system of finite elements with associated nodes
- Choose the most appropriate element type
- Decide what number, size and the arrangement of the elements
- The elements must be made small enough to give us able results and yet large enough to reduce computation effort

Step 2. Selection of the displacement function

- Choose displacement function within the element using nodal values of the element
- *Linear, quadratic, cubic polynomials can be used*
- The same displacement function can be used repeatedly for each element

Step 3. Define the strain/displacement and stress/strain relationships

- Strain/displacement and stress/strain relationships are necessary for deriving the equations for each element
- In case of 1-D, deformation, say in x-direction is given by, $\varepsilon = \frac{du}{dx}$
- Stress / strain law is , Hooke's law given by, $\sigma_x = E\varepsilon_x$

Step 4. Derive element stiffness matrix and equations

Following methods can be used

- Direct equilibrium method
- Work or energy methods
- Method of weighted residuals such as Galerkin's method

Any one of the above methods will produce the equations to describe the behavior of an element

The equations are written conveniently in matrix form as, $\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$

Step 5. Assemble the element equations to obtain the global or total equations and introduce boundary conditions

- The element equations generated in the step 4 can be added together using the method of superposition
- The final assembled or global equations will be of the matrix form $[K]\{u\}=\{F\}$
- Now introduce the boundary conditions or supports or constraints
- Invoking boundary conditions results in a modification of the global equation

Step 6. Solve for the unknown degrees of After introducing boundary conditions, we get a set of simultaneous algebraic equations and these equations can be written in the expanded form as

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ \\ \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} - - - - - - - K_{1n} \\ K_{21} & K_{22} & K_{23} - - - - - - - K_{2n} \\ K_{31} & K_{32} & K_{33} - - - - - - - K_{3n} \\ \\ \\ K_{n1} - - - - - - - - - - - - - K_{nn} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \\ \\ d_n \end{Bmatrix}$$

The above equations can be solved for unknown degrees of freedom by using an elimination method such as Gauss ‘s method or an iteration method such as the Gauss-Seidel method

Step7. Solve for the element strain and stress

Secondary quantities such as strain and stress , moment or shear force can now be obtained

Step 8. Interpret the results

- The final goal is to interpret and analyze the results for use in design / analysis process.
- Determine the locations where large deformations and large stresses occur in the structure
- Now make design and analysis decisions

03-Derive the stiffness matrix for one dimensional two nodded bar element.

Stiffness matrix of a 1-D bar element

Stiffness matrix [K]

$$= \int_v [B]^T [D][B]\, dv$$

$\underbrace{D = E}_{\text{young's modulus}}$

$[dv = A \cdot dx]$

Displacement function $U = N_1 u_1 + N_2 U_2$

$$N_1 = \frac{L - x}{L} \qquad N_2 = \frac{x}{L}$$

[B] Strain displacement matrix

$$= \begin{bmatrix} \frac{dN_1}{x} & \frac{dN_2}{x} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix}$$

$$[B]^T = \begin{bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{bmatrix}$$

Substitute [B], [D], $[B]^T$ values in Stiffness matrix equation.

$$K = \int_0^L \begin{bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{bmatrix} E \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} A\, dx$$

$$= \int_0^L \begin{bmatrix} \frac{1}{L^2} & -\frac{1}{L^2} \\ -\frac{1}{L^2} & \frac{1}{L^2} \end{bmatrix} EA \, dx$$

$$= AE \begin{bmatrix} \frac{1}{L^2} & -\frac{1}{L^2} \\ \frac{-1}{L^2} & \frac{1}{L^2} \end{bmatrix} \int_0^L dx$$

$$= AE \begin{bmatrix} \frac{1}{L^2} & -\frac{1}{L^2} \\ \frac{-1}{L^2} & \frac{1}{L^2} \end{bmatrix} [x]_0^L$$

$$= AE \begin{bmatrix} \frac{1}{L^2} & -\frac{1}{L^2} \\ \frac{-1}{L^2} & \frac{1}{L^2} \end{bmatrix} [L - 0]$$

$$= \frac{AE\cancel{L}}{\cancel{L^2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

The Properties of a stiffness matrix
- It is Symmetrix
- The Sum of elements in any Column is equal to zero

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04-List the advantages, disadvantages and applications of FEM.

Applications of FEM:

- ***Equilibrium problems or time independent problems***.

e. g. i) To find displacement distribution and stress distribution for a mechanical or thermal loading in solid mechanics. ii) To find pressure, velocity, temperature, and density distributions of equilibrium problems in fluid mechanics.

- ***Eigenvalue problems of solid and fluid mechanics***.

e. g. i) Determination of natural frequencies and modes of vibration of solids and fluids. ii) Stability of structures and the stability of laminar flows.

Time-dependent or propagation problems of continuum mechanics*. e.g. This category is composed of the problems that results when the time dimension is added to the problems of the first two categories.*

Engineering applications of the FEM:

- Civil Engineering structures
- Air-craft structures
- Heat transfer
- Geomechanics
- Hydraulic and water resource engineering and hydrodynamics
- Nuclear engineering
- Biomedical Engineering
- Mechanical Design- stress concentration problems, stress analysis of pistons, composite materials, linkages, gears, stability of linkages, gears and machine tools. Cracks and fracture problems under dynamic loads etc

Advantages of Finite Element Method

- *Model irregular shaped bodies quite easily*
- *Can handle general loading/ boundary conditions*
- *Model bodies composed of composite and multiphase materials because the element equations are evaluated individually*
- *Model is easily refined for improved accuracy by varying element size and type*
- *Time dependent and dynamic effects can be included*
- *Can handle a variety nonlinear effects including material behaviour, large deformation, boundary conditions etc.*

Disadvantages:

- *Needs computer programmes and computer facilities*
- *The computations involved are too numerous for hand calculations even when solving very small problems*
- *Computers with large memories are needed to solve large complicated problems*

Computer Programmes for the FEM

1. *Algor*
2. *ANSYS – Engineering Analysis System*
3. *COSMOS/M*
4. *STARDYNE*
5. *IMAGES-3D*
6. *MSC/NASTRAN- NASA Structural Analysis*
7. *SAP90- Structural Analysis Programme*
8. *GT- STRUDL – Structural Design Language*
9. *SAFE- Structural Analysis by Finite Elements*
10. *NISA- Non linear Incremental Structural Analysis etc.*