

UNIT - IV

TESTING OF HYPOTHESIS

Basic Definitions:

Population:

A population is used to refer any collection of individual it may be finite or infinite.

Sample:

A Sample is a small portion selected from the population and the process of drawing a sample from a population is called sampling.

Sample size:

The no. of individual in a selected sample is called the sample size.

Parameter and Statistics:

Any statistical method computed from population data is known as parameter and any statistical method computed from sample data is known as statistics.

Actual value of parameter

Estimated value of parameter

Estimated value of parameter

NOTATIONS :

| MEASURE | POPULATION | SAMPLE |
|--------------------|------------|-----------|
| Size | N | n |
| Mean | μ | \bar{x} |
| Standard deviation | σ | s |
| proportion | p | \hat{p} |
| Variance | σ^2 | s^2 |

Sampling Distribution:
The various value of statistics so obtained may be arrange as a frequency distribution which is known as sampling distribution.

Standard Error:

The standard deviation of sampling distribution of a statistics is known as its standard error, abbreviated as S.E (ie avg. amount of variability from the observation of sampling distribution.)

Statistical Hypothesis:

In attempting to each decision about population on the basis of sample observations, we make assumptions on the basis of sample observations,

We make assumptions about population, which are not necessarily true, are called statistical hypothesis.

Null hypothesis:

Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that is true and is denoted by H_0 .

[i.e] Hypothesis of no difference]

Alternative Hypothesis:

A hypothesis that is complementary to Null hypothesis is called alternative hypothesis and it is denoted by H_1 .

A procedure for deciding whether to accept or reject the null hypothesis is called the alternative hypothesis and is denoted by H_1 .

A procedure for deciding whether to accept or reject the null hypothesis is called the test of hypothesis.

level of Significance:

α is the probability level below which the null hypothesis is rejected generally 5% and 1% level of significance are used.

Critical region (or) Region of rejection:

The critical region of a test of statistical hypothesis is that region which leads to rejection of null hypothesis, H_0 . Those region which leads to the acceptance of H_0 is called acceptance region.

Error in Sampling:

Errors are type I, type II errors.

type I error : Reject H_0 when it is true

type II error : Accept H_0 when it is false.

$$P(\text{Type I error}) = \alpha \quad \& \quad P(\text{Type II error}) = \beta$$

One tail or two tail test:

If M_0 is population parameter > M
is the sample statistic, then the null hypothesis
is given by

$$H_0: \mu = M_0$$

Alternative hypothesis is given by,

$$H_1: \mu \neq M_0 \text{ (Two-tailed)}$$

$$H_1: \mu > M_0 \text{ (Right tailed) (one tail)}$$

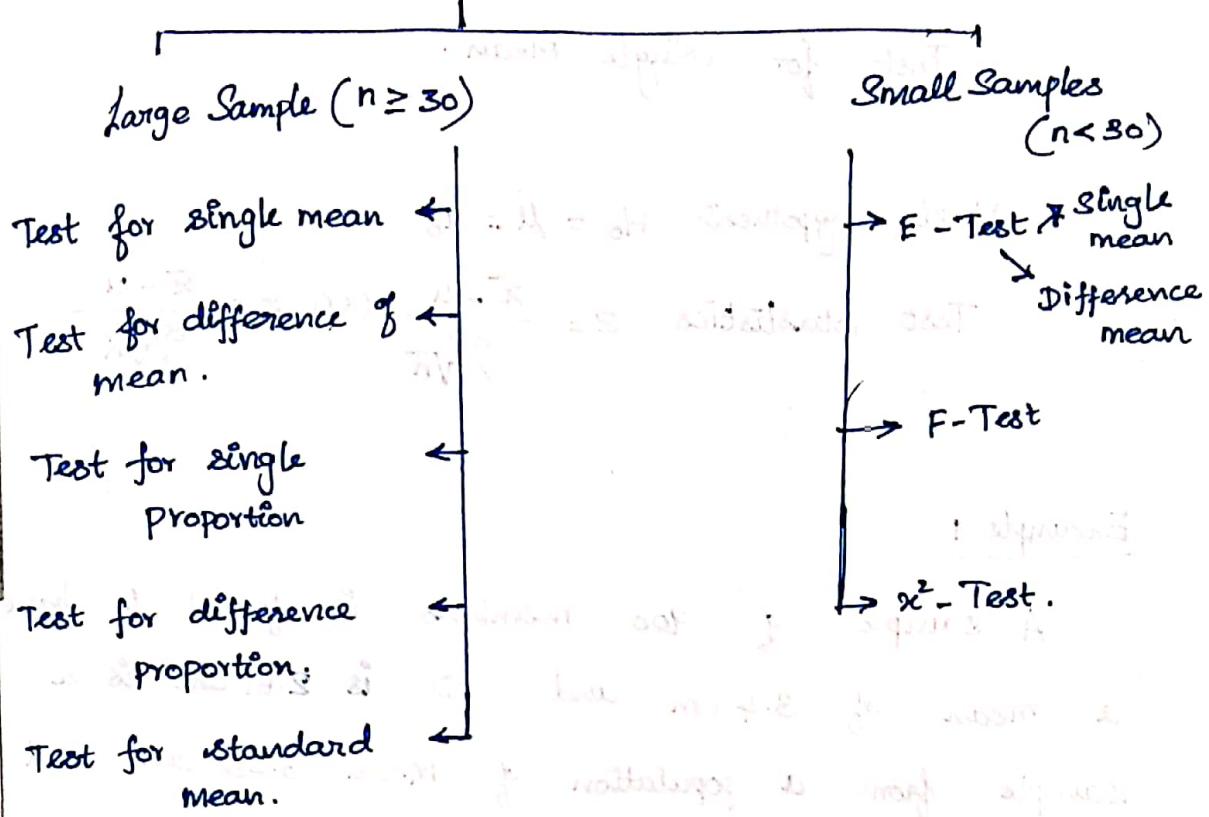
$$H_1: \mu < M_0 \text{ (Left tailed) (one tail)}$$

