SNS COLLEGE OF TECHNOLOGY

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DEPARTMENT OF AUTOMOBILE ENGINEERING

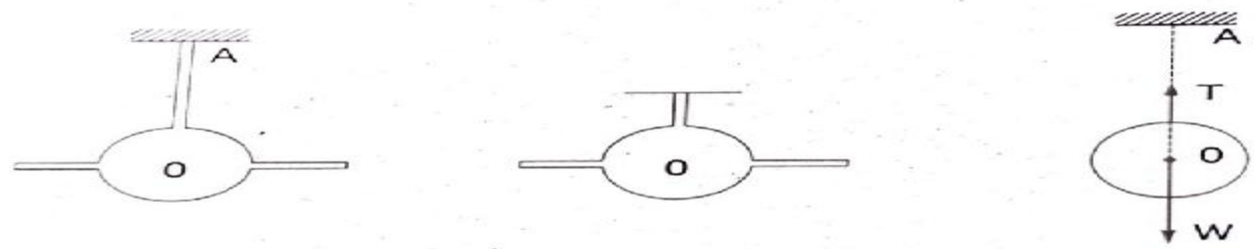
COURSE NAME : 23MET101 – ENGINEERING MECHANICS

I YEAR / I SEMESTER

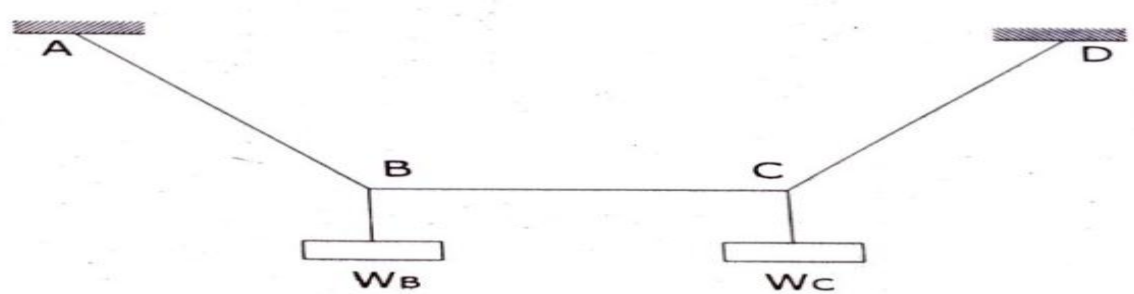
Topic – Free Body Diagram

Free body diagram:

such a body which has been seperated or isolated from the surrounding bodies is called free body



Similarly, we will draw the free body diagram at B and C , the figure shows two weight attached at B and C ,connected by a string ABCD,supported on ceiling at A and D



Free body diagram at B

The forces acting at B are:-

(i) Weight of the body attached at B, acting downwards, let it be W_B.

(ii) Tention on string AB; acting at B and towards A, let it be T_{BA}.

(iii) Tention on string BC, acting at B and towards C, let it be T_{BC}.

These forces are shown

Free body diagram at C

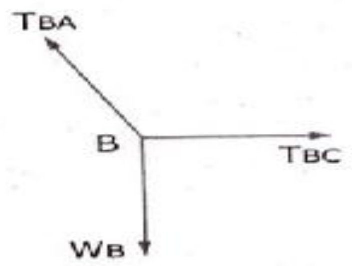
The forces acting at C are

(i) Weight of the body, attached at C, acting downwards let it be W_C.

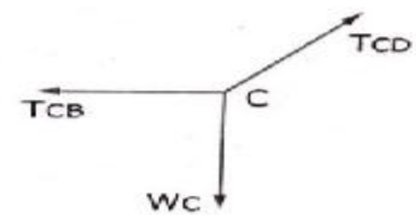
(ii) Tention on string CB; acting at C and towards B let it be T_{CB}.

(iii) Tention on string CD, acting at C and towards D, let it be T_{CD}.

