# SNS COLLEGE OF TECHNOLOGY

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**DEPARTMENT OF AUTOMOBILE ENGINEERING**

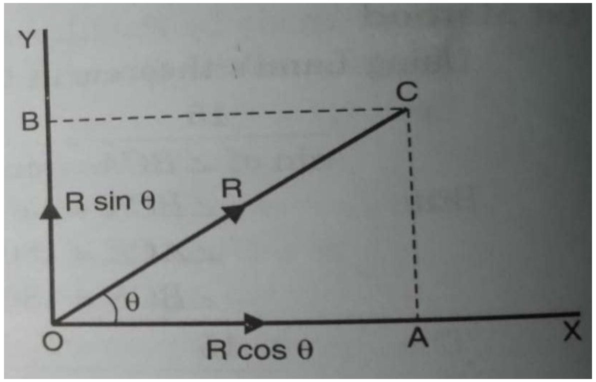
## COURSE NAME : 23MET101 – ENGINEERING MECHANICS

I YEAR / I SEMESTER

Topic – Resolution and Composition of forces

# RESOLUTION OF FORCES

- It is defined as the process of splitting up the given force into a number of components , without changing its effect on the body is called resolution of a force.
- A force is, generally, resolved along two mutually perpendicular directions.

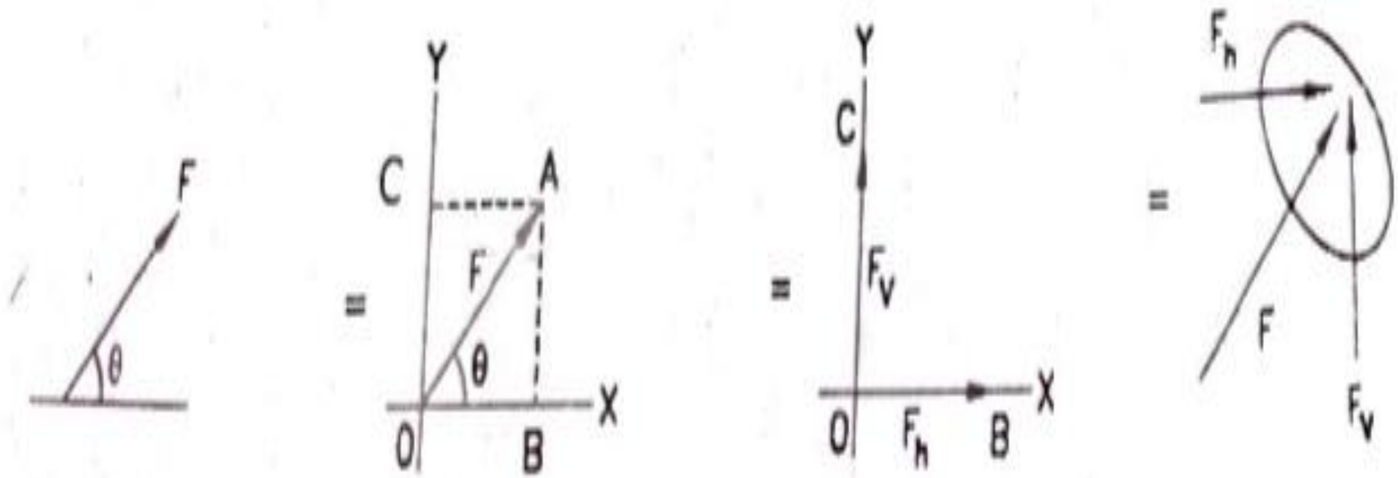



Components of R along x axis

Components of R along y axis

# Resolution of force

- Splitting up a force into components along the fixed reference axes is called resolution of a force.



# Magnitude of components

Let $F_h$ = Horizontal component of the force $F$

= $OB$ or $CA$ in the fig.

$F_v$ = Vertical component of the force $F$

= $OC$ or $BA$ in the fig.