Procedure for testing a hypothesis:

- 1) Formulate H_0 and H_1
- 2) choose the level of significance α
- 3) compute the test statistic, using the data available
- 4) Pick out the critical value from the tabulation
- 5) Conclusion:

Compare the computed value of the test statistic with the critical value at the given level of significance.

Sampling



Large Samples ($n \geq 30$)

Critical Values (or) Significant values:

The sample values of the statistic beyond which the null hypothesis will be rejected are called critical values (or) significant values.

| Matures of Test | Level of Significance 1%. | Significance 5%. | Significance 10%. |
|------------------------------------|------------------------------|---------------------|----------------------|
| Two tailed test ($Z_{\alpha/2}$) | 2.58 | 1.96 | 1.645 |
| One tailed test (Z_{α}) | 2.33 | 1.645 | 1.28 (Right) |
| | -2.33 | -1.645 | -1.28 (Left) |

Test of significance of large samples

Test for single mean.

Null hypothesis $H_0: \mu = \mu_0$

Test statistics $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ (or) $Z = \frac{\bar{x} - \mu}{S/\sqrt{n}}$

Example 1

A sample of 900 members is found to have a mean of 3.4 cm and SD is 2.61 cm is a sample from a population of Mean 3.25 cm and SD is 2.61 cm. If the population is normal & its mean is unknown. Find the 95% confidential limits of true mean.

Given:

$$n = 900$$

$$\bar{x} = 2.61 \text{ cm}$$

$$\mu = 3.25 \text{ cm}$$

$$\sigma = 2.61 \text{ cm}$$

Step 1:

formulate H_0 & H_1 .

$$H_0: \mu = 3.25 \text{ cm}$$

$$H_1: \mu \neq 3.25 \text{ cm} \quad (\text{Two tailed})$$

Step 2:

level of significance $\Rightarrow 5\% = 0.05$

Step 3:

Test the statistics

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \Rightarrow \frac{3.4 - 3.25}{2.61/\sqrt{900}} = \frac{0.15}{2.61/30} \\ = \frac{4.5}{2.61} \\ = 1.724$$

Step 4:

Critical value at 5% is $Z_{\alpha} = 1.96$

Step 5: Conclusion

$$|Z| = 1.724 < 1.96 = Z_{\alpha}$$

H_0 is accepted at 5% level of significance.

∴ The sample is taken from the population

Example 2:

A random sample of 200 employees at a large corporation, should their avg to be 42.8 yrs with a standard deviation of 6.89 yrs. Test the hypothesis

$H_0: \mu = 40$, $H_1: \mu > 40$ at $\alpha = 0.01$ (U) of significance.

Given:

$$n = 200$$

$$\bar{x} = 42.8 \text{ yrs. } \sigma = 6.89 \text{ yrs.}$$

$$\mu = 40.$$

Step 1: Formulate H_0 & H_1 ,

$$H_0: \mu = 40$$

$$H_1: \mu > 40 \text{ (right one tail) - Test}$$

Step 2: Level of significance $\alpha = 0.01$

Step 3: Test the statistics.

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{42.8 - 40}{6.89 / \sqrt{200}}$$

$$Z = 5.747$$

Step 4: Critical Value }
at 0.01 } $Z_\alpha = 2.33$

Step 5: Conclusion

$$|z| = 5 \cdot 747 > 2.33 - z_{\alpha}$$

∴ H_0 is Rejected at 1% level of significance

∴ The hypothesis $H_1: \mu > 40$ is accepted.