These forces are shown

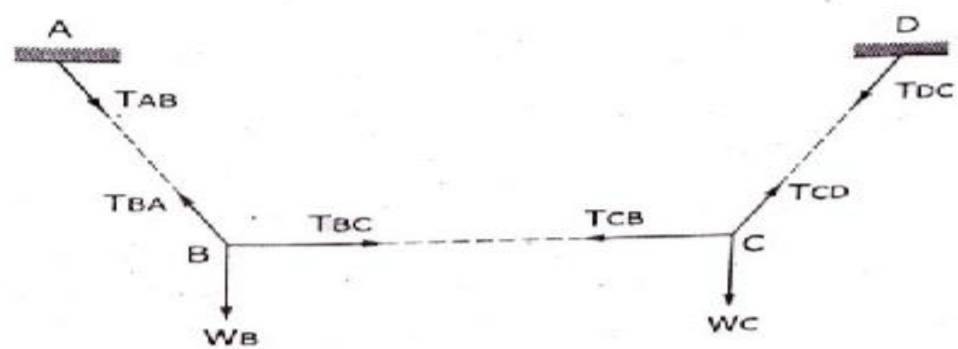


(a) FBD of B



(b) FBD of C

Now, all the forces acting on the string $ABCD$ are shown



1. In string AB, there are two forces T_{BA} and T_{AB}, one at each end. For the equilibrium condition of the string, these two forces should be equal, collinear and opposite to each other (as per two force equilibrium principle)

i.e., $T_{BA} = T_{AB}$

|||ly $T_{BC} = T_{CB}$ and

$T_{CD} = T_{DC}$.

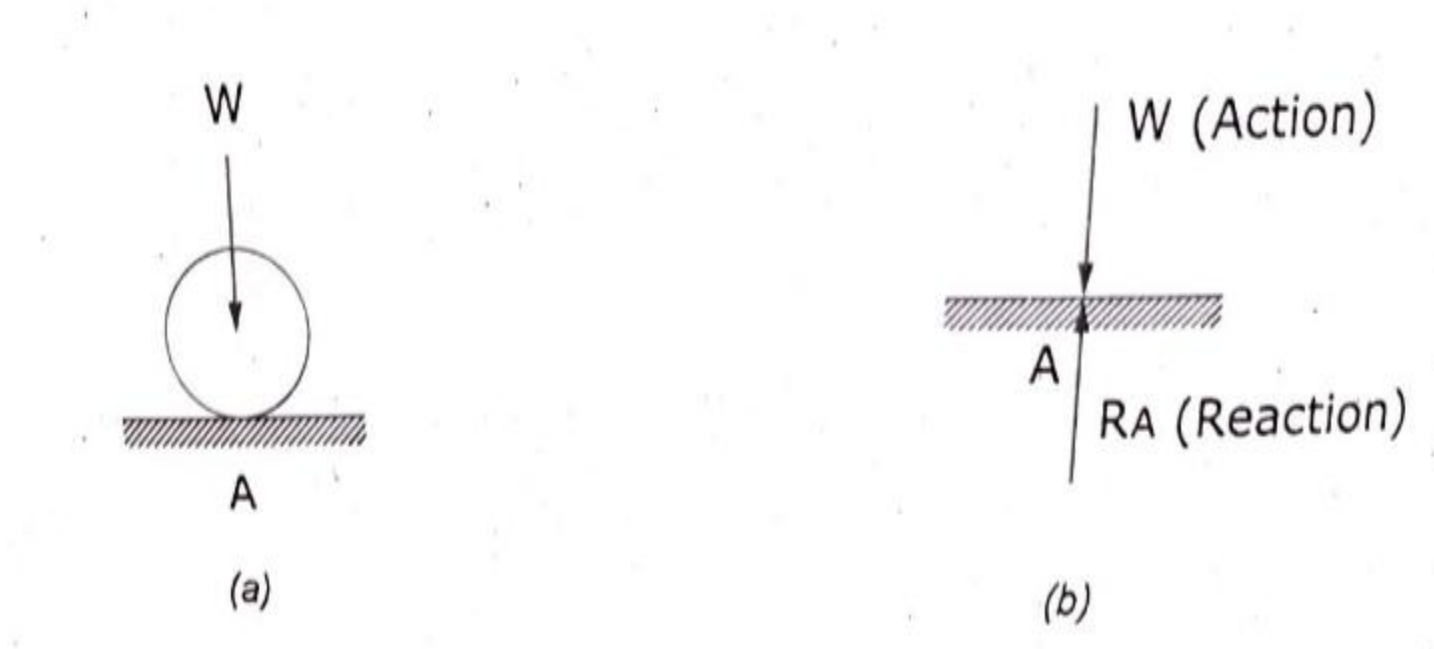
2. To find the unknown forces at B and C, apply the equations of equilibrium (or lami's therem) at B and C separately.

3. In fig 3.14, the reactions at A and D are not shown. To find the unknown forces at B and C, the reactions at A and D are not at all required.

