The second or short period mode is always so heavily damped that it is of very little consequence in itself. The pilot of the airplane is hardly ever cognizant of its existence. It is felt by the pilot only as a bump when a gust is encountered or in the response of the airplane to an abrupt control movement. The long period or phugoid mode has very weak damping, and many airplane designs today have negatively damped phugoid modes during part of the lift coefficient range through which they fly. The period of the phugoid mode is so long and the response of the airplane so slow that negative damping of this oscillation

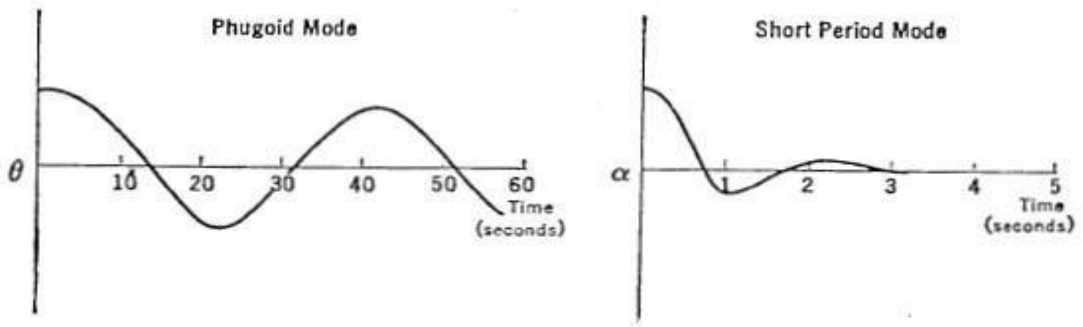

FIGURE 10–12. Typical longitudinal modes, controls locked.

has little bearing on the pilot's opinion of the flying qualities of the airplane. It is for this reason that neither the Air Force nor Navy has any requirements on the damping of the phugoid mode, and it is unimportant therefore to go into great detail on methods for designing the airplane to insure a damped phugoid.

The important things to understand about the two oscillatory modes of longitudinal motion are, first, their existence; second, a simple physical interpretation of them; and finally, a generalized picture of what the effects of the major airplane variables are on these modes.

The first step, that of their existence, has been developed theoretically and is well proved from flight test. Typical examples are given in Figure 10–12.

The second phase, that of obtaining a more simple physical explanation of this oscillation, is possible from flight test experience coupled with a knowledge of the dynamic system involved. The phugoid oscillation is one in which there is a large amplitude variation of air speed, pitch, and altitude, but during which the variation of angle of attack is very small and can be considered nearly constant. The motion is so slow that the effects of inertia forces and damping forces are very low. The whole phugoid oscillation can be thought of as a slow interchange of kinetic and potential energy about some equilibrium energy level, or as the attempt of the airplane to re-establish the equilibrium $C_L V^2 = K$, from which it has been disturbed.

Under the assumption of no change in angle of attack, no damping, no inertia, and no airplane drag, equations (10–89) reduce to the following simple form:

$$du + \frac{C_L}{2}\theta = 0$$

$$C_L u - d\theta = 0$$

Assumption of the solution in the form $u = u_1 e^{\lambda t/\tau}$ and substitution into (10–100) above give the following coefficient determinant:

$$\begin{vmatrix} \lambda & \frac{C_L}{2} \\ C_L & -\lambda \end{vmatrix} = 0$$

Expanding $$\lambda^2 + \frac{C_L^2}{2} = 0$$

solving $$\lambda = \pm i\sqrt{\frac{C_L^2}{2}}$$

and finally, $$u = u_1 e^{\pm i\sqrt{C_L^2/2}\,t/\tau}$$

This is an oscillation whose period is $2\pi/\sqrt{C_L^2/2}\ \tau$ seconds.

Substituting $C_L = 2(W/S)/\rho V^2$ and $\tau = (W/S)/\rho g V$, the period equals $.138V$ seconds, indicating a very slow oscillation at normal flight speeds.

The solution of the complete equations and experience from flight tests indicate that the linear variation of period with speed is verified, although the constant .138 is somewhat higher, nearly .178.

This approximation to the phugoid oscillation indicates no damping, with the oscillation continuing forever. If the assumption of zero drag is removed, equation (10–89) will reduce as follows:

$$(C_D + d)u + \frac{C_L}{2}\theta = 0$$

$$C_L u - d\theta = 0$$

Again assuming the solution in the form $u = u_1 e^{\lambda t/\tau}$, etc., and expanding the determinant of the algebraic equation in λ give:

$$\lambda^2 + C_D\lambda + \frac{C_L^2}{2} = 0$$

$$\lambda = -\frac{C_D}{2} \pm \sqrt{\left(\frac{C_D}{2}\right)^2 - \frac{C_L{}^2}{2}}$$

The solution of equation (10–102) then gives a damped oscillation which, as $C_L \gg C_D$, has a period, as before, equal to $.138V$ seconds. The time to damp the oscillation to ½ amplitude is $1.386/C_D\ \tau$ seconds. The damping of the phugoid is therefore a direct function of the airplane drag coefficient. Although the results of the complete equations show other factors influencing the damping somewhat, the drag is nevertheless a large factor and places the airplane designer in somewhat of a dilemma. The cleaner he makes the airplane, the more difficult it will be for him to insure damping of the phugoid mode.

Finally, the phugoid can be summed up as a long-period, slow oscillation of the airplane's velocity as it attempts to re-establish the equilibrium condition $C_L V^2 = K$, from which it has been disturbed. The lift coefficient is held essentially constant during the oscillation and the motion is damped, mainly because of the effects of the airplane's drag. The more drag, the better is the damping.

Flight test experience and analytical study of the short period mode indicate that this oscillation proceeds essentially at constant speed. This is due to the fact that the motion of the airplane in the short period mode proceeds so rapidly that the motion is completely damped out by the time the speed has time to change. Under the assumption of no change in speed ($u = 0$), equations (10–89) reduce as follows:

$$\begin{vmatrix} \frac{1}{2}C_{L_\alpha} + \lambda & -\lambda \\ C_{m_\alpha} + C_{m_{d\alpha}}\lambda & C_{m_{d\theta}}\lambda - h\lambda^2 \end{vmatrix} = 0$$