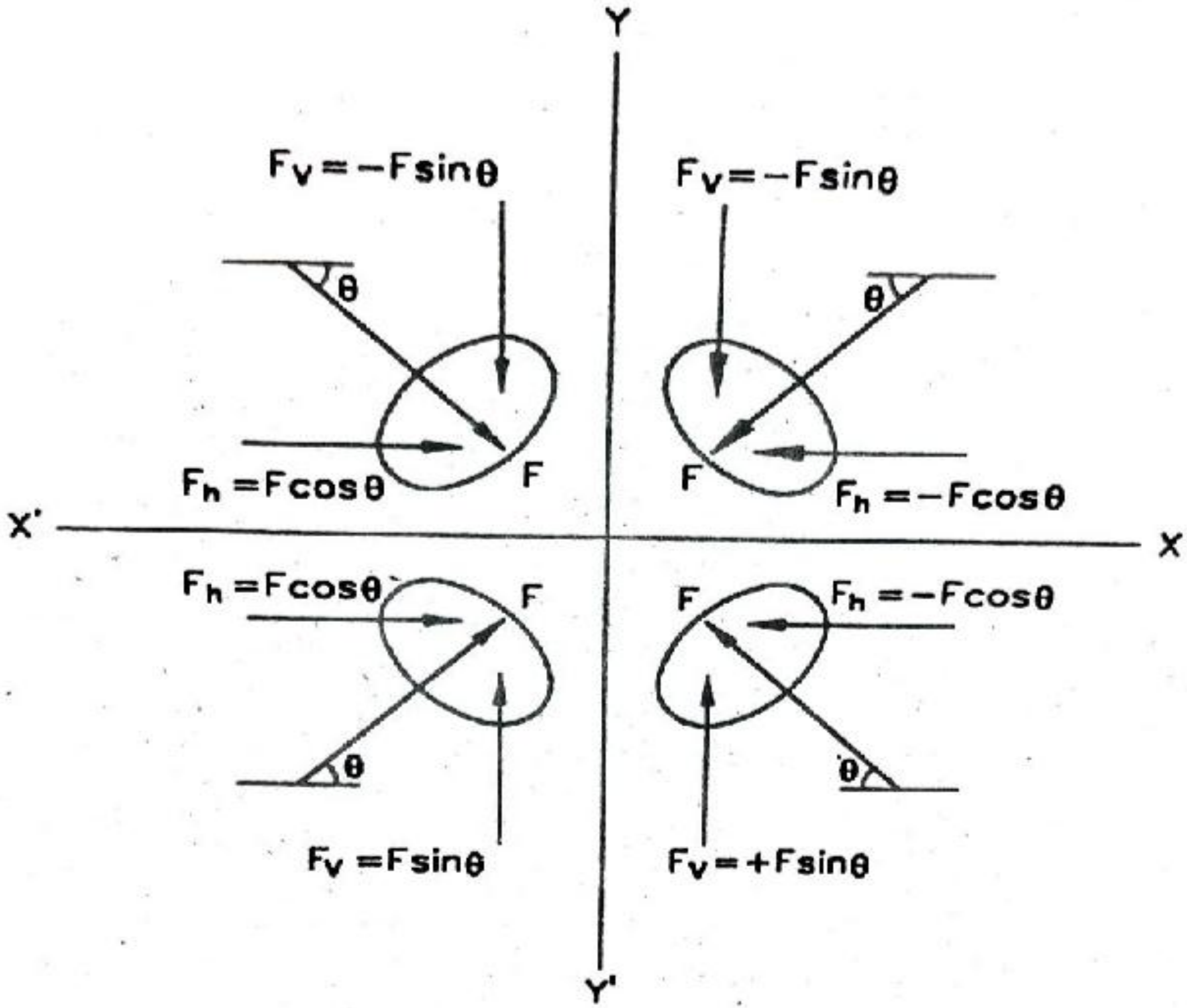
Consider the right angled triangle $OAB$.

$$\cos\theta = \frac{OB}{OA} = \frac{F_h}{F} \qquad \therefore F_h = F\cos\theta$$