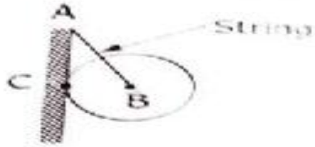
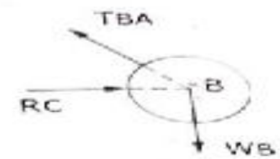
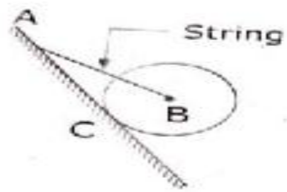
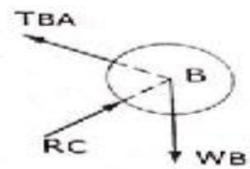
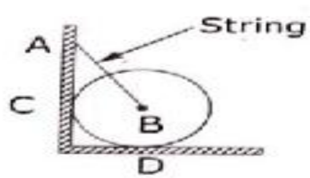
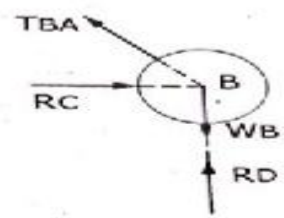
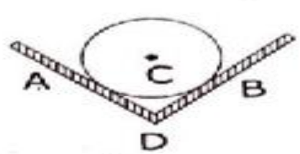
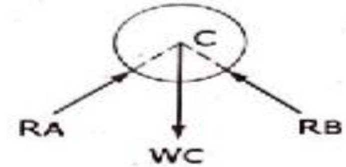
In next article, we will see how to find the reactions on a body, when it is subjected to some external forces.

Action and reaction

- For the condition of equilibrium , action and reaction are equal, collinear but acting in opposite direction,



Free body diagram examples

No.	Bodies under equilibrium	Free body diagram
1.		
2.		
3.		
4.		

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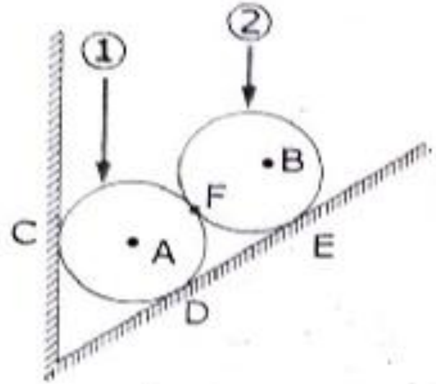
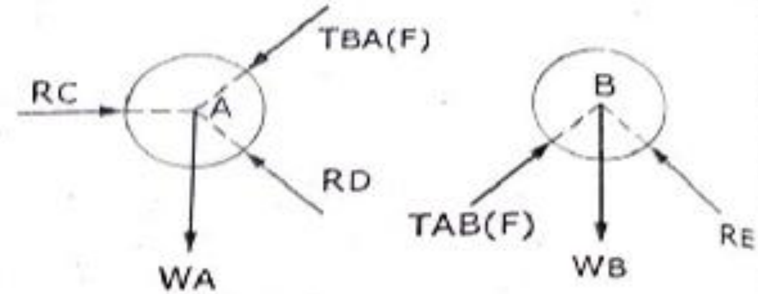
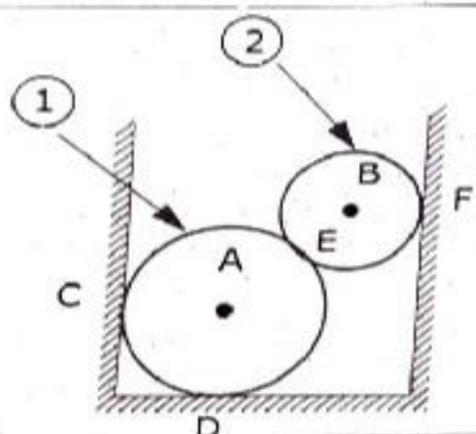
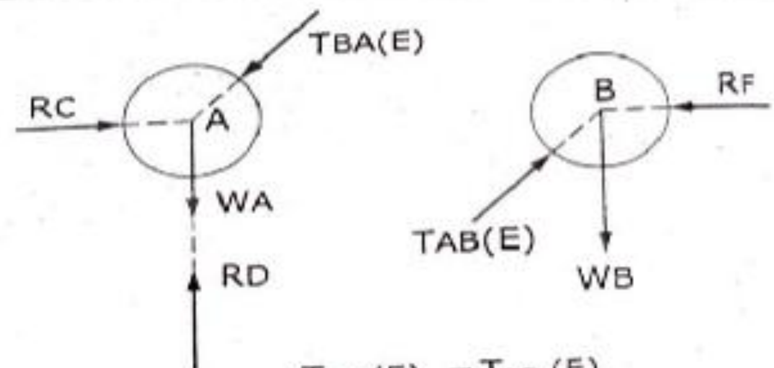




