Puzzles in Systems and Control

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Abstract. This tour of four puzzles is to highlight fundamental and surprising open problems in linear systems and control theory, ranging from system identification to robust control. The first puzzle deals with complications caused by fragility of poles and zeros in standard parameterizations of systems due to unavoidable parameter inaccuracies. The second and the third puzzle are concerned with difficulties in generalizing robust H_∞ control theory to realistic persistent signal setups. Surprisingly it seems very difficult to generalize the theory to such setups and simultaneously keep the terminology robust H_∞ control intact. These puzzles have implications also to model validation. The fourth puzzle, due to Georgiou and Smith, deals with difficulties in doubly-infinite time axis formulations of input-output stabilization theory.

1 Introduction

Linear systems have been claimed to be, at regular time intervals, a mature subject that can not yield new big surprises. There are, however, many misunderstandings and messy fundamental issues that need to be cleaned up. These are often known only to a small group of specialists, so the purpose of this paper is to discuss four such neglected topics, misunderstandings, or puzzles, in a tutorial fashion.

The 1970s witnessed the growth of stochastic system identification theory. The text booxs [13], [3], [25] summarize many of the developments obtained during this very dynamic phase that lasted to the mid 1980s. Developments in robust control dominated the 1980s. The text books [26], [9], [31] summarize many of the developments during the very dynamic phase of robust control research that lasted to the first part of the 1990s.

System identification and model validation for robust control design have been two popular research themes in the 1990s (see e.g. the special issues, IEEE TAC, July 1992; Automatica, December 1995; MMOS, January 1997, and the monograph [22]). So what could be more appropriate than to start our tour with an often overlooked puzzle that has deep implications for control, systems and system identification. This is the puzzle of the wandering poles and zeros. That is, the exponential explosion of the accuracy required, as a function of system order, in many standard system models and representations to reproduce system poles and zeros at least in a qualitatively correct manner. The sensitivity of poles and zeros is an old finding but we shall try to put the puzzle in a more quantitative form.

We are all familiar with least squares and prediction error identification of AR, ARX, and ARMAX models [13]. Such models are popular in pole placement and minimum variance type control design procedures [2]. The examples we provide should shake the reader's confidence in black box methodology and in such control desing methods that rely on accurate knowledge of system poles and zeros. Fortunately, modern robust control theory provides a partial rescue of black box identification methodology.

Recently several difficulties have surfaced in H_∞ control and systems theory. In the present work we shall discuss and study three such puzzles. One of them is the Georgiou-Smith puzzle in doubly-infinite time axis input-output H_∞ stabilization theory [8]. Georgiou and Smith [8] conclude their study by stating that in such a setting linear time-invariant systems with right-half plane poles can not be considered to be both causal and stabilizable. As this puzzle is very technical we shall leave it as the last puzzle of our tour. The second puzzle of our tour deals with difficulties with standard bounded power signal set formulations of H_∞ control [6],[31] due to e.g. lack of closedness with respect to addition of signals in such sets. The third puzzle has to do with the difficulty to define frequency domain concepts for general H_∞ functions and H_∞ uncertainty models, in contrast to what is assumed in many papers and books. These have also consequences for H_∞ model validation [23].

2 Puzzle A : Wandering Poles and Zeros

The sensitivity, or fragility, of the roots of polynomials to small coefficient perturbations is a well-recognized issue in numerical mathematics. This problem is sometimes discussed briefly in the connection of implementation issues for controllers [2]. This is, however, only a part of the puzzle of poles and zeros.

Let us study two examples. We have used in all computations the numerical and symbolic mathematics package MAPLE$^{\text{TM}}$. The computations have been directly latex documented within the Scientific Workplace$^{\text{TM}}$ package using its MAPLE facility, so as to minimize the risk for typographical errors in reproducing long expressions.

Example A.1 : System 1

Let us study the system

$$G_n(q^{-1}) = \frac{0.30003^n q^{-1}}{(1+0.69997q^{-1})^n} = \frac{0.30003^n q^{n-1}}{(q+0.69997)^n},\; n \geq 1. \tag{1}$$

Here q and q^{-1} denote the forward and the backward time shift operator, respectively (so that $(qu)(t) = u(t+1)$, $(q^{-1}u)(t) = u(t-1)$). This system is