SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

COIMBATORE-35.

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai.

DEPARTMENT OF AUTOMOBILE ENGINEERING

COURSE NAME : 23MET101 – ENGINEERING MECHANICS

I YEAR / I SEMESTER

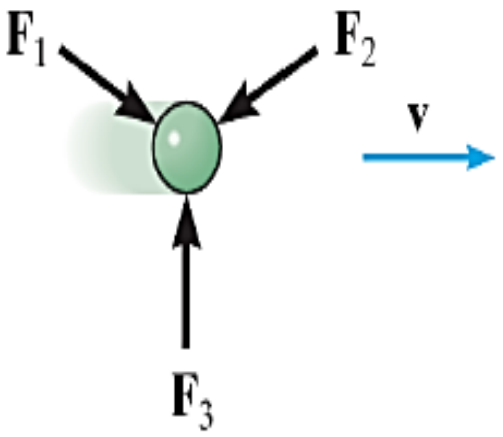
Topic – Law's of Mechanics, Lami's Theorem

Mechanics: Newton's Three Laws of Motion

Basis of formulation of rigid body mechanics.

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

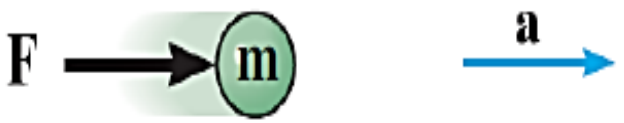
First law contains the principle of the equilibrium of forces → main topic of concern in Statics



Equilibrium

Mechanics: Newton's Three Laws of Motion

Second Law: A particle of mass "m" acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force.



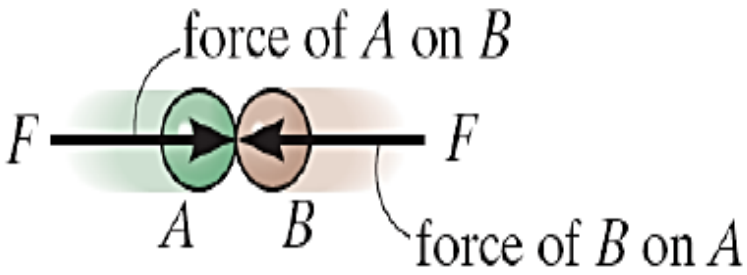
$$F = ma$$

Accelerated motion

Second Law forms the basis for most of the analysis in Dynamics

Mechanics: Newton's Three Laws of Motion

Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.

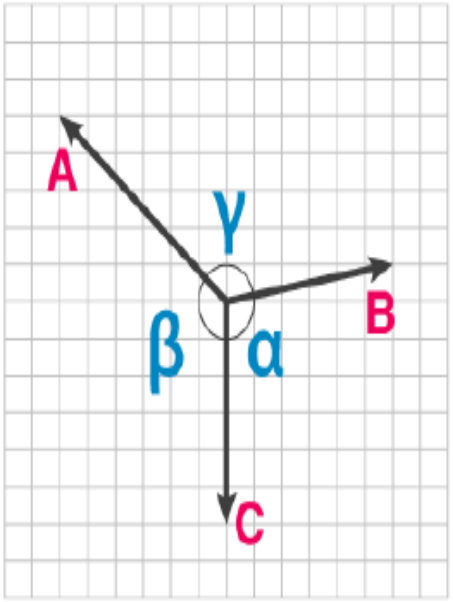


Action – reaction

Third law is basic to our understanding of Force → Forces always occur in pairs of equal and opposite forces.

Lami's Theorem

Lami's theorem relates the magnitudes of coplanar, concurrent and non-collinear forces that maintain an object in static equilibrium. The theorem is very useful in analyzing most of the mechanical as well as structural systems.



Lami's Theorem Statement

Lami's Theorem states, "When three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces". Referring to the above diagram, consider three forces A, B, C acting on a particle or rigid body making angles α, β and γ with each other.

In the mathematical or equation form, it is expressed as,

$$\frac{A}{sin\alpha} = \frac{B}{sin\beta} = \frac{C}{sin\gamma}$$

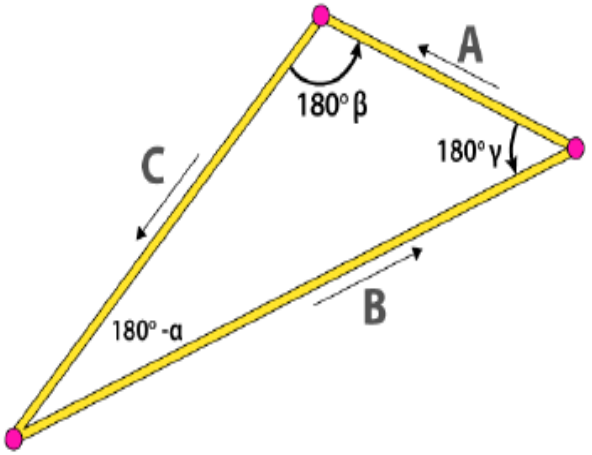
Lami's Theorem Derivation

Now, let's see how the theorem's equation is derived.