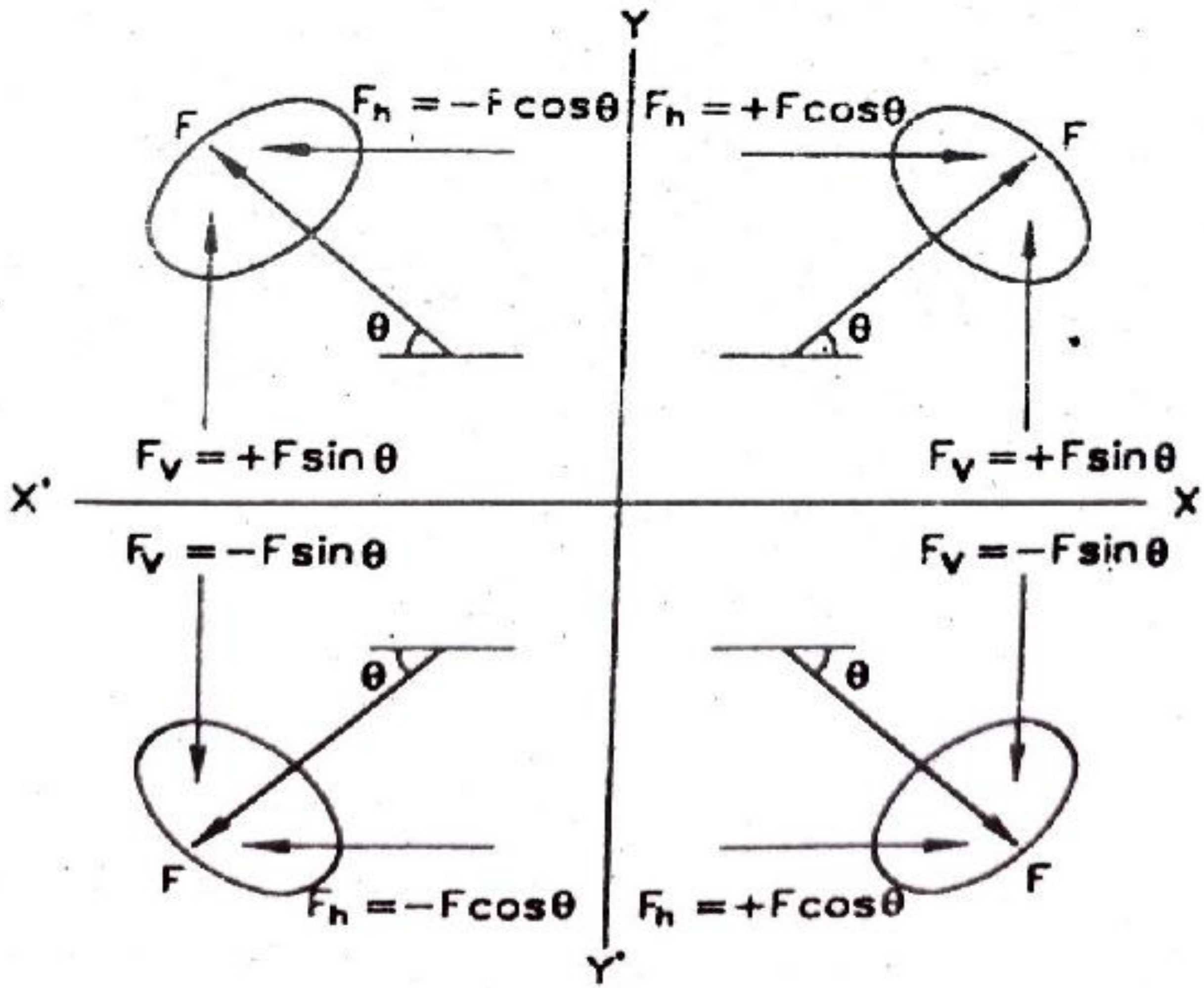
Similarly,

$$\sin\theta = \frac{AB}{OA} = \frac{F_v}{F} \qquad \therefore F_v = F\sin\theta$$

# For reference ,inclined forces ,acting towards the point of origin ,in all four quadrants are resolved below



Similarly, the forces acting in all the four quadrants ,but acts outwards from the point of origin are resolved



1. A vertical force has no horizontal component and its vertical component is the magnitude of the given force itself.


$\theta = 90°$

$$F_h = F\cos\theta \qquad F_v = F\sin\theta$$
$$= F\cos 90° \qquad = F\sin 90°$$
$$= 0 \qquad = F.$$

2. A Horizontal force has no vertical component and its horizontal component the magnitude of the given force itself.


$\theta = 0°$

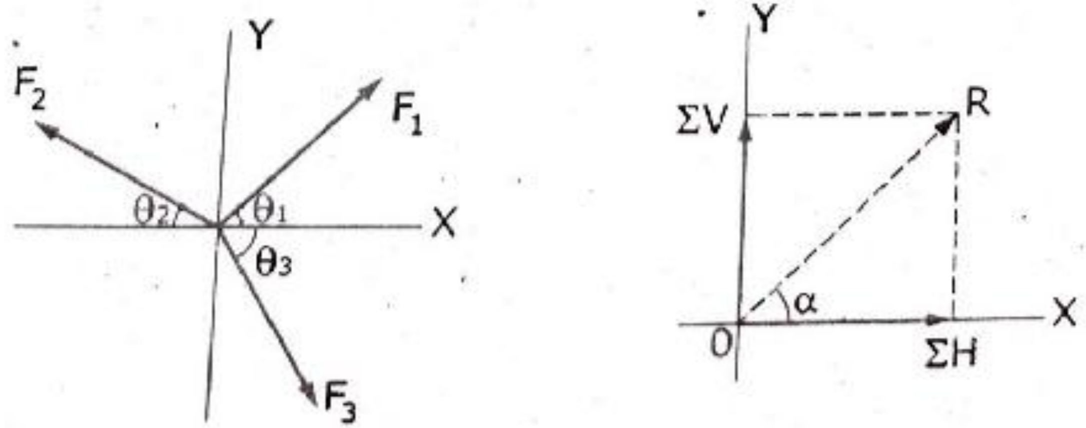
$$F_h = F\cos\theta \qquad F_v = F\sin\theta$$
$$= F\cos 0 \qquad = F\sin 0$$
$$= F \qquad = 0.$$

