Evaluation of Stability Derivatives

The four equations (10–66a–d) are simultaneous homogeneous differential equations with constant coefficients. The constant coefficients of these equations are made up of the airplane mass and inertia parameters and the so-called stability derivatives. In this section the evaluation of these derivatives will be studied in order that solution of the simultaneous equations can proceed.

For convenience all the angles in the sections on dynamics will be taken in radian measure.

1. C_{L_α}, the slope of the lift curve, a function of the airfoil section lift characteristics, a_0, and the wing aspect ratio. This derivative can be estimated from the curves given in Figure 5–5.

2. C_{D_α}, the rate of change of the drag coefficient with angle of attack. Obtained from the basic drag formula

$$C_D = C_{D_f} + \frac{{C_L}^2}{\pi e A}$$

3. C_{m_α}, the static longitudinal stability criterion $dC_m/d\alpha$

$$\frac{dC_m}{d\alpha} = \frac{dC_m}{dC_L} \cdot \frac{dC_L}{d\alpha}$$

$$C_{D_\alpha} = \frac{2C_L}{\pi e A} C_{L_\alpha}$$

or $C_{m_\alpha} = C_{L_\alpha}(x_{cg} - N_0)$, where N_0 is the stick-fixed neutral point.

4. $C_{m_{\dot{\alpha}}}$, the rate of change of pitching moment coefficient with rate of change of angle of attack with respect to t/τ. This derivative arises because of the lag in the wing downwash getting from wing to tail. For an airplane whose angle of attack is increasing at the rate $d\alpha/dt$, the tail angle of attack, at any instant of time, corresponding to the wing angle of attack, α_w, will be:

$$\alpha_t = \alpha_w - \epsilon - i_w + i_t$$

$$\epsilon = \frac{d\epsilon}{d\alpha}\left(\alpha_w - \frac{d\alpha}{dt}\Delta t\right)$$

where Δt is the time it takes an air particle to go from the wing to the tail, $\Delta t = l_t/V$.

The pitching moment coefficient due to the tail becomes

$$C_{mt} = -a_t\bar{V}\eta_t\left[\alpha_w - \frac{d\epsilon}{d\alpha}\left(\alpha_w - \frac{d\alpha}{dt}\,\frac{l_t}{V}\right) - i_w + i_t\right]$$

$$\frac{dC_m}{d(d\alpha/dt)} = -a_t\bar{V}\eta_t\frac{l_t}{V}\,\frac{d\epsilon}{d\alpha}$$

Dividing each side of (10–72) by the time parameter, τ, and making use of the relative airplane density factor, μ, we obtain:

$$\frac{dC_m}{d\left(\dfrac{d\alpha}{d(t/\tau)}\right)} = -a_t\bar{V}\eta_t\frac{l_t}{V}\,\frac{d\epsilon}{d\alpha}\,\frac{1}{\tau}$$

5. $C_{m_{d\theta}}$, the airplane's damping in pitch. There are several contributions to the longitudinal damping of the airplane, but the largest of these is the contribution of the horizontal tail. The wing, fuselage, and propeller all add to this derivative, but as in most normal airplane designs the horizontal tail is by far the largest factor; it is normal practice to evaluate the damping contribution of the horizontal tail and then lump all other contributions as a multiplying factor on the tail damping. This factor is usually taken equal to 1.10.

The damping contribution of the horizontal tail can be estimated by determining the change in tail angle of attack due to the airplane's angular velocity, $\dot{\theta}$.

$$\Delta\alpha_t = \dot{\theta}\,\frac{\bar{l}_t}{V}$$

7. The pitching moment due to the rate of elevator deflection with t/τ can be computed from the following formula:

$$\frac{dC_{mt}}{d\left(\dfrac{d\theta}{d(t/\tau)}\right)} = -a_t\bar{V}\eta_t\frac{l_t}{V\tau}$$

6. $C_{m\delta}$, the elevator power term discussed in Chapter 5,

$$C_{m\delta} = -a_t\bar{V}\eta_t\tau$$

$$C_{md\delta} = -\frac{1}{2\mu}[A + a_tB]\bar{V}\frac{c_t}{c_w}$$

8. C_{h_α}, the rate of change of elevator hinge moment coefficient with wing angle of attack. The variation of the elevator hinge moment with tail angle of attack, $C_{h_{\alpha t}}$, was discussed in Chapter 6. The derivative C_{h_α} can be developed in terms of $C_{h_{\alpha t}}$ by accounting for the wing downwash.

$$C_{h_\alpha} = C_{h_{\alpha t}}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

9. C_{h_δ}, the rate of change of elevator hinge moment with control deflection. Also treated in detail in Chapter 6.

10. $C_{h_{d\delta}}$, the elevator damping derivative, can be developed from the following formula:

$$C_{h_{d\delta}} = -\frac{1}{2\mu}[C + Da_t]\frac{c_t}{c_w}$$

11. $C_{h_{d\theta}}$, the rate of change of elevator hinge moment with rate of change of airplane pitch angle with t/τ. This derivative arises as a result of the change in angle of attack of the horizontal tail with airplane pitching velocity and the floating tendency of the elevator.

$$C_h = C_{h_{\alpha t}}\dot{\theta}\frac{l_t}{V}$$

$$\frac{dC_h}{d\left(\frac{d\theta}{dt}\right)} = C_{h_{\alpha t}}\frac{l_t}{V}$$

Dividing both sides of (10–86) by τ,

$$\frac{dC_h}{d\left(\frac{d\theta}{d(t/\tau)}\right)} = C_{h_{\alpha t}}\frac{l_t}{V\tau}$$

or

$$C_{h_{d\theta}} = C_{h_{\alpha t}}\frac{l_t}{\mu c}$$

It is assumed that the derivatives C_{m_μ}, C_{h_μ}, $C_{h_{d\alpha}}$, $C_{h_{d^2\alpha}}$, and $C_{m_{d^2\alpha}}$ are either zero or negligible.