$T_{BA}(F) = T_{AB}(F)$

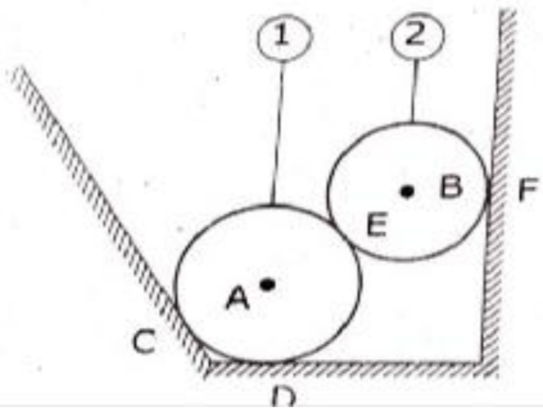
6.

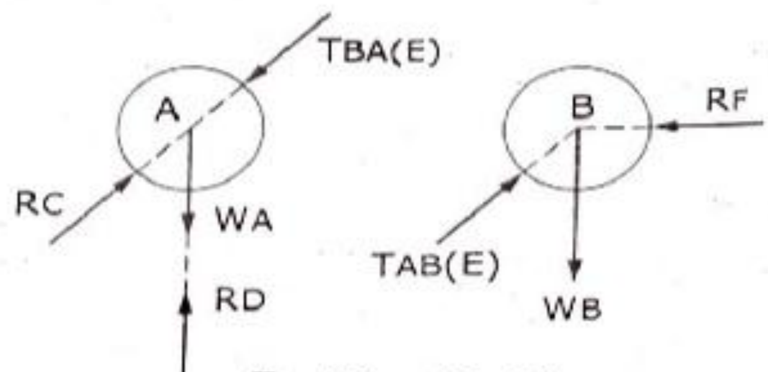




$T_{BA}(E) = T_{AB}(E)$

7.

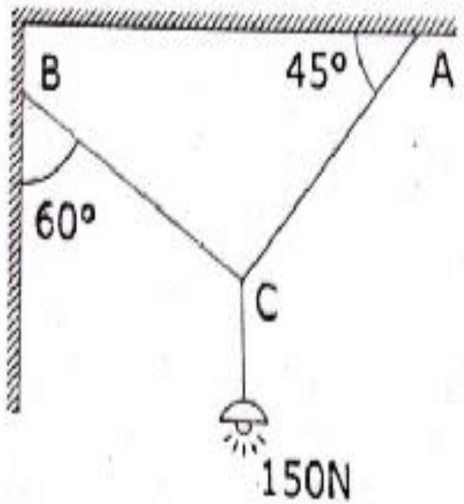


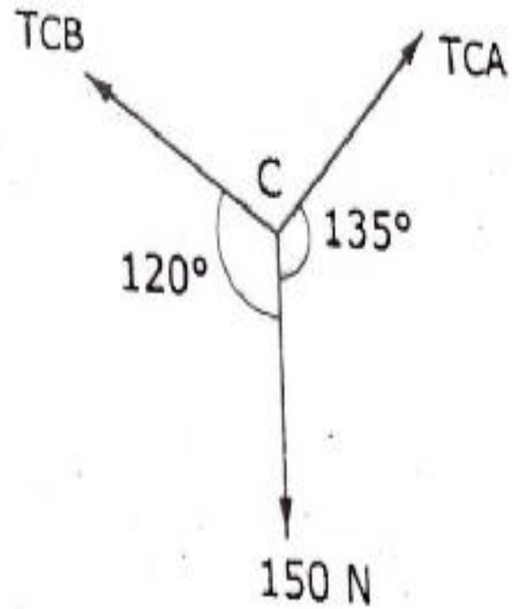


$T_{BA}(E) = T_{AB}(E)$

Example sums :

An electric light fixture weighing 150 newtons hangs from a point C, by two strings AC and BC as shown in fig   Determine the forces in the strings AC and BC.





Solution

The free body diagram at C is drawn in fig 3.17

T_{CB} = Tension in the string CB, from C to B

T_{CA} = Tension in the string CA, from C to A

From the geometry of the figure, the angle between T_{CB} force and 150 N is 120°; angle between T_{CA} force and 150 N is 135°; angle between T_{CA} and T_{CB} is, 360 – (120 + 135) = 105°.

now, by applying lami's equation at C,

$$\frac{T_{CB}}{\sin 135} = \frac{T_{CA}}{\sin 120} = \frac{150}{\sin 105}$$

$$\therefore \quad T_{CB} = \frac{150 \sin 135}{\sin 105} = 109.81 \text{ N}$$

$$\text{and} \quad T_{CA} = \frac{150 \sin 120}{\sin 105} = 134.49\text{N}$$

Aliter

The above problem can also be solved by applying equations of equilibrium at C.