# Procedure of finding the resultant force of more than two concurrent forces

**Step 1 : Find the Algebraic sum of the horizontal components.**

Resolve the forces horizontally and find the net horizontal force, taking right hand side force as positive. Let it be $\Sigma H$.

For example, for the concurrent force system




Resolving the forces horizontally. (i.e., along $X$ axis ), we get,

$$\Sigma H = F_1 \cos\theta_1 - F_2 \cos\theta_2 + F_3 \cos\theta_3$$

**Step 2 : Find the Algebraic sum of the vertical components.**

Resolve the forces vertically and find the net vertical force, taking upward force as positive. Let it be $\Sigma V$.

Resolving the forces vertically (i.e., along $Y$ axis), we get,

$$\Sigma V = F_1 \sin\theta_1 + F_2 \sin\theta_2 - F_3 \sin\theta_3.$$

**Step 3 : Find the Magnitude of Resultant force.**

The net horizontal force $\Sigma H$ (found in step 1) and the net vertical force $\Sigma V$ (found in step 2) can be drawn in the co-ordinate axes

(If $\Sigma H$ is positive draw towards right

$\Sigma H$ is negative draw towards left

$\Sigma V$ is positive draw upwards

$\Sigma V$ is negative draw downwards ).

Fig 2.11. is drawn, assuming both $\Sigma H$ and $\Sigma V$ are positive.

The Magnitude of Resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

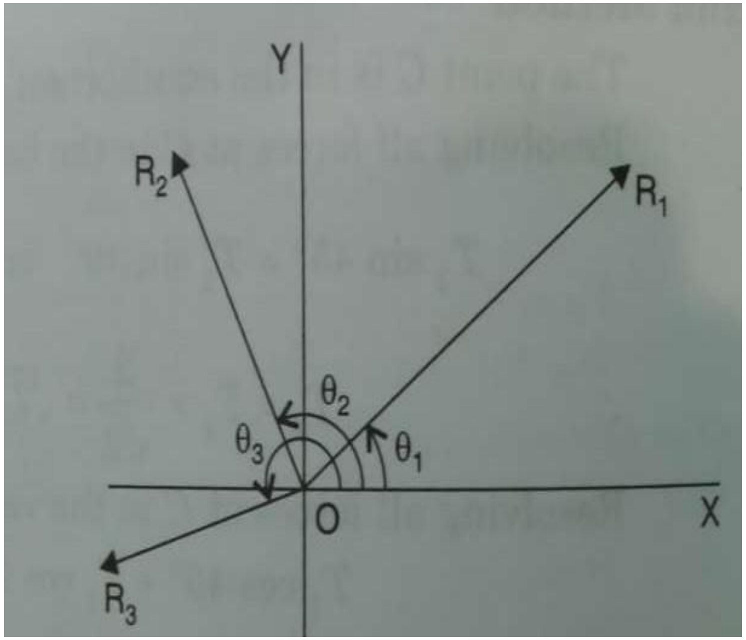
**Step 4 : Direction of Resultant force.**

Construct a rectangle, having $\Sigma H$ and $\Sigma V$ are the adjacent sides and draw the diagonal which originates from the point of origin. This diagonal of the rectangle will be the resultant force of $\Sigma H$ and $\Sigma V$ (or) the resultant force of the given concurrent force system.

Let $\alpha$ be the inclination of Resultant force with horizontal.

$$\tan\alpha = \frac{\Sigma V}{\Sigma H} \qquad \text{or} \qquad \alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right).$$

# Resolution of no. Of coplanar forces.




= Sum of components of all forces along $X$-axis.

$$H = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3 + \ldots$$

= Sum of components of all forces along $Y$-axis.

