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Unit 3-Differential Calculus

Evolutes

Evolutes :

The lows of the centre of worvature of the grn. wrive is called the evolute of the curre.

Method of floding Evolute:

- 1. We fite the parametric form of the $gv_{\underline{n}}$. $eq_{\underline{n}}$. $x = f(\underline{t})$; $y = g(\underline{t})$
- 2. Find the centre of worvariale c(x, y)
- 3. Elemente the parameter + & would the the eqn. noteins of x & y only.
- 4. Replace \overline{x} by $x \cdot 8 \overline{y}$ by $y \cdot 9n$ the obtained eqn., the evolute of the grn. curve.

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Evolutes

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Unit 3-Differential Calculus

J. Find the evolute of the parabola
$$y^2 = 4ax$$

Solow.
Take $x = at^2$ and $y = 2at$
 $\frac{dx}{dt} = 2at$
 $\frac{dy}{dt} = 2a$
Now $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dx}{dx} = 2a \cdot \frac{1}{2at}$
 $= \frac{1}{2}$
 $\frac{d^2y}{dx^2} = \frac{d}{dt} [\frac{dy}{dx}] \frac{dt}{dx} = \frac{d}{dt} [\frac{1}{2}] \frac{1}{2at}$
 $= -\frac{1}{2^2} \times \frac{1}{2at}$
 $\overline{x} = x - \frac{y}{y} \frac{[1+y]^2]}{\frac{y}{2}} = x - \frac{\frac{1}{2} [\frac{1+\frac{1}{2}}{2a}]}{\frac{1}{2a^2}^3}$
 $= x + \frac{1}{2} \int \frac{\frac{1}{2} \frac{x}{2at}}{\frac{1}{2a^2}^3}$
 $\overline{x} = x - \frac{y}{\frac{y}{2}} \frac{[1+y]^2}{\frac{y}{2}} = x - \frac{\frac{1}{2} [\frac{1+\frac{1}{2}}{2a}]}{\frac{1}{2a^2}^3}$
 $\overline{x} = x + 2a(\frac{x^2+1}{2a})$
 $\overline{y} = y + \frac{1+\frac{y}{2}}{\frac{y}{2}}$
 $= y - (\frac{x^2+1}{2a}) \times 2a^2^3$
 $\overline{x} = x + 2a(\frac{x^2+1}{2a})$
 $\overline{x} = 4x + \frac{1}{2}a(\frac{x^2+1}{2a})$
 $\overline{x} = 4x^2 + 2a(\frac{x^2+1}{2a}) = a^2 + 2a^2 + 2a$

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and
$$\tilde{y} = \Re \alpha t - \Re \alpha t \int t^{\varphi} + i \tilde{J}$$

 $= \Re \alpha t - \Re \alpha t \tilde{J}^{2} - \Re \alpha t$
 $\tilde{y} = -\Re \alpha t^{3}$
Here $\tilde{x} = 3\alpha t^{2} + \alpha \alpha \Rightarrow \tilde{x} - \alpha \alpha = 3\alpha t^{3}$
Taking cube on bethereds. $[\tilde{x} - \alpha \alpha \tilde{J}^{2}] = (3\alpha t^{3})^{3}$
 $(\tilde{x} - \alpha \alpha \tilde{J}^{2}] = t^{2} - \beta \alpha t^{3}$
Taking square on both states.
 $\tilde{y}_{t}^{R} = 4\alpha^{2} t^{4} \delta \Rightarrow \frac{y^{2}}{4\alpha^{2}} = t^{6} \longrightarrow (2)$
Fieron (1) and (2),
 $(\frac{\tilde{x} - \alpha \alpha}{2\pi \alpha^{3}}) = \frac{y^{2}}{4\alpha^{2}}$
 $4(\tilde{x} - \alpha \alpha)^{3} = \pi \alpha \tilde{y}^{2}$
Replace \tilde{x} by $x \ll \tilde{y}$ by y , we get $4(x - 2\alpha)^{3} = 2\pi \alpha y^{3}$.
 \tilde{x} the evolute $\hat{y} = 4(x - 2\alpha)^{3} = 2\pi \alpha y^{3}$.
 \tilde{x} the evolute $\hat{y} = 4(x - 2\alpha)^{3} = 2\pi \alpha y^{3}$.
 \tilde{x} the evolute $\hat{y} = 4(x - 2\alpha)^{3} = 2\pi \alpha y^{3}$.
 \tilde{x} the $x = \alpha \cos \theta$ $y = b \sin \theta$
 $\frac{dy}{d\theta} = -\alpha \sin \theta$ $\frac{dy}{d\theta} = b \cos \theta$
 $No(\infty), \frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$
 $= b \cos \beta\theta \cdot \frac{1}{-\alpha} \sin \theta$
 $\frac{dy}{dx} = -\frac{b}{\alpha} \cos \theta$