The angle of T_{CB} force with Horizontal is 30°; and the angle of T_{CA} force with horizontal is 45°

Applying $\Sigma H = 0$ $(\rightarrow +)$

i.e., $T_{CA} \cos 45 - T_{CB} \cos 30 = 0$

$$\therefore \quad T_{CA} \cos 45 = T_{CB} \cos 30$$

or $\quad 0.707\, T_{CA} = 0.866\, T_{CB} \therefore T_{CA} = 1.224\, T_{CB}$ ----------(i)

Applying $\Sigma V = 0$ $(\uparrow +)$ -----------(ii)

$T_{CA} \sin 45 + T_{CB} \sin 30 - 150 = 0$

solving the equations (1) and (2), two unknowns T_{CA} and T_{CB} can be determined.

Sub $T_{CA} = 1.224\, T_{CB}$ in eqn (ii)

$$\text{i.e.,} \quad 0.866\, T_{CB} + 0.5\, T_{CB} = 150$$

$$1.366\, T_{CB} = 150$$

$$T_{CB} = 1^{\frown}9.81 \text{ N}$$

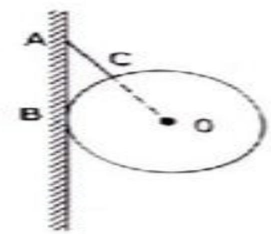
now $\quad T_{CA} = 1.224\, T_{CB}$

$$= 1.224 \times 109.81$$

$$= 134.41 \text{ N}$$

A smooth sphere of weight W is supported by a string to a point A on the smooth vertical wall the other end is in contact with the point B on the wall as in diagram, If the length of the string AC is equal to radius of the sphere find the tension in the string and reaction of the wall

=> *A smooth sphere of weight W is supported by a string fastened to a point A on the smooth vertical wall, the other end is in contact with point B on the wall as shown in fig 3.18. If the length of the string AC is equal to the radius of the sphere, find the tension in the string and reaction of the wall.*



Solution

Given data:

Let radius of sphere, $OB = OC = r$ and

Length of string, AC = radius $= r$

Weight of sphere $= W$

(a) (b)

Let,

T_{CA} = Tension in string, from C to A

R_B = Reaction of the wall at B, (at the point of contact), normal to the wall.

These forces are shown in fig ; the force system at O is coplanar concurrent as shown in fig.

Let the angl . of T_{CA} force with horizontal is θ.

In right angled tringle AOB,

$$OA = OC + CA = r + r = 2r \quad \text{and} \quad OB = r$$

$$\cos\theta = \frac{r}{2r} = \frac{1}{2}$$

$$\therefore \theta = \cos^{-1}(½) = 60°$$

Applying $\Sigma H = 0$ at O,

i.e., $R_B - T_{CA}\cos\theta = 0$ ..... (i)

|||ly Applying $\Sigma V = 0$

i.e., $T_{CA}\sin\theta - W = 0$

$$\therefore \quad T_{CA} = \frac{W}{\sin\theta} = \frac{W}{\sin 60} = 1.155W$$

Sub T_{CA} = 1.155 W in equation (i)

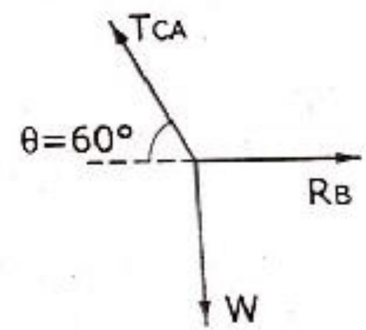
i.e., $R_B - (1.155W \times \cos 60) = 0$

$$\therefore \quad R_B = 0.577\ W$$

∴ Tension in the string = 1.155 W

and Reaction of the wall = 0.577 W

(Aliter)



The above problem can be solved by Lami's theorom also. But, in Lami's theorem, three concurrent forces, must act outwards. Hence, the forces shown in fig 3.19(b) is taken as below.