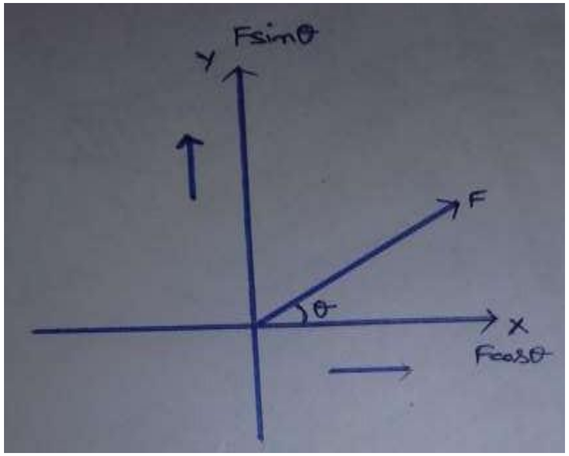
$$V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3 + \ldots$$

$$R = \sqrt{H^2 + V^2}$$

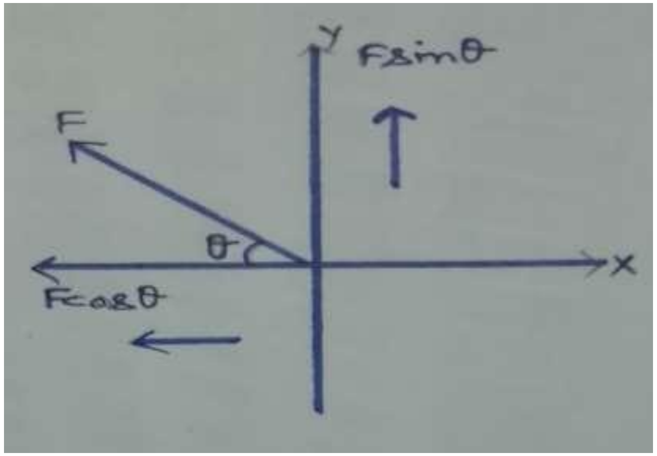
$$\tan \theta = \frac{V}{H}$$

# Principle of Resolution

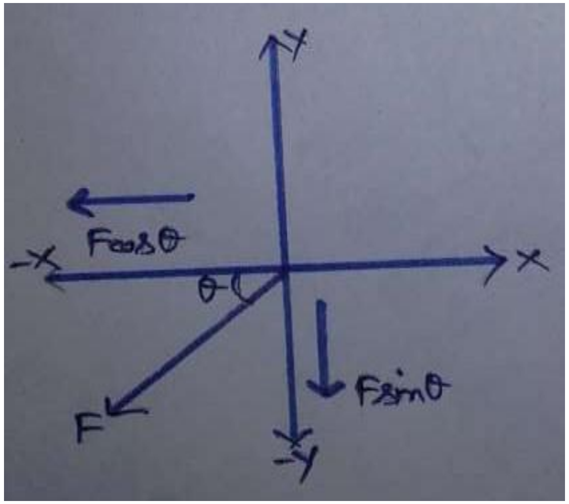
It states, “The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in the same direction.”



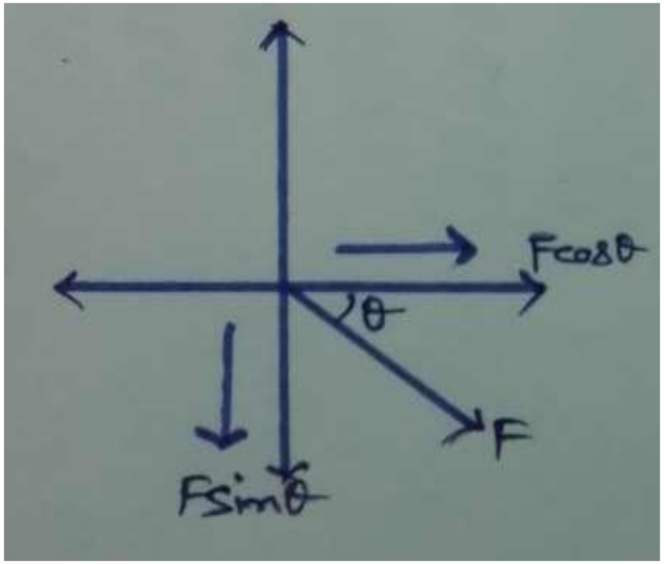

1st quadrant




2nd quadrant




3rd quadrant




4th quadrant

# METHOD OF RESOLUTION FOR THE RESULTANT FORCE.

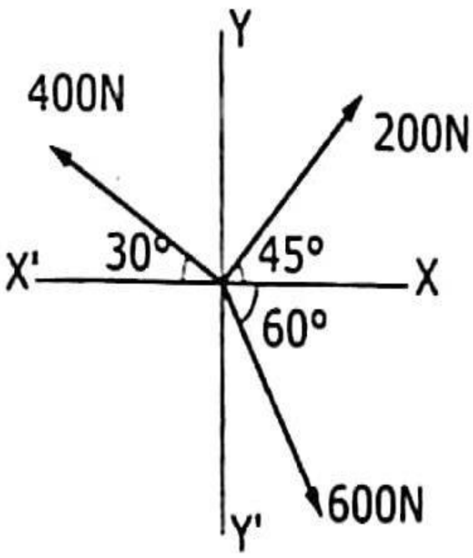
- Resolve all the forces horizontally and find the algebraic sum of the horizontal components.
- Resolve all the forces vertically and find the algebraic sum of all the vertical components.
- The resultant R of the given forces will be given by the equation :