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$$\begin{aligned} \frac{d^2 y}{dx^2} &= \frac{d}{de} \begin{bmatrix} dy \\ dx \end{bmatrix} \frac{de}{dx} \\ &= \frac{d}{de} \begin{bmatrix} -\frac{b}{a} (\cot \theta) \end{bmatrix} \frac{1}{-a, \sin \theta} \\ &= -\frac{b}{a^2} (\sin^2 \theta) (\sin \theta) \end{bmatrix} \\ &= -\frac{b}{a^2} (\sin^2 \theta) (\sin^2 \theta) \end{bmatrix} \\ &= -\frac{b}{a^2} (\sin^2 \theta) (\sin^2 \theta) \end{bmatrix} \\ &= \frac{a}{a^2} (\sin^2 \theta) = -\frac{b}{a^2} (\sin^2 \theta) \end{bmatrix} \\ &= \frac{a}{a^2} (\sin^2 \theta) \begin{bmatrix} 1 + \frac{b^2}{a^2} (\sin^2 \theta) \end{bmatrix} \\ &= \frac{a}{a} (\cos^2 \theta) \end{bmatrix} \\ &= \frac{a}{a} (\cos^2 \theta) + \frac{b}{a} (\sin^2 \theta) \begin{bmatrix} 1 + \frac{b^2}{a^2} (\sin^2 \theta) \end{bmatrix} \\ &= \frac{a}{a} (\cos^2 \theta) - \frac{a}{a} (\cos^2 \theta) \begin{bmatrix} \alpha^2 + b^2 (\sin^2 \theta) \end{bmatrix} \\ &= \frac{a}{a} (\cos^2 \theta) - \frac{1}{a} (\cos^2 \theta) \sin^2 \theta \begin{bmatrix} \alpha^3 \sin^2 \theta + b^2 (\sin^2 \theta) \end{bmatrix} \\ &= a (\cos^2 \theta) - \frac{1}{a} (\cos^2 \theta) \sin^2 \theta \begin{bmatrix} \alpha^3 \sin^2 \theta + b^2 (\sin^2 \theta) \end{bmatrix} \\ &= a (\cos^2 \theta) - \frac{1}{a} (\cos^2 \theta) \sin^2 \theta + b^2 (\sin^2 \theta) \end{bmatrix} \\ &= a (\cos^2 \theta) - \frac{1}{a} (\cos^2 \theta) \sin^2 \theta + b^2 (\sin^2 \theta) \end{bmatrix} \\ &= a (\cos^2 \theta) - \frac{1}{a} (\cos^2 \theta) \sin^2 \theta + b^2 (\sin^2 \theta) \end{bmatrix} \\ &= a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) = a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) \\ &= a (\cos^2 \theta) (\sin^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) = a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) \\ &= a (\cos^2 \theta) (\sin^2 \theta) - \frac{b^2}{a} (\sin^2 \theta) = a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) \\ &= a (\cos^2 \theta) (\sin^2 \theta) - \frac{b^2}{a} (\sin^2 \theta) = a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) \\ &= a (\cos^2 \theta) (\sin^2 \theta) - \frac{b^2}{a} (\sin^2 \theta) = a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) \\ &= a (\sin^2 \theta) [1 - \sin^2 \theta] - \frac{b^2}{a} (\sin^2 \theta) = a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) \\ &= a (\cos^2 \theta) (\sin^2 \theta) - \frac{b^2}{a} (\sin^2 \theta) = a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) \\ &= a (\sin^2 \theta) [1 - \sin^2 \theta] - \frac{b^2}{a} (\sin^2 \theta) = a (\cos^2 \theta) - \frac{b^2}{a} (\cos^2 \theta) \\ &= a (\cos^2 \theta) (\sin^2 \theta) - \frac{b^2}{a} (\sin^2 \theta) \\ &= b \sin^2 \theta + \frac{[1 + \frac{b^2}{a^2} (\sin^2 \theta)] \\ &= b \sin^2 \theta + \frac{[1 + \frac{b^2}{a^2} (\sin^2 \theta)] \\ &= \frac{b}{a^2} (\sin^2 \theta) + \frac{[1 + \frac{b^2}{a^2} (\sin^2 \theta)] \\ &= \frac{b}{a^2} (\sin^2 \theta) + \frac{b^2}{a^2} (\sin^2 \theta) \end{bmatrix}$$