; the angle between T_{CA} and W is $(90+\theta) = (90+60)$
$= 150°$

|||ly the angle between T_{CA} and R_B is $(180-\theta) = (180-60) = 120°$

and the angle between R_B and W is 90°

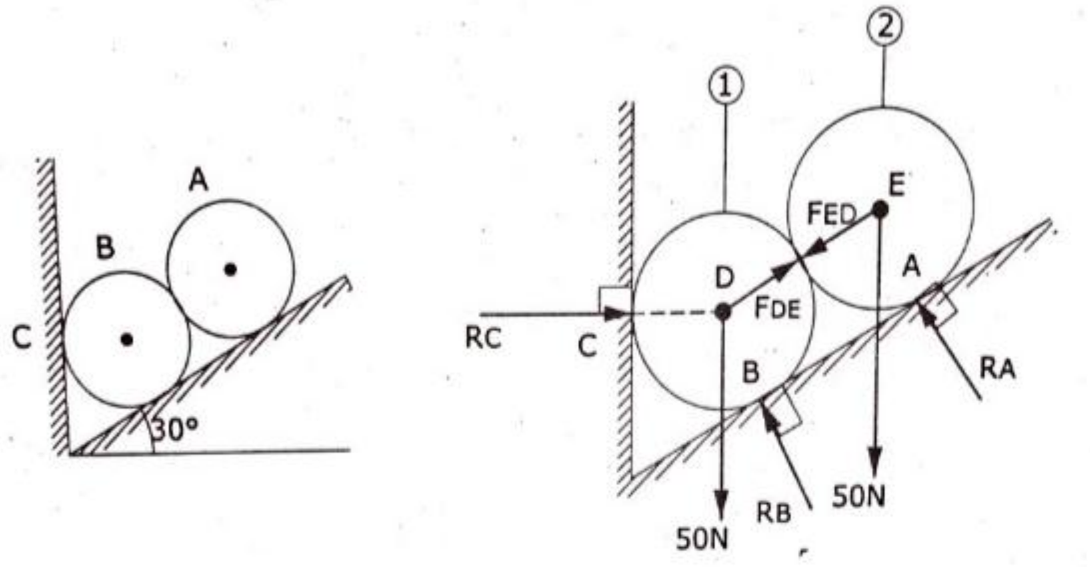
applying Lami's theorem at **O**

$$\frac{T_{CA}}{\sin 90} = \frac{R_B}{\sin 150} = \frac{W}{\sin 120}$$

$$\therefore \quad T_{CA} = \frac{W \sin 90}{\sin 120} = 1.155\ \text{W}$$

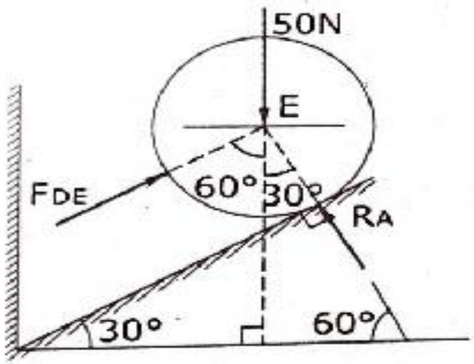
$$\text{and} \quad R_B = \frac{W \sin 150}{\sin 120} = 0.577\text{W}$$

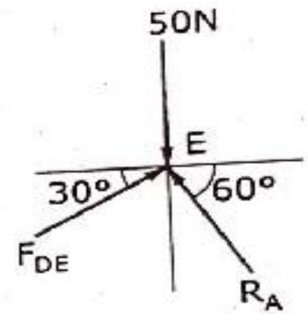
Two identical rollers, each of weight 50N, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth.



Free body diagram of roller (2)

The forces acting on roller (2) are shown in





The angles of F_{DE} and R_A with horizontal are 30° and 60° respectively

now, applying the equations of equilibrium at E,

$$\Sigma H = 0$$

i.e., $$F_{DE} \cos 30 - R_A \cos 60 = 0 \qquad \text{..... (i)}$$

$$\Sigma V = 0$$

i.e., $$F_{DE} \sin 30 + R_A \sin 60 - 50 = 0 \qquad \text{..... (ii)}$$

Solving the equations (i) and (ii) F_{DE} and R_A can be determined.

From eqn. (i)

$$F_{DE} \cos 30 = R_A \cos 60$$

or $$F_{DE} = \frac{R_A \cos 60}{\cos 30} = 0.577\ R_A$$

$$\text{Sub } F_{DE} = 0.577\, R_A \text{ in eqn (ii)}$$

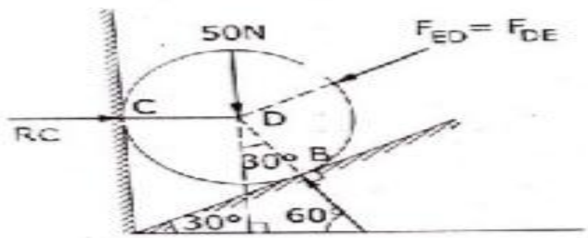
$$(0.577\, R_A \times 0.5) + (0.866\, R_A) = 50$$

$$1.154\, R_A = 50 \qquad \therefore R_A = 43.32 \text{ N}$$

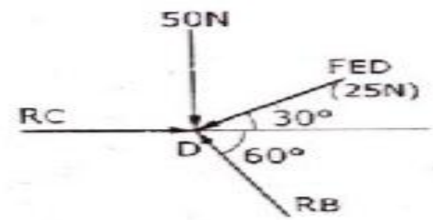
$$\therefore F_{DE} = 0.577\, R_A$$

$$= 0.577\,(43.32) = 25 \text{ N}$$

Free body diagram of roller (1)