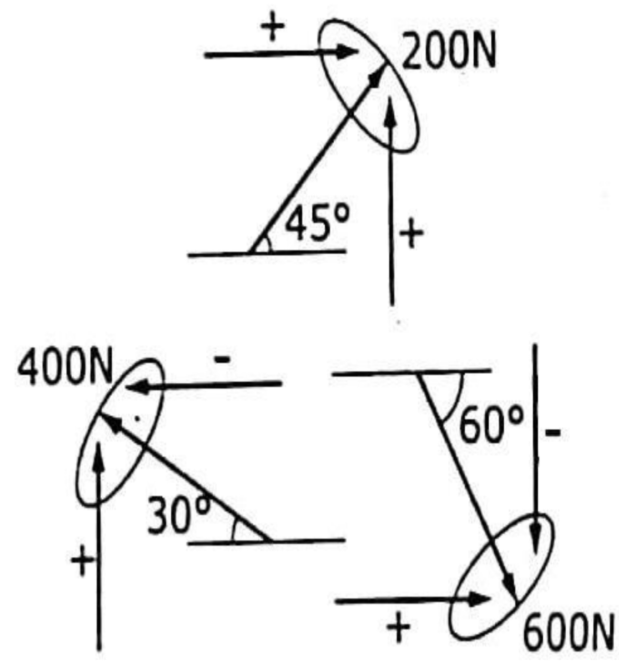
$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

- The resultant force will be inclined at an angle ,with the horizontal such that

$$\tan\theta = \frac{\sum V}{\sum H}$$

*Three coplanar concurrent forces are acting at a point as shown in fig 2.12. Determine the resultant in magnitude and direction.*



**Step 1 : Algebraic sum of Horizontal forces: (i.e., $\Sigma H$)**

Resolving the forces horizontally (i.e., along $XX'$ axis ) we get,

$$\Sigma H = 200\cos 45^\circ - 400\cos 30^\circ + 600\cos 60^\circ$$

$$= 141.42 - 346.41 + 300 = 95.01N$$

**Step 2 : Algebraic sum of vertical forces. (i.e. $\Sigma V$ )**

Resolving the forces vertically (i.e. along $YY'$ axis ) we get,

$$\Sigma V = 200\sin 45^\circ + 400\sin 30^\circ - 600\sin 60^\circ$$

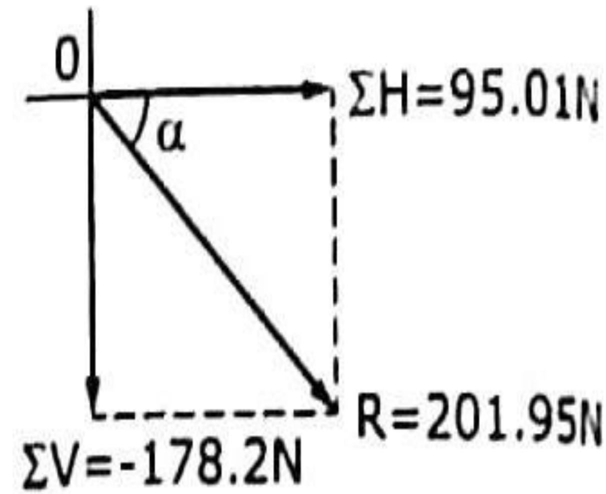
$$= 141.42 + 200 - 519.62 = -178.2N$$

**Step 3 : Magnitude of Resultant force.**

Magnitude of Resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$\therefore R = \sqrt{(95.01)^2 + (-178.2)^2}$$