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$$= b S f D B - \frac{d^{2} + \frac{b^{2}}{a^{2}} \frac{\cos^{2} \theta}{S f D^{2} \theta}}{\frac{b}{a^{2}} \cos^{2} \theta}$$

$$= b S f D B - \frac{a^{2} S D^{2} \theta + b^{2} \cos^{2} \theta}{(b^{2} S D^{2} \theta)} \times \frac{a^{2}}{b} \frac{S D^{2} \theta}{S^{2} D^{2}} B$$

$$= b S f D B - \frac{a^{2} \delta f D^{3} \theta}{(b^{2} S D^{2} \theta)} - b \cos^{2} \theta \delta f D B$$

$$= b S f D B - \frac{a^{2} \delta S D^{3} \theta}{(b^{2} S D^{2} \theta)} - b \cos^{2} \theta \delta f D B$$

$$= b S f D B - \frac{a^{2} \delta S D^{3} \theta}{b} - b \cos^{2} \theta \delta f D B$$

$$= b S f D B - \frac{a^{2} \delta S D^{3} \theta}{b} - \frac{a^{2} \delta S D^{3} \theta}{b} = b S f D^{3} \theta - \frac{a^{2} \delta S D^{3}}{b} G$$

$$= b S f D B - \frac{a^{2} \delta S D^{3} \theta}{b} - \frac{a^{2} \delta S D^{3} \theta}{b} = b S f D^{3} \theta - \frac{a^{2} \delta S D^{3}}{b} G$$

$$= b S f D B - \frac{a^{2} \delta S D^{3} \theta}{b} - \frac{a^{2} \delta S D^{3} \theta}{b} = b S f D^{3} \theta - \frac{a^{2} \delta S D^{3}}{b} G$$

$$= \left[b - \frac{a^{2}}{b} \right] S f D^{3} \theta$$

$$= \left[\frac{b^{2} - a^{2}}{b} \right] S f D^{3} \theta$$

$$\Rightarrow S f D^{3} \theta = \frac{b U}{b^{2} - a^{2}} \right] S f D^{3} \theta$$

$$\Rightarrow S f D^{3} \theta = \frac{b U}{b^{2} - a^{2}} \right]$$

$$= b S f D S \theta = \left[\frac{a \overline{x}}{a^{2} - b^{3}} \right]^{\frac{3}{3}} \Rightarrow S f D^{3} \theta = \left[\frac{b \overline{y}}{b^{2} - a^{2}} \right]^{\frac{3}{3}} \right]$$

$$= b S f D S \theta = \left[\frac{a \overline{x}}{a^{2} - b^{2}} \right]^{\frac{3}{3}} = 1 \Rightarrow (a \overline{x})^{\frac{2}{3}} + (b \overline{y})^{\frac{2}{3}} = (a^{2} - b^{2})^{\frac{2}{3}} \right]$$

$$= b S f D S \theta = \frac{a^{2} \delta D^{2}}{b^{2} - a^{2}} = 1 \Rightarrow (a \overline{x})^{\frac{2}{3}} + (b \overline{y})^{\frac{2}{3}} = (a^{2} - b^{2})^{\frac{2}{3}}$$

$$= b S f D S \theta = \frac{a^{2} \delta D^{2}}{b^{2} - a^{2}} = 1 \Rightarrow (a \overline{x})^{\frac{2}{3}} + (b \overline{y})^{\frac{2}{3}} = (a^{2} - b^{2})^{\frac{2}{3}}$$

$$= b S f D S \theta = \frac{a^{2} \delta D^{2}}{(a^{2} - b^{2})^{\frac{2}{3}}} = 1 \Rightarrow (a \overline{x})^{\frac{2}{3}} + (b \overline{y})^{\frac{2}{3}} = (a^{2} - b^{2})^{\frac{2}{3}}$$

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