Let F_A, F_B, and F_C be the forces acting at a point. As per the statement of the theorem, we take the sum of all forces acting at a given point which will be zero.

i.e. $F_A + F_B + F_C = 0$

The angles made by force vectors when a triangle is drawn are,



We write angles in terms of complementary angles and use triangle law of vector addition. Then, by applying the sine rule we get,

$$\frac{A}{sin(180-\alpha)} = \frac{B}{sin(180-\beta)} = \frac{C}{sin(180-\gamma)}$$

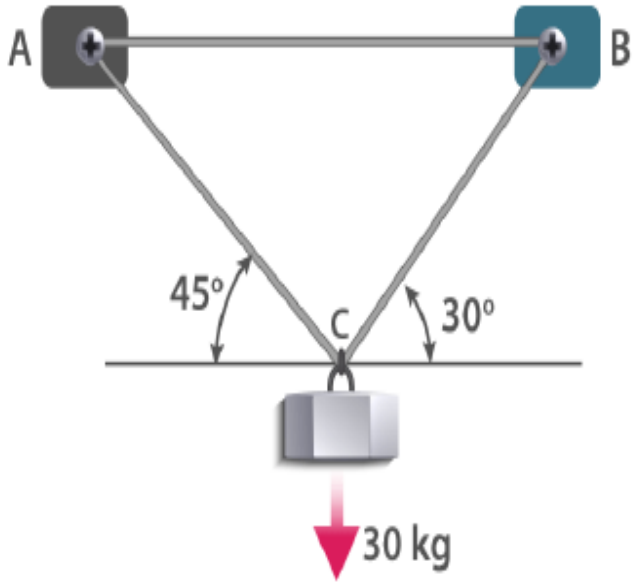
So, we have,

$$\frac{A}{sin\alpha} = \frac{B}{sin\beta} = \frac{C}{sin\gamma}$$

Hence, it is clearly seen that by applying sine rule to complementary angles we arrive at the required result for Lami's theorem.

Now, we will see how Lami's theorem is useful to determine the magnitude of unknown forces for the given system.

An iron block of mass 30 kg is hanging from the two supports A and B as shown in the diagram. Determine the tensions in both the ropes.

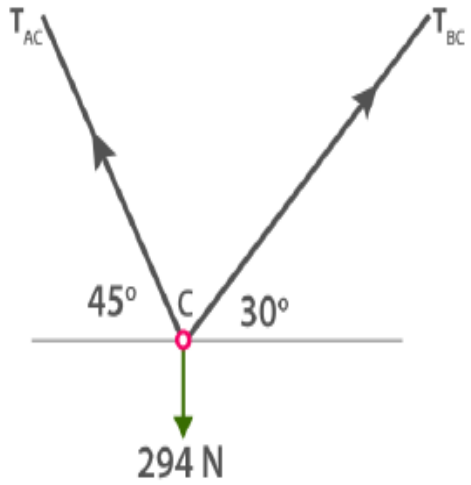


Solution:

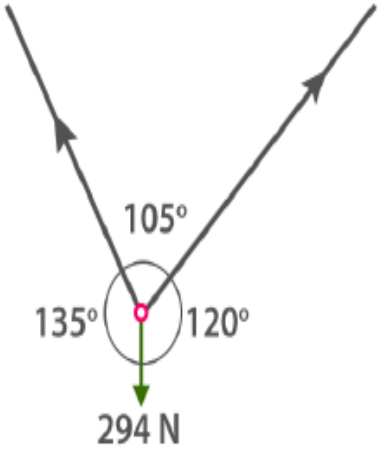
Given,

m = 30 kg, W = mg = 30 x 9.8 = 294 N

Let's draw an FBD i.e. Free Body Diagram for the given condition. C be the point of suspension from where the iron block is hanging.



We get required angles to apply Lami's Theorem as,



Using Lami's Formula we get,

$$\frac{T_{AC}}{sin(120^o)} = \frac{T_{BC}}{sin(135^o)} = \frac{294}{sin(105^o)}$$

Therefore,

$$\frac{T_{AC}}{sin(120^o)} = \frac{294}{sin(105^o)}$$

$$\text{i.e. } T_{AC} = \frac{294\ X\ sin(120^o)}{sin(105^o)} = \frac{294\ X\ 0.866}{0.966} = 263.566N$$

Similarly,

$$\frac{T_{BC}}{sin(120^o)} = \frac{294}{sin(105^o)}$$

$$\text{i.e. } T_{BC} = \frac{294\ X\ sin(135^o)}{sin(105^o)} = \frac{294\ X\ 0.707}{0.966} = 215.2N$$

So, the required tensions along the shown directions are 263.566 N and 215.2 N respectively.

Thank You..