$$R = 201.95N$$

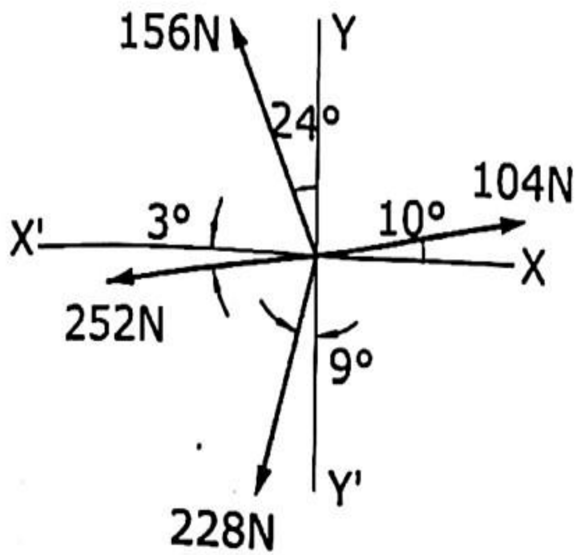



**Step 4 : Direction of Resultant force.**

then, $\tan\alpha = \dfrac{\Sigma V}{\Sigma H}$

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{178.2}{95.01}\right) = 61.93^\circ$$

*The four coplanar forces are acting at a point as shown in fig 2.14. (a). Determine the resultant in magnitude and direction.*




**Solution.**

Let $F_1 = 104N$; $F_2 = 156N$; $F_3 = 252N$ and $F_4 = 228N$

Angle of inclination of the forces $156N$ and $228N$ are given with reference to $y$ axis. Hence θ has to be found with $x$ axis.

Therefore,

$$\theta_1 = 10°$$

$$\theta_2 = (90 - 24) = 66°$$

$$\theta_3 = 3°$$

and $\theta_4 = (90 - 9) = 81°$

Resolving the forces horizontally, we get

$$\Sigma H = 104 \cos 10° - 156 \cos 66° - 252 \cos 3° - 228 \cos 81°$$

$$= 102.4 - 63.44 - 251.64 - 35.66$$

$$= -248.32 N$$

Resolving the forces vertically, we get.

$$\Sigma V = 104 \sin 10° + 156 \sin 66° - 252 \sin 3° - 228 \sin 81°$$

$$= 18.06 + 142.5 - 13.18 - 225.2$$

$$= -77.82$$

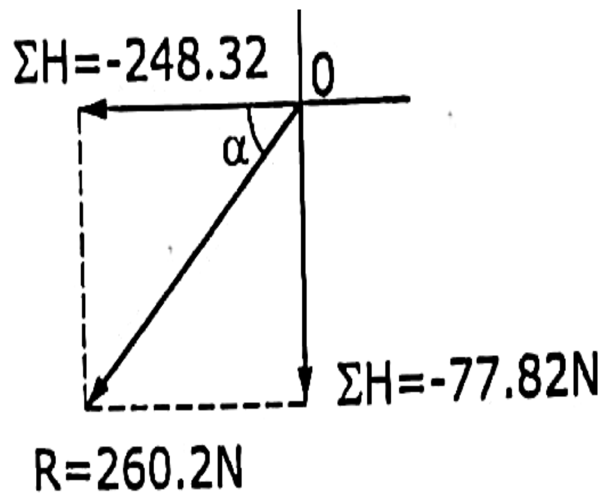
Magnitude of Resultant force. $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$\therefore R = \sqrt{(-248.32)^2 + (-77.82)^2} = 260.2 N$$

Direction of Resultant force,

$$\tan\alpha = \frac{\Sigma V}{\Sigma H}$$

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{77.82}{248.32}\right) = 17.4°$$



*The forces 10N, 20N, 30N and 40N are acting on one of the vertices of a regular pentagon, towards the other four vertices taken in order. Find the Magnitude and direction of the resultant force R.*

We know,

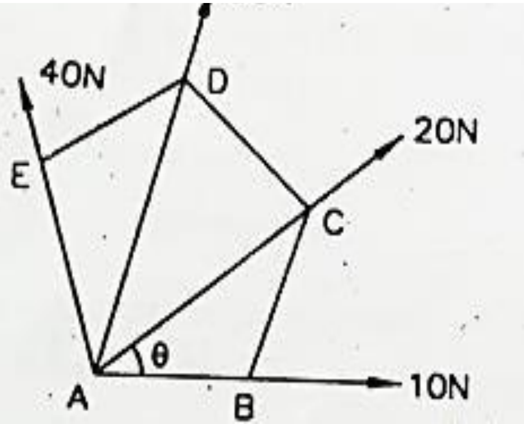
Sum of the interior angles of any regular polygon is $(2n - 4) \times 90°$, where $n$ = number of sides. So, for Regular Pentagon,

$$\text{Sum of interior angles} = (2 \times 5 - 4) \times 90 = 540°$$

$$\therefore \text{ Each included angle} = \frac{540}{5} = 108°$$

Joining the vertices $B$, $C$, $D$ and $E$ with the vertex $A$, we get 3 equal angles θ, as shown in fig 2.15. (a).