(a)



(b)

The forces, acting on roller (1) are shown in fig. The angles of F_{ED} (or F_{DE}) and R_B with horizontal are 30° and 60° respectively (shown in fig. 3.31b). Note that, F_{ED} = F_{DE} = 25N.

now, applying the equations of equilibrium at D.

$$\Sigma H = 0 \Rightarrow R_C - 25\cos 30 - R_B \cos 60 = 0 \quad \text{..... (i)}$$

$$\Sigma V = 0 \Rightarrow R_B \sin 60 - 25 \sin 30 - 50 = 0 \quad \text{..... (ii)}$$

From eqn. (ii),

$$R_B \sin 60 = 50 + 25 \sin 30 = 62.5$$

$$\therefore R_B = \frac{62.5}{\sin 60} = 72.17 \text{ N}$$

$$\text{Sub } R_B = 72.17N \text{ in eqn (i)}$$

$$R_C - 25\cos 30 - 72.17 \cos 60 = 0$$

$$\therefore R_C = 57.73 \text{ N}$$

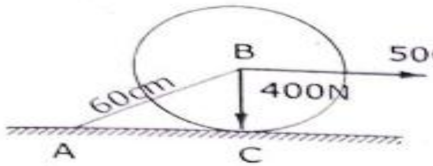
Result:

Reaction at A = 43.32 N

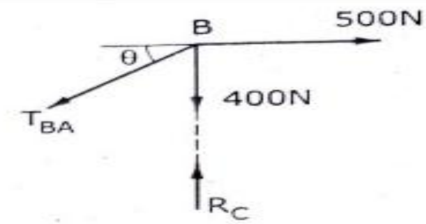
Reaction at B = 72.17 N

Reaction at C = 57.73 N

A circular roller of radius 20cm and of weight 400N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 60cm as in diagram. A horizontal force of 500N is acting on B. find the tension in the bar AB and reaction C



(a)



(b)

Solution:

In right angled triangle ABC,

$$\text{Length of bar } AB = 60 \text{ cm}$$

$$\text{Radius } BC = 20 \text{ cm}$$

$$\therefore \quad \text{Let } \angle BAC = \theta$$

$$\sin\theta = \frac{BC}{AB} = \frac{20}{60}$$

$$\text{or} \quad \theta = \sin^{-1}\left(\frac{20}{60}\right) = 19.47°$$

The Forces acting at centre of roller are shown in fi

T_{BA} = Tension in the Bar AB, acting at B and towards A.

Applying equations of equilibrium at B.

$$\Sigma H = 0$$

$$500 - T_{BA}\cos 19.47 = 0$$

$$\therefore \quad T_{BA} = \frac{500}{\cos 19.47} = 530.32\text{N}$$

$$\Sigma V = 0$$

$$R_C - 400 - T_{BA}\sin 19.47 = 0$$

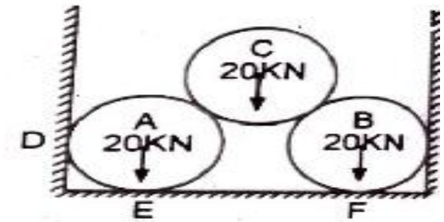
$$\text{or} \quad R_C - 400 - (530.32 \sin 19.47) = 0$$

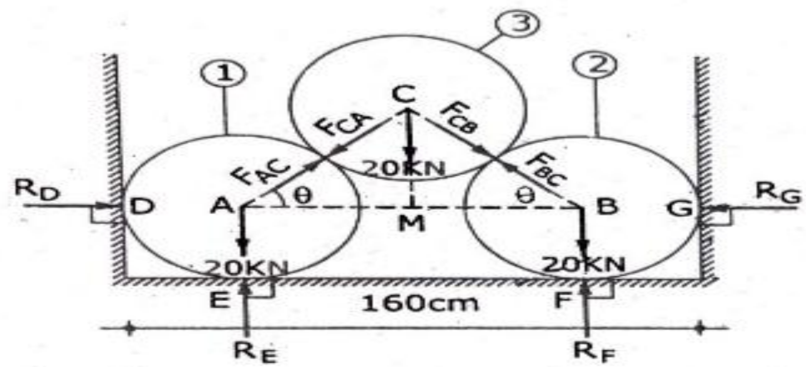
$$\therefore \quad R_C = 576.76\text{N}$$

$\therefore$ Tension in the bar = 530.32N, and

Reacuon at C = 576.76N.

