Test of significance of small samples:

Variance ratio test (or) F- test for equality of variance

Null hypothesis : $H_0 : \sigma_1^2 = \sigma_2^2$

Test statistic $F = \dfrac{S_1^2}{S_2^2}$ where $S_1^2 > S_2^2$

Where $S_1^2 = \dfrac{n_1 s_1^2}{n_1 - 1}$ (or) $S_1^2 = \dfrac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1}$

$S_2^2 = \dfrac{n_2 s_2^2}{n_2 - 1}$ (or) $S_2^2 = \dfrac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}$

Degrees of freedom : $(V_1 : V_2)$

Where $V_1 = (n_1 - 1)$, $V_2 = (n_2 - 1)$

Note : Always F Greater than one

Suppose $S_2^2 > S_1^2$ then $F = \dfrac{S_2^2}{S_1^2}$

with d.o.f $V_1 = (n_2 - 1)$, $V_2 = (n_1 - 1)$

Example ①

Two random sample of 11 and 9 items show that the sample standard deviation of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions are normal test the hypothesis that the true variance are equal, against the alternative hypothesis that they are not.

Given :

$n_1 = 11$ $n_2 = 9$

$s_1 = 0.8$ $s_2 = 0.5$

Step 1: Formulate H_0 & H_1

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$ (Two tailed test)

Step 2: LOS $\alpha = 0.05$

Step 3: Test statistic

$$F = \frac{S_1^2}{S_2^2}$$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{11(0.8)^2}{10} = \frac{7.04}{10} = 0.704$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{9(0.5)^2}{9-1} = \frac{2.25}{8} = 0.28125$$

$$F = \frac{\overset{0.704}{\cancel{7.04}}}{0.2812} \Rightarrow 2.503$$

Step 4: Critical value

$$D.o.f = (V_1 \cdot V_2)$$

$$V_1 = (n_1 - 1) \Rightarrow 10$$

$$V_2 = (n_2 - 1) \Rightarrow 8$$

$$D.o.f = (\overset{C}{10} \cdot \overset{R}{8})$$

$$F_{tab} = 3.35$$

Step 5: Conclusion

$$F = 2.503 < 3.35 = F_{tab}$$

$\therefore H_0$ is accepted.

Example 2:

Two random sample have the following results

Sample	Size	sample mean	sum of square of deviation from the mea
1	12	14	108
2	10	15	90.

Test whether the samples came from the same population.

Given:

$n_1 = 12$, $n_2 = 10$

$\bar{x}_1 = 14$, $\bar{x}_2 = 15$

$\Sigma(x_1 - \bar{x}_1)^2 = 108$, $\Sigma(x_2 - \bar{x}_2)^2 = 90$.

$$S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{108}{12-1} = \frac{108}{11} = 9.818$$

$$S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{90}{10-1} = 10$$

$\therefore$ $S_1^2 < S_2^2$

Step 1: Formulate H_0 & H_1

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Step 2: LOS at $\alpha = 5\% = 0.05$

Step 3: Test Statistic,

$$F = \frac{S_2^2}{S_1^2} = \frac{10}{9.818}$$

$F = 1.018$

Step 4: Degrees of freedom $= (V_1 \cdot V_2)$

$= (n_2 - 1), (n_1 - 1)$

$= (9, 11)$

Critical value, $F_\alpha = 2.90$

Step 5: Conclusion:

$F = 1.018 < 2.90 = F_\alpha$

$\therefore$ H_0 is accepted