$$\therefore \text{ angle } \theta = \frac{108°}{3} = 36°$$



Hence the forces and their corresponding angles with $x$ axis are designated as below

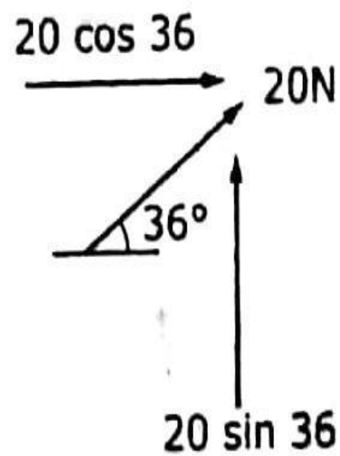
$F_1 = 10N; \quad \theta_1 = 0°$

$F_2 = 20N; \quad \theta_2 = 36°$

$F_3 = 30N; \quad \theta_3 = (2 \times 36) = 72°$

$F_4 = 40N; \quad \theta_4 = 180 - (3 \times 36) = 72°$

Resolved components of each force are given below:

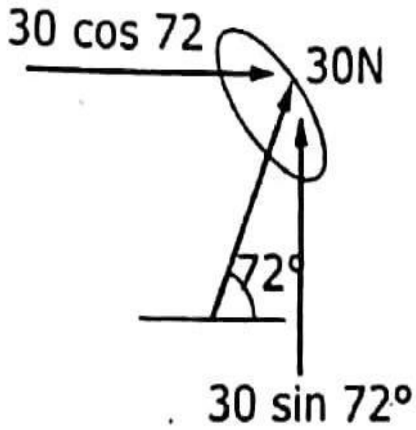



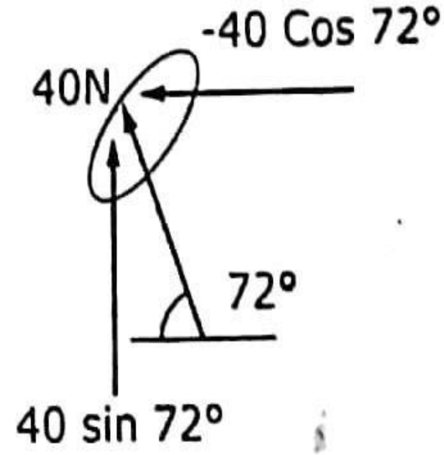

$F_h = 10 \cos 0° = 10$

$F_v = 10 \sin 0° = 0$

$F_h = 20 \cos 36°$

$F_v = 20 \sin 36°$




$F_h = 30\cos 72°$ $\quad$ $F_h = -40\cos 72°$

$F_v = 30\sin 72°$ $\quad$ $F_v = 40\sin 72°$

Resolving the forces horizontally,

$$\Sigma H = 10 + 20\cos 36° + 30\cos 72° - 40\cos 72°$$

$$= 10 + 16.18 + 9.27 - 12.36$$

$$= 23.09N$$

Resolving the forces vertically,

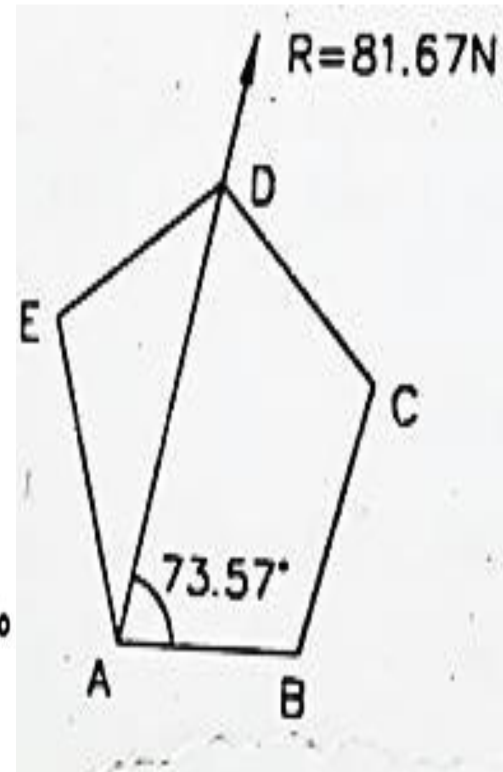
$$\Sigma V = 0 + 20\sin 36° + 30\sin 72° + 40\sin 72°$$

$$= 0 + 11.76 + 28.53 + 38.04$$

Magnitude of Resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$\therefore R = \sqrt{(23.09)^2 + (78.33)^2}$$

$$= 81.67N$$




Direction of Resultant force,

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{78.33}{23.09}\right) = 73.57^\circ$$

Thank You..