SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

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STATE TRANSITION MATRIX

INTRODUCTION

We can write the solution of the ***homogeneous*** state equation

$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$ Laplace transform $s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s)$

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{x}(0) \quad \Rightarrow \quad \mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(0)$$

The inverse Laplace transform $\mathbf{x}(t) = \mathcal{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]\mathbf{x}(0)$

Note that $(s\mathbf{I} - \mathbf{A})^{-1} = \frac{\mathbf{I}}{s} + \frac{\mathbf{A}}{s^2} + \frac{\mathbf{A}^2}{s^3} + \cdots$

$$\mathcal{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}] = \mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \cdots = e^{\mathbf{A}t}$$

INTRODUCTION

Hence, the inverse Laplace transform of $(s\mathbf{I} - \mathbf{A})^{-1}$

$$\mathcal{L}^{-1}\left[(s\mathbf{I} - \mathbf{A})^{-1}\right] = \mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \cdots = e^{\mathbf{A}t}$$

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0)$$

State-Transition Matrix $\mathbf{x}(t) = \mathbf{\Phi}(t)\mathbf{x}(0)$

where $\mathbf{\Phi}(t)$ is an $n \times n$ matrix and is the unique solution of

$$\dot{\mathbf{\Phi}}(t) = \mathbf{A}\mathbf{\Phi}(t), \qquad \mathbf{\Phi}(0) = \mathbf{I}$$

Where

$\mathbf{\Phi}(t) = e^{\mathbf{A}t} = \mathcal{L}^{-1}\left[(s\mathbf{I} - \mathbf{A})^{-1}\right]$ Note that $\mathbf{\Phi}^{-1}(t) = e^{-\mathbf{A}t} = \mathbf{\Phi}(-t)$

Properties of State Transition Matrix

1. $\Phi(0) = e^{\mathbf{A}0} = \mathbf{I}$
2. $\Phi(t) = e^{\mathbf{A}t} = \left(e^{-\mathbf{A}t}\right)^{-1} = \left[\Phi(\ \ t)\right]^{-1}$ or $\Phi^{-1}(t) = \Phi(\ \ t)$
3. $\Phi(t_1 + t_2) = e^{\mathbf{A}(t_1+t_2)} = e^{\mathbf{A}t_1}e^{\mathbf{A}t_2} = \Phi(t_1)\Phi(t_2) = \Phi(t_2)\Phi(t_1)$
4. $\left[\Phi(t)\right]^n = \Phi(nt)$
5. $\Phi(t_2 - t_1)\Phi(t_1 - t_0) = \Phi(t_2 - t_0) = \Phi(t_1 - t_0)\Phi(t_2 - t_1)$