



DEPARTMENT OF MATHEMATICS

UNIT-IV FUNCTIONS OF SEVERAL VARIABLES

TAYLOR SERIES EXPANSION.

Taylor's expansion for the function $f(x, y)$ at the pt. a, b is given by

$$f(x, y) = f(a, b) + \frac{1}{1!} [(x-a)f_x(a, b) + (y-b)f_y(a, b)]$$

$$+ \frac{1}{2!} [(x-a)^2 f_{xx}(a, b) + 2(x-a)(y-b)f_{xy}(a, b) + (y-b)^2 f_{yy}(a, b)]$$

$$+ \frac{1}{3!} [(x-a)^3 f_{xxx}(a, b) + 3(x-a)^2(y-b)f_{xxy}(a, b) + 3(x-a)(y-b)^2 f_{xyy}(a, b) + (y-b)^3 f_{yyy}(a, b)] + \dots$$

① Find the Taylor's series for the function $e^x \sin y$ at $(0, \pi/2)$ upto second degree.

$$f = e^x \sin y$$

$$f_x = e^x \sin y$$

$$f_{xx} = e^x \sin y$$

$$f_{xy} = e^x \cos y$$

$$f_y = e^x \cos y$$

$$f_{yy} = e^x (-\sin y)$$

$$f(0, \pi/2) = e^0 \sin \pi/2 = 1$$

$$f_x(0, \pi/2) = e^0 \sin \pi/2 = 1$$

$$f_{xx}(0, \pi/2) = e^0 \sin \pi/2 = 1$$

$$f_{xy}(0, \pi/2) = e^0 \cos \pi/2 = 0$$

$$f_y(0, \pi/2) = e^0 \cos \pi/2 = 0$$

$$f_{yy}(0, \pi/2) = e^0 (-\sin \pi/2) = -1$$



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$$f(x, y) = f(a, b) + \frac{1}{1!} [(x-a) f_x(a, b) + (y-b) f_y(a, b)] + \frac{1}{2!} [(x-a)^2 f_{xx}(a, b) + 2(x-a)(y-b) f_{xy}(a, b) + (y-b)^2 f_{yy}(a, b)]$$

$$\begin{aligned} f(x, y) &= f(0, \pi/2) + \frac{1}{1!} [(x-0) f_x(0, \pi/2) + (y-\pi/2) f_y(0, \pi/2)] + \\ &\quad \frac{1}{2!} [(x-0)^2 f_{xx}(0, \pi/2) + 2(x-0)(y-\pi/2) f_{xy}(0, \pi/2) + (y-\pi/2)^2 f_{yy}(0, \pi/2)] \\ &= 1 + \frac{1}{1!} [x \cdot (1) + (y - \pi/2) (0)] + \frac{1}{2!} [x^2 \cdot (1) + 2(x)(y - \pi/2) (0) + \\ &\quad (y - \pi/2)^2 \cdot (-1)] \\ &= 1 + \frac{1}{1!} (x) + \frac{1}{2!} [x^2 - (y - \pi/2)^2] + \dots \end{aligned}$$