SNS COLLEGE OF TECHNOLOGY

AN AUTONOMOUS INSTITUTION

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TAYLOR'S SERIES EXPANSION

point (a,b), $h = x-a$; $k = y-b$

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[h f_x + k f_y\right]$$

$$+ \frac{1}{2!}\left[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}\right]$$

$$+ \frac{1}{3!}\left[h^3 f_{xxx} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy} + k^3 f_{yyy}\right]$$

$$+ \cdots$$

1 Expand $e^x \cos y$ as a taylor series in powers of x and y upto third degree

$f(x,y) = e^x \cos y$

point is not given directly but $h = x - a \Rightarrow h = x$
$k = y - b \Rightarrow k = y$
In qn: given that in powers of x and y.
so, $h = x$, $k = y$
$a = 0$, $b = 0$

$\Rightarrow (a,b) = (0,0)$

$h = x$

$k = y$

$f(a,b) = f(0,0)$

$= e^0 \cos 0$

$f(a,b) = 1$

$f(x,y) = e^x \cos y$	At pt $(0,0)$
$f_x = e^x \cos y$	1
$f_y = -e^x \sin y$	0
$f_{xx} = e^x \cos y$	1
$f_{xy} = -e^x \sin y$	0
$f_{yy} = -e^x \cos y$	-1
$f_{xxx} = e^x \cos y$	1
$f_{xxy} = -e^x \sin y$	0

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$f_{xyy} = -e^x \cos y$ | $\cancel{-e^x \cos y}$ -1

$f_{yyy} = +e^x \sin y$ | $\cancel{+e^x \sin y}$ 0

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[h f_x + k f_y\right]$$
$$+ \frac{1}{2!}\left[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}\right]$$
$$+ \frac{1}{3!}\left[h^3 f_{xxx} + 3h^2k f_{xxy} + 3hk^2 f_{xyy} + k^3 f_{yyy}\right]$$

$$= 1 + \frac{1}{1!}\left[x(1) + y(0)\right] + \frac{1}{2!}\left[x^2(1) + 2xy(0) + y^2(-1)\right]$$
$$+ \frac{1}{3!}\left[x^3(1) + 3x^2y(0) + 3xy^2(-1) + y^3(0)\right]$$
$$= 1 + \frac{1}{1!}[x] + \frac{1}{2!}[x^2 - y^2] + \frac{1}{3!}[x^3 - 3xy^2] + \cdots$$
$$= 1 + x + \frac{1}{2}(x^2 + y^2) + \frac{1}{6}(x^3 - 3xy^2) + \cdots$$

2. $e^x \sin y$ at $(0,0)$

$f(x,y) = e^x \sin y$

$(a,b) = (0,0)$

$h = x$, $k = y$

$f(a,b) = f(0,0)$

$= e^0 \sin 0$

$f(a,b) = 0$

$f(x,y) = e^x \sin y$	At pt $(0,0)$
$f_x = e^x \sin y$	0
$f_y = e^x \cos y$	1
$f_{xx} = e^x \sin y$	0
$f_{xy} = e^x \cos y$	1
$f_{yy} = -e^x \sin y$	0
$f_{xxx} = e^x \sin y$	0
$f_{xxy} = e^x \cos y$	1
$f_{xyy} = -e^x \sin y$	0
$f_{yyy} = -e^x \cos y$	-1

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[h f_x + k f_y\right]$$
$$+ \frac{1}{2!}\left[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}\right]$$
$$+ \frac{1}{3!}\left[h^3 f_{xxx} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy} + k^3 f_{yyy}\right]$$

$$= 0 + \frac{1}{1!}\left[x(0) + y(1)\right] + \frac{1}{2!}\left[x^2(0) + 2xy(1) + y^2(0)\right] + \frac{1}{3!}\left[x^3(0) + 3x^2y(1) + 3xy^2(0) + y^3(-1)\right] + \cdots$$

$$= \frac{1}{1!}[y] + \frac{1}{2!}[2xy] + \frac{1}{3!}[3x^2y - y^3] + \cdots$$

$$f(x,y) = y + \frac{1}{2}(2xy) + \frac{1}{6}(3x^2y - y^3) + \cdots$$

$$f(x,y) = y + xy + \frac{1}{6}(3x^2y - y^3) + \cdots$$

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Taylor's series

3. Expand $e^x \log(1+y)$ in power of x & y upto 2nd degree.

$f(x,y) = e^x \log(1+y)$

$(a,b) = (0,0)$

$h = x$

$k = y$

$\boxed{\log(1) = 0}$

$f(a,b) = f(0,0)$

$= (1) \log(1)$

$= 1 (0)$

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$f(a,b) = 0$

$f(x,y) = e^x \log(1+y)$	At pt $(0,0)$
	0
$f_x = e^x \log(1+y)$	1
$f_y = e^x \frac{1}{(1+y)}(1)$	
$f_{xx} = e^x \log(1+y)$	0
$f_{xy} = e^x \frac{1}{(1+y)}$	1
$f_{yy} = e^x \left(\frac{-1}{(1+y)^2}\right)(1)$	-1

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[hf_x + kf_y\right] + \frac{1}{2!}\left[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}\right] + \cdots$$

$$= 0 + \frac{1}{1!}\left[x(0) + y(1)\right] + \frac{1}{2!}\left[x^2(0) + 2xy(1) + y^2(-1)\right] + \cdots$$

$$= \frac{1}{1}(0+y) + \frac{1}{2}\left[0 + 2xy - y^2\right] + \cdots$$

$$= y + \frac{1}{2}(2xy - y^2) + \cdots$$

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4. Expand $xy^2+2x-3y$ in powers of $(x+2)$ & $(y-1)$ upto 2nd degree.

$f(x,y) = xy^2+2x-3y$

$h = x - a \Rightarrow a = -2$

$k = y - b \Rightarrow b = 1$

$(a,b) = (-2,1)$

$h = x+2$

$k = y-1$

$f(a,b) = f(-2,1)$

$= (-2)(1) + 2(-2) - 3(1)$

$= -2-4-3$

$f(a,b) = -9$

$f(x,y) = xy^2+2x-3y$	At $(-2,1)$
$f_x = y^2+2$	3
$f_y = 2xy-3$	-7
$f_{xx} = 0$	0
$f_{xy} = 2y$	2
$f_{yy} = 2x$	-4

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$$f(x,y) = f(a,b) + \frac{1}{1!}\left[h f_x + k f_y\right] + \frac{1}{2!}\left[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}\right] + \ldots$$

$$= -9 + \frac{1}{1!}\left[(x+2)3 + (y-1)-7\right] + \frac{1}{2!}\left[(x+2)^2 0 + 2(x+2)(y-1)2 + (y-1)^2(-4)\right] + \ldots$$

$$= -9 + 1\left[3x + 6 - 7y + 7\right] + \frac{1}{2}\left[4(x+2)(y-1) - 4(y-1)^2\right] + \ldots$$

$$= -9 + [3x - 7y + 13) + \frac{1}{2}\left[4(x+2)(y-1) - 4(y-1)^2\right] + \ldots$$

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