Several methods are available for extracting the roots from quartic equations. One of these is the normal analytical treatment given in any book on algebra. Another method,* which at times is much faster, especially for the case where all roots are complex, is a graphical method and is the one described herein. The graphical method first assumes the quartic equation broken down into two quadratic equations.

$$(\lambda^2 + a_1\lambda + b_1)\ (\lambda^2 + a_2\lambda + b_2) = 0$$

Expanding these quadratics and equating the like powers of λ yield the following:

$$A = 1$$

$$B = a_1 + a_2$$

$$C = b_1 + a_1a_2 + b_2$$

$$D = a_2b_1 + a_1b_2$$

$$E = b_1b_2$$

As these equations are symmetric in a and b, it is possible by elimination to form the following two general equations:

$$a = \frac{B}{2} \pm \sqrt{\left(\frac{B}{2}\right)^2 - C + b + \frac{E}{b}}, \quad \text{and} \quad a = \frac{b^2B - bD}{b^2 - E}$$

If these equations are plotted on rectangular coordinates a versus b, there will be two intersections, (a_1, b_1) and (a_2, b_2). These values are placed in the quadratics (10–96) which can readily be solved for the roots. After experience is gained in this method, the roots can be obtained quite rapidly and with any degree of accuracy required.

If the aerodynamic characteristics of a typical airplane are substituted into equations (10–95), the coefficients (A–E) of the stability quartic in λ are obtained. The roots of this quartic for a statically stable airplane in almost all cases combine into two complex pairs.

$\lambda_{1,2} = \xi_1 \pm i\eta_1$

$\lambda_{3,4} = \xi_2 \pm i\eta_2$

The combining of the roots into two complex pairs indicates that the airplane's longitudinal motion, after a disturbance with the elevator locked, has two oscillatory modes. The period and damping of these modes can be obtained by making use of the relations given below:

$$\text{Period} = \frac{2\pi}{\eta}\tau \text{ seconds}$$

$$\text{Time to damp to } \tfrac{1}{2} \text{ amplitude} = \frac{.693}{\xi}\tau \text{ seconds}$$

Typical values for the roots are as follows:

$$\lambda_{1,2} = -.02 \pm i\,0.30$$
$$\lambda_{3,4} = -2.0 \pm i\,2.5$$

These indicate the following oscillations for an airplane with a time characteristic $\tau = 1.5$.

$\lambda_{1,2}$—Period 31.5 seconds, time to damp to half amplitude 52 seconds

$\lambda_{3,4}$—Period 3.77 seconds, time to damp to half amplitude .52 second

The characteristic modes of stick-fixed longitudinal motion for nearly all airplanes are two oscillations, one of long period with poor damping and the other of short period with heavy damping. The first of these oscillations is usually referred to as the phugoid mode or long period mode, while the second is referred to merely as the short period mode or second mode.