Three smooth pipes each weighing 20KN and of diameter 60 cm are to be placed in a rectangular channel with horizontal base as shown in fig 3.33. Calculate the reactions at the points of contact between the pipes and between the channel and the pipes. Take, width of channel as 160 cm.





In triangle ABC,

$$\text{Side } AB = 160 - DA - BG \qquad DA = BG = \text{radius} = 30 \text{ cm}$$

$$= 160 - 30 - 30$$

$$= 100 \text{ cm}$$

$$AC = BC = 2 \times radius$$

$$= 2 \times 30 = 60 \text{ cm}$$

Draw a vertical line through C, perpendicular to AB, to intersect at M.

now, in right angled triangle BMC

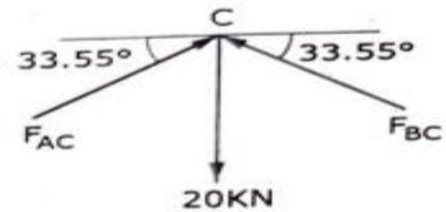
$$BM = \frac{AB}{2} = \frac{100}{2} = 50 \text{ cm}$$

$$\therefore \cos\theta = \frac{BM}{BC} = \frac{50}{60}$$

$$\text{or} \quad \theta = \cos^{-1}\left(\frac{50}{60}\right) = 33.55^\circ$$

Free-body diagram of pipe (3)

The forces acting on pipe ... only the forces acting over the pipe (3). i.e., F_{AC}, F_{BC} and self weight 20 KN are drawn; F_{CB} and F_{CA} should not be considered).



Now, applying the equations of equilibrium at C,

$\Sigma H = 0$

$$F_{AC} \cos 33.55 - F_{BC} \cos 33.55 = 0$$

$$\text{or} \quad F_{AC} = F_{BC} \quad \text{..... (i)}$$

$\Sigma V = 0$

$$F_{AC} \sin 33.55 + F_{BC} \sin 33.55 - 20 = 0 \quad \text{..... (ii)}$$

Sub $F_{AC} = F_{BC}$ in equation (ii)

$$F_{BC} \sin 33.55 + F_{BC} \sin 33.55 - 20 = 0$$

$$\text{or} \quad 2 F_{BC} \sin 33.55 = 20$$

$$\therefore \quad F_{BC} = 18.09 \text{ KN}$$

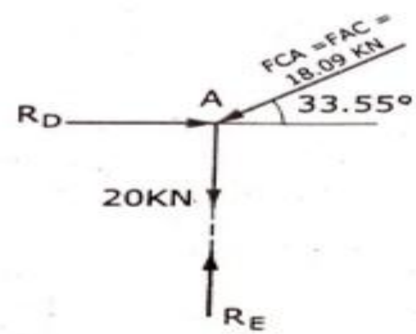
From eqn (i) $\quad F_{AC} = 18.09$ KN

Freebody diagram of pipe (1)

The forces acting on pipe (1) are shown in fig (Here, the force acting over the pipe, ie., F_{CA} is considered; F_{AC} should not be taken)

But $F_{CA} = F_{AC} = 18.09$ KN

Now, applying the equations of equilibrium at A,



$$\Sigma H = 0$$

$$R_D - F_{CA} \cos 33.55 = 0$$

$$\text{or} \quad R_D - 18.09 \cos 33.55 = 0$$

$$\therefore \quad R_D = 15.07 \text{ KN}$$

$$\Sigma V = 0$$

$$R_E - 20 - F_{CA} \sin 33.55 = 0$$

$$\text{or} \quad R_E - 20 - 18.09 \sin 33.55 = 0$$

$$\therefore \quad R_E = 30 \text{KN.}$$

Similarly, the free body diagram of pipe (2) is analysed for unknown forces R_G and R_F.

But, due to symmetry, we get

$$R_G = R_D = 15.07 \text{ KN} \quad \text{and} \quad R_F = R_E = 30\text{KN}$$

Result.

Reaction at D = 15.07 KN ($\rightarrow$)

Reaction at G = 15.07 KN ($\leftarrow$)

Reaction at E = 30KN ($\uparrow$)

and, Reaction at F = 30 KN ($\uparrow$).

Thank You..